El:m 44 ~~A56 A70~~

NATURAL COSECANT. SUBTRACT.

0	1	2	3	4	5	6	7	8	9	6′ ·1	12′ ·2	18′ ·3	24′ ·4	30′ ·5	36′ ·6	42′ ·7	48′ ·8	54′ ·9
+∞	57·3	28·7	19·1	14·3	11·5	9·57	8·21	7·19	6·39	–	–	–	–	–	–	–	–	–
5·76	5·24	4·81	4·45	4·13	3·86					–	–	–	–	–	–	–	–	–
						3·63	3·42	3·24	3·07	2	3	5	7	9	11	12	14	16
2·924	790	669	559	459						10	21	32	43	54	65	77	88	100
					366	281	203	130	063	7	14	21	29	36	43	51	58	66
2·000	1·942	1·887	1·836	1·788						5	10	15	20	25	30	36	41	46
					1·743	1·701	1·662	1·624	1·589	4	7	11	15	18	22	26	30	34
1·556	524	494	466	440	414	390	367	346	325	2	5	7	9	12	14	17	19	22
1·305	287	269	252	236	221	206	192	179	167	1	3	4	6	7	9	10	12	14
1·155	143	133	122	113	103	095	086	079	071	1	2	3	4	4	5	6	7	8
1·064	058	051	046	040	035	031	026	022	019	0	1	1	2	2	3	3	4	4
1·015	012	010	008	006	004	002	001	001	000	–	–	–	–	–	–	–	–	–

NATURAL SECANT. ADD.

0	1	2	3	4	5	6	7	8	9	6′ ·1	12′ ·2	18′ ·3	24′ ·4	30′ ·5	36′ ·6	42′ ·7	48′ ·8	54′ ·9
1·000	000	001	001	002	004	006	008	010	012	–	–	–		–	—	–	–	–
1·015	019	022	026	031	035	040	046	051	058	0	1	1	2	2	3	3	4	4
1·064	071	079	086	095	103	113	122	133	143	1	2	3	4	4	5	6	7	8
1·155	167	179	192	206	221	236	252	269	287	1	3	4	6	7	9	10	12	14
1·305	325	346	367	390	414	440	466	494	524	2	5	7	9	12	14	17	19	22
1·556	589	624	662	701						4	7	11	15	18	22	26	30	34
					743	788	836	887	942	5	10	15	20	25	30	36	41	46
2·000	063	130	203	281						7	14	21	29	36	43	51	58	66
					366	459	559	669	790	10	21	32	43	54	65	77	88	100
2·92	3·07	3·24	3·42	3·63						2	3	5	7	9	11	12	14	16
					3·86	4·13	4·45	4·81	5·24	–	–	–	–	–	–	–	–	–
5·76	6·39	7·19	8·21	9·57	11·5	14·3	19·1	28·7	57·3	–	–	–	–	–	–	–	–	–

NATURAL COTANGENT. SUBTRACT.

0	1	2	3	4	5	6	7	8	9	6′ ·1	12′ ·2	18′ ·3	24′ ·4	30′ ·5	36′ ·6	42′ ·7	48′ ·8	54′ ·9
+∞	57·3	28·6	19·1	14·3	11·4	9·51	8·14	7·12	6·31	–	–	–	–	–	–	–	–	–
5·67	5·14	4·70	4·33	4·01	3·73					–	–	–	–	–	–	–	–	–
						3·49	3·27	3·08	2·90	2	4	6	8	10	12	14	16	18
2·75	2·61	2·48	2·36	2·25	2·14	2·05	1·96	1·88	1·80	1	2	3	4	5	6	7	8	9
1·732	664	600	540	483						6	12	18	24	30	36	42	48	54
					428	376	327	280	235	5	9	14	19	23	28	33	38	42
1·192	1·150	1·111	1·072	1·036						4	7	11	15	19	23	27	30	34
					1·000	·966	·933	·900	·869	3	6	9	13	16	19	22	26	29
·839	810	781	754	727	700	675	649	625	601	3	5	8	11	13	16	18	21	24
·577	554	532	510	488	466	445	424	404	384	2	4	6	9	11	13	15	17	19
·364	344	325	306	287	268	249	231	213	194	2	4	6	8	9	11	13	15	17
·176	158	141	123	105	087	070	052	035	017	2	4	5	7	9	11	12	14	16

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PREFACE

THE object of this book is to embody the improvements which have been effected in the teaching of Arithmetic in recent years. Public discussions and the investigations of committees since the War have thrown much light on the relative values of different methods and on the grouping and co-ordinating of various subjects. We have taken advantage of these enquiries in evolving the plan and character of the present volume.

It has been assumed that the earlier part of the subject has been already learned, and here we have confined ourselves to indicating the best methods to adopt in simple computation and to the provision of examples for purposes of revision.

We have endeavoured to introduce the pupil to the quickest method of solving any given type of example when he first meets with it, and to avoid a treatment which, though perhaps more easily grasped, will afterwards be discarded. This will explain the early introduction of the methods of Practice and Ratio. Many types of questions such as reduction and profit and loss, which are usually relegated to the section in which they are specifically treated, will be found at an earlier stage among examples of which they are obvious illustrations.

Examples in Arithmetic serve two different purposes. They may afford practice in rapid computation or they may be of a problem type and so call for a certain amount of ingenuity in their solution. This distinction has been recognised in the two classes of papers throughout the book, Revision Papers and Problem Papers.

There are two points which we think should be impressed upon all pupils:

1. That simple subsidiary calculations should be done mentally, *e.g.* the number of shillings in a £ should be written down at once without writing 20 and drawing a line.

2. That calculations should be carried out in terms of the highest convenient unit, thus reducing the number of figures involved, *e.g.* work in £ rather than in pence, in feet rather than in inches.

Part I deals mainly with whole numbers. Part II opens with decimals and fractions, and includes proportion, ratio and percentage, and harder examples on Areas and Volumes.

Part III includes an introduction to Trigonometry as an illustration of the ratio principle, but mainly deals with logarithms and mensuration. Practical work in the latter is highly desirable. It makes the pupil realise the limitations in the accuracy of his results due to imperfections of measurement, and an inspection of the object measured will, in time, check the tendency to produce answers which are obviously absurd.

In Part IV the financial topics are of a practical character (*e.g.* Chapter XXIII on Shares and Stocks), and an opportunity has been provided for acquiring facility in dealing with questions on percentage. Our thanks are due to Mr H. C. Beaven of Clifton College for permission to use some of his original problems. The Tables of Logarithms, Reciprocals, and Square Roots are taken from Mr Castle's Tables by kind permission of Messrs Macmillan & Co., but the Square Root Tables are arranged in a new order. The 3-figure Trigonometrical Tables (Part III) are taken from Mr Durell's *School Mechanics*.

C. V. D.
R. C. F.

CONTENTS

PAGE
Tables and Constants xiii

PART I

CHAP.

I. Addition and Subtraction

Historical Note 1
Addition 2
Subtraction 4
Negative Numbers 7
Brackets 8

II. Multiplication and Division

Multiplication 9
Division 13
Short Methods 16
Miscellaneous Examples M (I) 18

III. Compound Quantities and Averages

Reduction 22
Addition and Subtraction 28
Multiplication 32
Division 34
Averages 36

IV. Fractional Parts and Practice

Fractional Parts 39
Simple Practice 41
Compound Practice 43
Invoices 45
Miscellaneous Examples M (II) 46

CHAP. PAGE
V. **Areas and Volumes** (Whole Numbers)
Areas 49
Volumes 52
Revision Papers (*R* 1–10) 55
Problem Papers (*P* 1–10) 59

PART II

VI. **Decimals**
Notation 65
Significant Figures 67
Metric System 69
Addition and Subtraction 72
Multiplication 74
Division 76
British and Metric Units 79
Decimalisation of Money, etc. 80

VII. **Prime Factors: H.C.F. and L.C.M.**
Prime Numbers 85
Indices 86
H.C.F. and L.C.M. 88
Tests of Divisibility 93

VIII. **Fractions**
Addition 95
Subtraction 96
Multiplication 98
Division 100
Miscellaneous Examples M (*III*) 103
Revision Papers (*R* 11–20) 106
Problem Papers (*P* 11–20) 110

IX. **Unitary Method**
Direct Proportion 114
Inverse Proportion 117

CHAP. PAGE
X. **Ratio and Percentage**
Ratio 119
Percentage 129

XI. **Areas** 131

XII. **Volumes** 140
Miscellaneous Examples M (IV) 145
Revision Papers (*R* 21–30) 148
Problem Papers (*P* 21–30) 152

PART III

XIII. **Graphs and Direct Variation**
Graphs 157
Direct Variation 162

XIV. **Harder Fractions** 166

XV. **Significant Figures and Approximations**
Degree of Accuracy 172
Absolute and Relative Errors 173
Multiplication and Division, approximate data . . 177
Formal Contracted Methods 186

XVI. **Squares and Square Roots**
Squares 189
Rule for Square Root 190
Use of Tables : Squares, Square Roots, Reciprocals . 193
Pythagoras' Theorem 196
Miscellaneous Examples M (V) 199
Revision Papers (*R* 31–40) 202
Problem Papers (*P* 31–40) 207

XVII. **Indices and Logarithms**
Positive Integral and Fractional Indices . . . 212
Logarithms (Positive Characteristics) 215
Negative Indices. 218
Logarithms (Negative Characteristics) 219

CHAP. PAGE

XVIII. **Mensuration**

Parallelogram, Triangle, Trapezium, Prism, Pyramid . 225
Circle 231
Cylinder, Cone, Sphere 235

XIX. **Density and Specific Gravity**

Definitions and Table of Values 240
Principle of Archimedes 242

Revision Papers (*R* 41–50) 247
Problem Papers (*P* 41–50) 252

XX. **Trigonometrical Ratios**

Tangent of an Angle 257
Sine and Cosine 261
Cosecant, Secant, and Cotangent 266

Miscellaneous Examples M (*VI*) 269

PART IV

XXI. **Interest and Discount**

Simple Interest 273
Compound Interest by Computation 279
Compound Interest by Tables or Logarithms . . 282
Compound Interest Law 284
Cash Discount, Banker's Discount, True Discount . . 286

XXII. **Harder Percentages, Profit and Loss**

Miscellaneous Applications of Percentage . . . 289
Profit and Loss per cent. 294
Harder Problems on Percentage 298

XXIII. **Shares and Stocks**

Meaning of Terms : Procedure 302
Shares 302
Stocks 309

Miscellaneous Examples M (*VII*). 313

CHAP. PAGE

XXIV. **Further Variation**

Inverse Variation 317
Other Types of Variation 319
Determination of Laws ($y=ax+b$, $y=ax^n$) . . . 325

XXV. **Miscellaneous Problems**

Relative Velocity, Rate of Working, Mixtures, Etc. . 328
Miscellaneous Examples M (VIII) 337
Revision Papers (R 51–60) 344
Problem Papers (P 51–60) 348
Exercises in Rapid Computation (A–J) 354
Tables 359
Answers . . . . *at end*

TABLES

MONEY

English

4 farthings = 1 penny (d.).
12 pence = 1 shilling (s.).
20 shillings = 1 pound (£) or sovereign.

A florin = 2s.
A half-crown = 2s. 6d.
A crown = 5s.
A guinea = £1, 1s.

French

100 centimes (c.) = 1 franc (fr.).

American

100 cents = 1 dollar ($).

Since the War, the exchange value of foreign money has varied a great deal. The rates of exchange are quoted daily in the newspapers.

LENGTH

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
22 yards = 1 chain (ch.).
10 chains = 1 furlong (fur.).
8 furlongs = 1 mile (m.).
3 miles = 1 league.

5½ yards = 1 pole (p.), rod, or perch.
4 poles = 1 chain.
40 poles = 1 furlong.

220 yards = 1 furlong.
1760 yards = 1 mile.

100 links = 1 chain.

6 feet = 1 fathom.
100 fathoms = 1 cable.
69 miles = 1 degree (approx.).

6080 feet = 1 nautical mile (Admiralty).
1 nautical mile per hour = a speed of 1 knot.

4 inches = 1 hand.

AREA

144 (=12^2) sq. inches (sq. in.) = 1 sq. foot (sq. ft.).
9 (=3^2) sq. feet = 1 sq. yard (sq. yd.).
484 (=22^2) sq. yards = 1 sq. chain.
10 sq. chains = 1 acre (ac.).
640 acres = 1 sq. mile (sq. m.).

4840 sq. yd. = 1 ac.
$30\frac{1}{4}$ (=$5\frac{1}{2}^2$) sq. yds. = 1 sq. pole.
40 sq. p. = 1 rood (r.).
4 roods = 1 acre.

Land-agents use pole or perch for sq. pole.

VOLUME

1728 (=12^3) cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 (=3^3) cubic feet = 1 cubic yard (cu. yd.).

CAPACITY

Liquid Measure

4 gills = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

Dry Measure

2 pints = 1 quart.
4 quarts = 1 gallon.
2 gallons = 1 peck (pk.).
4 pecks = 1 bushel (bush.).
8 bushels = 1 quarter (qr.).

Apothecaries' Measure

60 minims = 1 drachm.
8 drachms = 1 fluid ounce.
20 fluid ounces = 1 pint.

1 gallon = 277·4 cu. inches approx.
= ·1605 cu. foot approx.
1 cu. foot = 6·23 gallons approx.

WEIGHT

Avoirdupois

16 drams (dr.). = 1 ounce (oz.).
16 ounces = 1 pound (lb.).
28 pounds = 1 quarter (qr.).
4 quarters = 1 hundredweight (cwt.).
20 hundredweights = 1 ton (t.).

7000 grains (gr.) = 1 pound.

14 pounds = 1 stone.
2 stone = 1 quarter.

112 pounds = 1 hundredweight.
2240 pounds = 1 ton.

100 pounds = 1 cental or quintal.

1 gallon of water weighs 10 lb.
" 1 pint of water weighs a pound and a quarter."
1 cu. foot of water weighs 62·3 lb. approx., or 1000 ounces approx.

Troy (for precious metals)

24 grains (gr.) = 1 pennyweight dwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound (lb.).

Apothecaries' Weights

20 grains = 1 scruple (scr.).
3 scruples = 1 drachm (dr.).
8 drachms = 1 ounce.
12 ounces = 1 pound.

1 grain (Avoirdupois) = 1 grain (Troy) = 1 grain (Apothecaries').
1 ounce (Troy) = 1 ounce (Apothecaries').

1 ounce (Avoirdupois) = 437½ grains.
1 ounce (Troy) = 480 grains.

1 pound (Avoirdupois) = 7000 grains.
1 pound (Troy) = 5760 grains.

TIME

60 seconds (sec.). = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (d.).
7 days = 1 week (wk.).
365 days = 1 ordinary year (yr.).
366 days = 1 leap year.
100 years = 1 century.

ANGLE

60 seconds (60″) = 1 minute (1′).
60 minutes = 1 degree (1°).
90 degrees = 1 right angle.

1 radian = 57·3° approx.
π radians = 180 degrees.

TABLES

NUMBER

12 units = 1 dozen (doz.).
12 dozen = 1 gross.

20 units = 1 score.

PAPER MEASURE

24 sheets = 1 quire.
20 quires = 1 ream.

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METRIC SYSTEM

LENGTH

10 millimetres (mm.) = 1 centimetre (cm.).
10 centimetres = 1 decimetre (dm.).
10 decimetres = 1 metre (m.).
10 metres = 1 dekametre (Dm.).
10 dekametres = 1 hectometre (Hm.).
10 hectometres = 1 kilometre (Km.).

1 metre = 39·37 inches approx.
1 kilometre = ·6214 mile approx.
1 inch = 2·54 cm. approx.
1 yard = ·9144 m. approx.
1 mile = 1·609 Km. approx.

1 kilometre is often taken as = $\frac{5}{8}$ mile.

The tables for AREAS and VOLUMES are derived from the table for Lengths, e.g.

100 (= 10^2) sq. mm. = 1 sq. cm., etc.
and
1000 (= 10^3) cu. mm. = 1 cu. cm., etc.

1 sq. cm. = ·155 sq. in. approx.
1 sq. in. = 6·45 sq. cm. approx.

1 cu. cm. = ·061 cu. in. approx.
1 cu. in. = 16·39 cu. cm. approx.

1 sq. decametre = 1 are (a.).

100 centiares (ca.) = 1 are.
100 ares = 1 hectare (Ha.).

1 are = 119·6 sq. yd. approx.
1 hectare = 2·47 acres approx.

1 hectare is often taken as $2\frac{1}{2}$ acres.

CAPACITY

10 centilitres (cl.) = 1 decilitre (dl.).
10 decilitres = 1 litre (l.).
10 litres = 1 decalitre (Dl.).
10 decalitres = 1 hectolitre (Hl.)
10 hectolitres = 1 kilolitre (Kl.).

1 litre = 1 cubic decimetre.

1 litre = 1·76 pints approx.
1 gallon = 4·546 litres approx.
1 litre = 61·03 cu. inches approx.

1 litre is often taken as $1\frac{3}{4}$ pints.

WEIGHT

10 milligrams (mg.) = 1 centigram (cg.).
10 centigrams = 1 decigram (dg.).
10 decigrams = 1 gram (gm.).
10 grams = 1 decagram (Dg.).
10 decagrams = 1 hectogram (Hg.)
10 hectograms = 1 kilogram (Kg.).
000 kilograms = 1 metric ton or tonne.

100 kilograms = 1 quintal.

1 gram = 15·4 grains approx.
1 kilogram = 2·205 lb. approx.
1 metric ton = ·9842 ton (avoirdupois)
1 lb. = ·4536 Kg. approx.

1 kilogram is often taken as $2\frac{1}{5}$ lb.

SUMMARY OF CONSTANTS

1 gallon = 277·4 cu. in. = ·1605 cu. ft. = 4·546 litres = 4·546 cu. dm.
1 litre = 1000 cu. cm. = 61·03 cu. in. = 1·76 pints.

1 inch = 2·54 cm.
1 yard = ·9144 m.
1 mile = 1·609 Km.

1 sq. in. = 6·45 sq. cm.
1 cu. in. = 16·39 cu. cm.

1 lb. = ·4536 Kg.
1 ton = 1·016 metric tons.

1 cm. = ·394 in.
1 metre = 39·37 in.
1 kilometre = ·6214 mile.

1 sq. cm. = ·155 sq. in.
1 cu. cm. = ·061 cu. in.
1 are = 119·6 sq. yd.
1 hectare = 2·47 acres.

1 Kg. = 2·205 lb.
1 metric ton = ·9842 ton.
1 gram = 15·4 grains.

1 gallon of water weighs 10 lb.
1 cu. foot of water weighs 62·3 lb.
1 cu. cm. of water weighs 1 gram.
1 litre of water weighs 1 kilogram.

MENSURATION FORMULÆ

Area of Parallelogram = base × height.

Area of Triangle $= \frac{1}{2}$ base × height
$= \sqrt{\{s(s-a)(s-b)(s-c)\}}$ where $s = \frac{1}{2}(a+b+c)$.

Area of Trapezium $= \frac{1}{2}$ (sum of parallel sides) × distance between them.

Volume of Prism = base-area × height.

Volume of Pyramid $= \frac{1}{3}$ base-area × height.

Circumference of Circle $= 2\pi \times$ radius $= \pi \times$ diameter.

Area of Circle $= \pi \times (\text{radius})^2 = \frac{1}{4}\pi \times (\text{diameter})^2$.

Area of Sector of a Circle $= \frac{1}{2}$ radius × length of arc.

Area of Curved Surface of Circular Cylinder = perimeter of base × height $= 2\pi rh$.

Volume of Circular Cylinder = base-area × height $= \pi r^2 h$.

Area of Curved Surface of Cone $= \pi \times$ base-radius × slant side $= \pi rl$.

Volume of Cone $= \frac{1}{3}$ base-area × height $= \frac{1}{3}\pi r^2 h$.

Area of Surface of Sphere $= 4\pi \times (\text{radius})^2 = 4\pi r^2$.

Volume of Sphere $= \frac{4}{3}\pi \times (\text{radius})^3 = \frac{4}{3}\pi r^3$.
$= \frac{1}{6}\pi \times (\text{diameter})^3 = \frac{\pi}{6} d^3$.

ARITHMETIC

PART I

CHAPTER I

ADDITION AND SUBTRACTION

Historical Note :

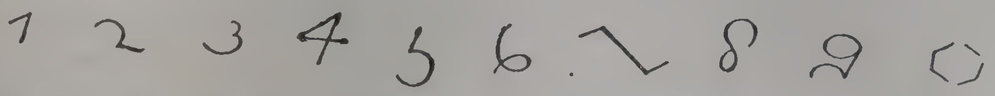
The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called **digits.**

The word "digit" comes from the Latin *digitus*, meaning a finger. It is used for these symbols because numbers were originally indicated by holding up the fingers of the hand.

Numbers were originally written by making strokes, one for each unit, but these soon become confusing, and the Greeks used the letters of their alphabet to denote numbers, α for 1, β for 2, etc.

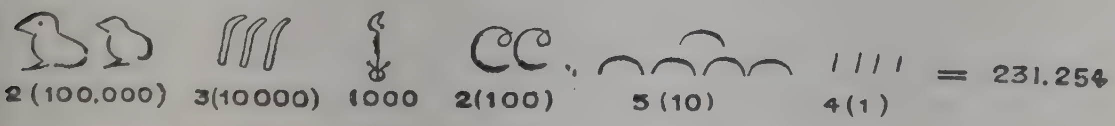
Our digits are of Hindu origin, and reached Europe during the twelfth century through the Arabs.

They were probably letters of an ancient alphabet somewhat transformed in the course of ages, as will be seen from the fact that in Caxton's time, 1480, they appeared as



The most ancient mathematical handbook is an Egyptian papyrus, now in the British Museum, written by Ahmes about 2000 B.C.

The Egyptians used a decimal system, and represented numbers by hieroglyphics thus :



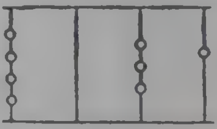
The use of a decimal scale is due to the fact that we have ten fingers, but there are many indications of the use of other scales ; for instance, the Roman system shows signs of a quinary (5) system :

I II III IV V, VI . . . IX X, XI . . . XV.

The Romans also used letters for large numbers—L for 50, C for 100, D for 500, M for 1000. Thus : XL=40, LX=60, XC=90, CM=900, MDV=1505, MCMXCIX=1999.

Celtic remnants of a vigesimal (20) system occur in the French *quatre vingts* and in the English "score."

The decimal notation had been in use for 1000 years before it was perceived that its usefulness could be increased by the adoption of the principle of position, and there is evidence that the Hindu notation of the second century A.D. did not include the zero.

The principle of position developed from the use of the abacus, a system of vertical rods with sliding buttons thus,  represents 4031.

When the numeral was written, this became

M	C	X	I
4		3	1

The disappearance of the columns led to the necessity for a symbol to indicate the absence of a digit ; at first a dot (·) was used, but this could easily be effaced and a zero sign, in the form of a polygon ⬡, was used by the Florentine merchants in the twelfth century. The number was written in the modern form 4031 about the fourteenth century.

The word zero is from *zephirum*, the Latin form of the Arabic *sifr* (empty)—the symbol being used at first to indicate the existence of an empty space. In English the word has taken the form cipher.

Addition

A **unit** is a single thing, such as " a ton," " a foot," etc. When a number is used with a unit, we call the expression a *quantity*.

3 feet is a quantity, so is 5 shillings.

Mental Work.—Much mental practice in addition should be given to beginners. Such expressions as $6+8+7+5$ should be evaluated by reading off the successive sums, 6, 14, 21, 26.

A useful example for oral work is to start with a number and ask each member of the class in turn to add a given fixed number to the last number named : thus, start with 3 and add 7 successively, 3, 10, 17, 24, 31, etc.

Example.—

Add

$$\begin{array}{r} 6,473 \\ 5,817 \\ 934 \\ 6,809 \\ \underline{5,380} \\ \underline{\underline{25,413}} \\ {\scriptstyle 2\;3\;2\;2} \end{array}$$

Note.—When large numbers are added together, put down the carrying figure underneath each column.

Practice should be given in the addition of figures in *rows* as well as in columns.

EXERCISE I (a)

Add the following groups of numbers both vertically and horizontally :

	1.	2.	3.	4.
13.	234	439	397	476 =
14.	617	538	468	987 =
15.	528	326	778	123 =
16.	315	217	513	868 =

	5.	6.	7.	8.
17.	2,167	3,257	528	1,732 =
18.	384	984	9,236	7,464 =
19.	1,592	7,631	8,715	867 =
20.	6,173	4,693	1,574	9,693 =

	9.	10.	11.	12.
21.	63,154	32,614	62,496	617 =
22.	8,217	153,128	3,286	28,431 =
23.	16,943	7,926	14,785	7,893 =
24.	7,238	18,435	819	652,467 =

25. What are the totals of the vertical and horizontal results ? Do they agree ?

Add the following groups, 5 at a time, both in rows and in columns :

	26.	27.	28.	29.	30.
36.	5,632	4,815	72,643	7,458	39,268 =
37.	7,446	683	5,249	48,377	6,849 =
38.	9,734	26,467	69,683	368,926	67,785 =
39.	35,893	3,269	87,649	73,256	1,347 =
40.	2,378	86,492	8,327	5,784	641,283 =

	31.	32.	33.	34.	35.
41.	862,493	76,285	73,642	3,568	627,487 =
42.	67,348	8,977	624,765	38,296	93,843 =
43.	527,846	836,254	72,859	847	968,325 =
44.	8,317	92,865	735,468	863,752	737,645 =
45.	84,378	7,864	7,289	968,438	725,984 =

The number of passengers carried by different railway companies is given below. Add the rows and the columns. Check by adding the totals at the foot of the first three columns and the total of the fourth column.

	46.	47.	48.	
	First Class.	*Second Class.*	*Third Class.*	*Total.*
49.	1,528,436	2,684,719	98,236,425	
50.	1,468,397	2,354,627	89,137,218	
51.	1,348,625		78,286,173	
52.	2,817,614	283,519	73,274,615	
53.	1,928,417	2,817,618	60,437,218	
54.	1,173,428		59,713,847	

55. In 1911 the occupations of the inhabitants of England and Wales were :

	Men.	*Women.*
Industrial	7,015,605	2,452,533
Commercial	2,062,710	151,321
Domestic service	387,677	1,734,040
Agriculture and fishing	1,165,654	94,822
Professional	367,578	347,043
Government	248,624	50,975
Defence	205,817	
Unoccupied	2,208,535	10,026,379

Add vertically and horizontally to find the total population.

Subtraction

To subtract 5 from 9 is to find the difference between 9 and 5, or, in other words, to find what number added to 5 will make 9.

Oral Work.—A useful example is to start with a number and ask each member of the class in turn to subtract a given fixed number from the last number named : thus, start with 120 and subtract 7 successively, 120, 113, 106, 99, 92, etc.

Another useful method is to write up on the blackboard about 20 numbers, and, pointing to them haphazard, to ask what must be added to make up some constant number. A similar method may be used to give practice in complementary addition in money sums.

Example.—Subtract 38 *from* 64.

Method I

$$\begin{array}{rl} 64 = & 50+14 \\ \underline{38} = & \underline{30+\ \ 8} \\ \underline{\underline{26}} & \underline{\underline{20+\ \ 6}} \end{array}$$

Method II

$$\begin{array}{rcr} 64 & \text{gives the} & 60+14 \\ \underline{38} & \text{same} & \underline{40+\ \ 8} \\ \underline{\underline{26}} & \text{result as} & \underline{\underline{20+\ \ 6}} \end{array}$$

Note.—The working on the right in each method gives the process of reasoning: it is not, of course, meant that it should be written down.

Experiment has shown that Method II leads to more accurate results than are obtained by Method I.

The statement that we pay back 10 to the bottom line which we have borrowed from the top line should be avoided, as it is as immoral as it is untrue.

EXERCISE I (b)

Find the difference between the following pairs of numbers:

1. $\begin{array}{r} 64 \\ \underline{32} \end{array}$ **2.** $\begin{array}{r} 213 \\ \underline{101} \end{array}$ **3.** $\begin{array}{r} 1357 \\ \underline{624} \end{array}$ **4.** $\begin{array}{r} 853 \\ \underline{768} \end{array}$ **5.** $\begin{array}{r} 1527 \\ \underline{939} \end{array}$

6. $\begin{array}{r} 6428 \\ \underline{3576} \end{array}$ **7.** $\begin{array}{r} 17{,}248 \\ \underline{6{,}839} \end{array}$ **8.** $\begin{array}{r} 400{,}038 \\ \underline{237{,}146} \end{array}$ **9.** $\begin{array}{r} 369{,}284 \\ \underline{174{,}895} \end{array}$

Find the value of

10. 62 + 47. **11.** 8174 + 6239. **12.** 72,635 + 14,289. **13.** 37 − 16.
14. 234 − 178. **15.** 63,284 − 59,759. **16.** 732,618 − 648,937.

17. By what number is 62,417 greater than 38,417?

18. What number must be added to 48,419 to make 73,624?

19. What is the sum of 28,416 and 73,519?

20. From 63,214 take 49,638.

21. The sum of two numbers is 63,412; the greater is 57,483. What is the other?

22. The difference between two numbers is 37 and the greater is 642. What is the smaller?

23. When 2613 is subtracted from a certain number the result is 3849. Find the number.

24. The difference between two numbers is 6214, and the smaller is 6837. What is the greater?

25. By what number does 3219 exceed 1847?

26. What number must be subtracted from 62,413 to leave 21,847?

27. By how much does the sum of 6214 and 5197 exceed their difference?

28. By what number is 72,614 greater than 5819?

29. The sum of two numbers is 26,214: the smaller is 9817. Find the other.

30. What number must be subtracted from 86,419 to leave 27,865?

31. How many years are there from the beginning of A.D. 627 to the end of A.D. 1924?

32. A has 45 stamps, B has 67. How many must B give A, so that both may have the same number?

33. A firm employs 2642 men and 1848 women. If the total number is to be unaltered, how many men must be dismissed and how many women engaged to make the numbers of each sex the same?

Find the balance required in the following to make the given totals:

	34.	**35.**	**36.**	**37.**
	714	4,363	8,764	63,217
	862	8,271	3,217	8,965
	3145	396	6,578	54,789
Balance				
	5286	15,218	27,126	182,063

38. The output of gold from the Transvaal for 6 months in 1920 and 1921 is given below.

Show with a plus sign the increase each month for 1921 above the corresponding month of 1920, and the decrease with a minus sign.

	1920.	1921.		1920.	1921.
May .	2,970,924	2,898,048	Aug. .	2,983,852	3,023,086
June .	3,042,817	2,883,583	Sept. .	2,899,236	2,937,158
July .	3,128,421	2,930,609	Oct. .	2,815,506	3,008,256

39. The number of unemployed is given as:
June 1921, 2,044,655; June 1922, 1,405,293; June 1923, 1,319,912.
Find the decrease each year and the total decrease.

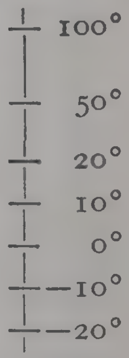
40. Find the total increase in the two Services given:

	Fighting Services.	*Civil Service.*
1914 . .	86,027,992	55,005,722
1923 . .	167,127,000	188,976,000

41. The dead-weight debt in 1920 was £7,434,949,429 and in 1921 it was £7,831,744,300, but there was a net decrease of £246,334,610 on total liabilities. What change had taken place in the other liabilities apart from the dead-weight debt?

Negative Numbers

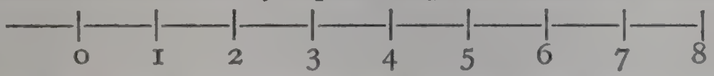
On the scale of a centigrade thermometer the temperature of freezing water is marked 0, and the temperature of boiling water is marked 100. The distance between these two marks is divided into 100 equal parts called degrees (symbol °). To indicate temperatures below 0°, the divisions are numbered −1°, −2°, etc., so that a temperature 10° below the freezing-point of water is numbered −10°.



In a similar way the height of a place above sea-level may be 500 ft., but if the place is 500 ft. below sea-level we may say that its height is −500 ft.

If we use a letter a to represent one number and b to represent a different number, the result obtained by adding a and b is $a+b$.

We can represent numbers by equal lengths measured along a line thus:



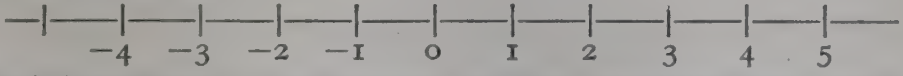
The addition of 5 to 3 is represented by starting from 0 and moving 3 divisions to the right to the figure 3, and then 5 divisions further to the right, thus arriving at 8.

The result obtained by subtracting b from a is written $a-b$.

If a is greater than b, *e.g.* $a=5$ and $b=3$, the result is 2. This is obtained on our scale by moving 5 divisions *to the right* from 0, as far as the figure 5, and then 3 divisions *to the left* to the figure 2.

If a is less than b, the result can be interpreted by an extension of our scale.

We mark off a series of divisions *to the left* of 0, and call these −1, −2, −3 . . . . Such numbers are called *negative numbers*. 1, 2, 3, etc., are called *positive* numbers.



To find the value of $a-b$ when $a=3$ and $b=5$, we start from 0 and measure 3 divisions *to the right* as before; then, moving 5 divisions *to the left*, we arrive at the number −2.

Thus $3-5=-2$.

EXERCISE I (c)

1. Find the value of $a+b$ when $a=6$, $b=2$; $a=4$, $b=3$, and illustrate from a diagram.

2. Find the values of $a-b$ when $a=6$, $b=2$; $a=2$, $b=6$; $a=3$, $b=4$, and illustrate from a diagram.

3. The height of Jerusalem above sea-level is +2500 ft., the Sea of Galilee is −680 ft., and the Dead Sea −1300 ft. How much higher is Jerusalem than (i) the Sea of Galilee, (ii) the Dead Sea? What is the drop from the Sea of Galilee to the Dead Sea?

4. A thermometer reads 49° at 8 a.m.: it is read every 6 hours, and the *changes* are +16°, −12°, −8°, +6°. What does it read finally?

HIGH
BANGALORE-1

5. A freezing mixture has a temperature of $-22°$. It is melted and warmed to $+46°$. How many degrees has its temperature risen ?

6. A liquid, X, is at a temperature of $-14°$, and another, Y, is at $-3°$. Which is at the higher temperature ? How much must they rise before they reach $+10°$?

7. A liquid at a temperature of $-4°$ is mixed with another at $+12°$. The temperature of the mixture is $-2°$. What change has occurred in each ?

8. A traveller climbs from a place whose height is 620 ft. to another 520 ft. above, and then descends 1340 ft. What is the height of the last place ?

9. There is a ladder on the wall of a quay. The top is 16 ft. and the bottom is -14 ft. above the water. If the water-level now rises 10 ft., how much is the bottom of the ladder above the new water-level ? What is the length of the ladder ?

10. If a stone is thrown vertically upwards its velocity increases by -32 ft. per second during each second. It is thrown upwards with velocity 96 ft. per second. How long will it be before (i) it comes to rest, (ii) it is moving downwards with velocity 96 ft. per second ?

Brackets

If a man's gains are represented as $+$, and his losses as $-$, his record might be

$$+62-140+210+50-180.$$

His gains are $+62+210+50=322.$
His losses are $140+180=320.$
His net gain is $322-320=2.$

If we use brackets to separate the gains from the losses we can write his record as

$(62+210+50)-(140+180)$ and evaluate the contents of the brackets separately.

EXERCISE I (d)

1. Simplify (i) $16+12-8+5-22$; (ii) $14+3-(7+9)$; (iii) $16-23+47-(50-18)$.

2. Discuss whether $18-(12-6)$ is the same as $18-12-6$ or $18-12+6$.

3. Discuss whether $16-(5+4)$ is the same as $16-5+4$ or $16-5-4$.

4. Write the following again with a bracket round the second and third terms. Evaluate before and after putting the bracket. $12-8-4$, $12-8+4$, $12+8-4$, $12+8+4$.

5. Find the values of $a-(b-c)$ when (i) $a=6$, $b=4$, $c=2$; (ii) $a=6$,

$b=3$, $c=5$; (iii) $a=4$, $b=7$, $c=3$. Find also the values of $a-b+c$ in each case.

6. Find the values of $a-(b+c)$ when (i) $a=7$, $b=3$, $c=2$; (ii) $a=4$, $b=1$, $c=5$; (iii) $a=2$, $b=3$, $c=4$. Find also the values of $a-b-c$ in each case.

Write the answers to Ex. 7–17 by using brackets—*do not work out the results.*

7. Find what is left after subtracting the sum of 26 and 12 from 43.

8. What change is received from 20 shillings after paying two bills, one of 4 shillings and one of 7 shillings ?

9. Subtract the difference between 80 and 14 from 73.

10. What is your height above sea-level if you start at 6000 ft. and, after descending 2500 ft., climb another 1000 ft.?

11. Express your gain after the following transactions : gain the difference between £20 and £13, then lose the difference between £16 and £12.

12. Subtract the sum of 13 and 9 from the difference of 44 and 17.

13. What must be added to the difference of 23 and 14 to give 15 ?

14. What is the excess of the sum of 17 and 19 over the difference of 32 and 15 ?

15. Subtract a from b; subtract the result from $c+d$.

16. What must be added to $a-b$ to give $c-d$?

17. From a subtract the sum of $b+c$ and then subtract $d+e$ from the result.

18. Show by using brackets that subtracting 999 from 8200 is the same as adding 1 and subtracting 1000.

CHAPTER II

MULTIPLICATION

MULTIPLICATION by an integer is repeated addition : thus 3 times 4 means $4+4+4=12$; 3 times 4 is written 4×3.

The result of the multiplication is called the **product** : thus 12 is the product of 3 and 4; and the two numbers which are multiplied together are called **factors** : thus 3 and 4 (or 6 and 2) are factors of 12.

When three or more numbers are multiplied together, the result is called their **continued product** : thus the continued product of 2, 3, 4, 5 is 120.

Letters for Numbers.—When the number represented by a is multiplied by the number represented by b, the operation is written $a \times b$ or simply ab; $5a$ means $a \times 5$.

Note the difference between the arithmetical notation "23" and the algebraic notation "ab," when $a = 2$, $b = 3$.

23 means "two tens $+3$," but ab means 2×3 or 6.

Squares, Cubes, etc.—When the number 3 is multiplied by itself, we obtain the **square** of 3, which is written 3^2; $3^2 = 3 \times 3 = 9$. Similarly $a \times a = a^2$.

In like manner the cube of $4 = 4^3 = 4 \times 4 \times 4 = 64$,
and the cube of $x = x^3 = x \times x \times x$.

EXERCISE II (a) (mainly Oral)

Multiply :—**1.** 17 by 10, 100, 10,000. **2.** 203 by 10, 1000, 20.

3. Start with 7 and double it, and continue doubling till you get a number of 3 figures.

4. Start with 2 and treble it, and continue trebling it up to 200.

Multiply :—**5.** 19 by 2, 200, 30, 40, 500. **6.** 13 by 3, 4, 5, 7, 8, 10, 12, 600, 110. **7.** 103 by 5, 7, 90, 12, 13, 17. **8.** 209 by 9, 70, 11, 12, 300. **9.** 144 by 5, 2, 50, 25. **10.** 13 by 2, 4, 8, 16, 32.

11. Read off the values of 7×13, 70×130, 700×13, 14×13.

12. Read off the values of 125×8, 125×32, 125×40.

13. What are the squares of the first twenty integers ?

14. What are the cubes of the first ten integers ?

15. What are the continued products of 2, 3, 5 ; 5, 6, 7 ; 4, 5, 11 ; 3, 5, 8, 9 ?

16. Give all possible factors of 42, 45, 49, 95, 91.

17. Give all the factors of 105, 143, 126, 175, 333.

18. Multiply 4^2 by 5^2 and 2^3 by 5^3.

19. Read off the values of $4 \times 13 \times 25$, $4 \times 7 \times 125$, $2^3 \times 3^3 \times 5^3$.

20. Read off the values of 2×99, 11×99, 7×999, 37×99.

21. When $a = 4$ write down the values of $5a$, a^2, $3a$, $16a$.

22. When $b = 3$ write down the values of $2b$, b^3, $3b$, $10b + 4$, $10b + 2$, $b^2 + b$, $2b + 4$, $100b + 10b + b$.

23. When $a = 2$ and $b = 3$ find the values of $3a + 4b$, $4a - 2b$, $a^2 + b^2$, $b^2 - a^2$, $a^2 - b^2$, $(a + b)^2$, $(b - a)^2$, $100a + 10b + 4$.

24. When $a = 4$, $b = 3$, $c = 2$ find the values of $5a - (2b + c)$, $3a - (4b - 2c)$, $a^2 - (b^2 + c^2)$, $3a^2 - (2b^2 - c^2)$.

25. When $a = 100$ find the value of $62(a - 1)$, also of $62a - 62$. Hence state a simple way of multiplying by 99.

26. Multiply 327 by 999.

27. What is the value of $37 - ab$ if $a = 3$, $b = 7$? **28.** $275 - abc$ if $a = 2$, $b = 7$, $c = 5$? **29.** $73 - (10a + b)$ if $a = 7$, $b = 3$? **30.** $329 - (10a + b)$ if $a = 32$, $b = 9$? **31.** $329 - (100a + 10b + c)$ if $a = 3$, $b = 2$, $c = 9$?

Multiplication by numbers of more than one digit should always **start with the figure on the left.** Later on, when contracted methods are taught, very few of the figures obtained by multiplying with the figures on the right are retained in the working.

Example.—Multiply 6347 *by* 278.

$$\begin{array}{r} 6347 \\ \underline{278} \\ 1{,}269{,}4 \\ 444{,}29 \\ \underline{50{,}776} \\ \underline{\underline{1{,}764{,}466}} \end{array}$$

Example.—Multiply 623 *by* 204.

The dots indicate the missing line resulting from multiplication by 0. They fix the proper position of the next line.

$$\begin{array}{r} 623 \\ \underline{204} \\ 124{,}6 \\ \cdot\cdot\;\cdot\cdot \\ \underline{2{,}492} \\ \underline{\underline{127{,}092}} \end{array}$$

A useful mental exercise for the more able members of a class is to require them to multiply by two figures at once.

Example.—Multiply 4836 *by* 43.

$$\begin{array}{r} \cdot\;\cdot\;\cdot \\ 4836 \\ \underline{43} \\ \underline{\underline{207{,}948}} \end{array}$$

$3 \times 6 = 18$, put down 8, carry 1.
$3 \times 3 = 9$, add 1, $= 10$ and $4 \times 6 = 24$, add 10; total 34.
Put down 4, tick the 6 in the top line, carry 3.
$3 \times 8 = 24$, add 3, $= 27$ and $4 \times 3 = 12$, add 27; total 39.
Put down 9, tick the 3 in the top line, carry 3.
And so on.

Letters for Numbers

Example.—Multiply $a + b$ by $c + d$.

$$\begin{array}{l} a + b \\ \underline{c + d} \\ ac + bc \\ \underline{ad + bd} \\ \underline{\underline{\mathbf{ac + bc + ad + bd}}} \end{array}$$

First multiply $a + b$ by c, then multiply $a + b$ by d; add the results.

Note the special case $(a + b)^2 = (a + b)(a + b)$
$= a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$.

If $a = 20$, $b = 3$ we have $23^2 = (20 + 3)^2 = (20)^2 + 2(20)(3) + 3^2$
$= 400 + 120 + 9 = 529$.

We may also evaluate squares from the form $(a + b)^2 = a^2 + 2ab + b^2$
$= a(a + 2b) + b^2$.

Thus $53^2 = 50 \times 56 + 3^2 = 2809$
or $86^2 = 80 \times 92 + 6^2 = 7360 + 36 = 7396$.

EXERCISE II (b)

Find the product of

1. 32 × 17
2. 45 × 19
3. 27 × 23
4. 142 × 26
5. 362 × 13
6. 358 × 19
7. 462 × 21
8. 732 × 18
9. 463 × 37
10. 589 × 53
11. 362 × 286
12. 784 × 683
13. 1246 × 68
14. 2357 × 73
15. 8647 × 98
16. 5236 × 99
17. 6348 × 998
18. 7258 × 645
19. 3682 × 587
20. 7706 × 508
21. 8827 × 609
22. 1374 × 1028
23. 2647 × 2083
24. 2479 × 8076
25. 6007 × 503
26. 9030 × 1001
27. 7009 × 8060
28. 50607 × 3900
29. 990 × 7306
30. 999 × 6006

EXERCISE II (c)

1. Find the product of 213 and 175.

2. Multiply 6023 by 3107, also 3107 by 6023.

3. If a charabanc carries 28 people, how many can be taken by 46 of them ?

4. A man whose estate was valued at £642,430 left 5 bequests of £20,000, 4 of £1500, 6 of £1200. If the taxes amounted to £174,624, how much was left to be divided after paying the bequests ?

5. A book contains 324 pages, and on each page there are 61 lines. How many lines are there in the book, and how many words at 14 to the line ?

6. What would be the cost of 125 motors at £340 each ?

7. A firm pays a man £3 a week. What would it spend in wages on 1634 men for a year of 52 weeks ?

8. Light travels 186,000 miles in 1 second. How far does it go in 1 hour ?

9. There are 1760 yd. in a mile. How many yards in 124 miles ?

10. Find the square of 623. **11.** Find the cube of 32.

12. 960 farthings make £1. How many farthings in £524 ?

13. Find the total cost of 62 rings at £47 each and 54 bracelets at £37 each.

14. A library has 428 shelves each holding 43 books, 836 holding 57, and 1243 holding 132. Find the total number of books.

15. Find the value of 2614^2.

16. If motors are parked in a row with 2 ft. between each, what space is required for a row of 142 cars each of length 15 ft., leaving 2 ft. at each end of the row ?

17. If the distance between consecutive stations is 542 yd., how many yards does a man travel on a return journey from the first to the ninth station ?

18. A register of names has 34 names on each page. How many names are there in the book if they begin on page 4 and end on page 26?

19. Cabbages are planted in a garden 2 ft. apart and 2 ft. from the edge of the garden. If there are 56 cabbages in a row, how wide is the garden?

20. A cage is lifted by a rope passing round a wheel. If the circumference of the wheel is 53 ft., how much will the cage descend in 24 turns of the wheel?

21. A bridge has 37 spans, each of which measures 48 yd. What is the length of the bridge?

22. The circumference of a wheel is 76 in. How many inches will it travel in 134 revolutions?

23. Blocks of the same size are packed in a box. There are 17 rows, each containing 13 blocks on the bottom of the box, and there are 33 layers. How many blocks are there?

24. Express without brackets (1) $2x(a+b)$, (2) $a(3x+2y)$, (3) $4(a-2b)$.

25. Simplify (1) $(x+y)(a+b)$, (2) $(2x+y)(a+3b)$.

26. Since $(a+b)^2=a(a+2b)+b^2$, we can write down the square of a number such as 54 by evaluating $50(58)+4^2=2916$. Here $a=50$, *i.e.* $54-4$, and $(a+2b)=54+4$, *i.e.* $b=4$. In a similar way write down the values of 26^2, 33^2, 61^2, 75^2.

27. Use $(a+b)(a-b)=a^2-b^2$ to write down the values of 44^2-43^2, 85^2-83^2, 49×51, 78×82.

28. Given that $97^2=9409$, write down the values of 98^2 and 96^2.

Division

Division by an integer is equivalent to repeated subtraction; thus $12-3-3-3-3=0$, therefore 3 is contained in 12 four times, or $12\div3=4$. The number to be divided is called the **dividend**, the number by which it is divided is called the **divisor**, and the number of times that the divisor is contained in the dividend is called the **quotient**.

In the example $12\div3=4$, the dividend is 12, the divisor is 3, and the quotient is 4.

If the divisor is not contained an exact number of times in the dividend, the process of division leaves a **remainder**; thus if 14 is divided by 3, the quotient is 4 and the remainder is 2.

Note.—**Never** say 3 into 12 goes 4, but say **either** 12 divided by 3 equals 4, or 3 is contained 4 times in 12. The word "into" leads to confusion with multiplication in connection with brackets.

Example.—Divide 5334 *by* 20.

$$\begin{array}{r|l} 2{,}0 & \underline{533{,}4} \\ & \underline{266} - 14 \end{array}$$

Quotient 266.
Remainder 14.

Example.—Divide 6055 *by* 35.

$$\begin{array}{r|l} 7 & 6055 \\ 5 & \underline{865} \\ & \underline{173} \end{array}$$

Quotient 173.

EXERCISE II (d)

Evaluate : **1.** $434 \div 7$. **2.** $30,756 \div 12$. **3.** $4030 \div 10$.
4. $\frac{236,400}{100}$. **5.** $67,606 \div 14$. **6.** $6804 \div 21$. **7.** $23,016 \div 42$.
8. $7704 \div 36$. **9.** $\frac{532,400}{100}$. **10.** $\frac{64,170}{30}$. **11.** $\frac{724,200}{300}$.
12. $\frac{1,862,000}{1000}$. **13.** $96,200 \div 100$.

14. Find the quotient and remainder if (i) 734 is divided by 10, (ii) 8651 is divided by 20, (iii) 4723 is divided by 100, (iv) 6793 is divided by 300.

15. Find what number multiplied by 7 will make 21.

16. Find what number multiplied by 21 will make 777.

17. Find the quotient when 8477 is divided by 49.

18. How many times is 63 contained in 2646 ?

19. How many pounds are there in 68,720 shillings ?

20. How many shillings are there in 19,344 pence ?

21. How many pence are there in 7846 shillings ?

22. How many 7-lb. jars can be filled from 2534 lb. of jam ?

23. Find one-half of 528 and one-sixth of 1254.

24. If a wheel travels 20 ft. in one turn, how many turns will it make in travelling 68,460 ft. ?

25. I distribute 5240 pence among some men, giving 20 pence to each. How many men were there ?

26. I distribute £1 among some men, giving each man 1s. 3d. How many men were there ?

27. How many lb. of sugar are required to make 432 packages each containing 21 lb. ?

28. How many minutes in 544,680 seconds ?

29. How many right angles in 5220 degrees ?

30. How many parcels each containing 30 lb. can be made from 5229 lb., and how much is left over ?

31. How many times is 16 contained in 11,248 ?

32. If 24 is subtracted from 1152, then 24 from the remainder, and so on, how many subtractions can be made before the remainder is zero ?

33. How many matches are there in 645 boxes each containing 144 ?

34. If $a=12$ and $b=6$, find the values of $\frac{9a}{6}$, $\frac{3a}{4}$, $\frac{4a+6b}{2}$, $\frac{2a^2+3b^2}{12}$.

35. A wheel is 15 ft. in circumference. How many times will it turn in rolling 1650 ft.?

36. A cage is raised by a rope which passes over a wheel whose circumference is 42 ft. How many turns of the wheel are required to raise the cage 1134 ft. ?

37. A shed is built for sheltering bicycles. If 12 in. is allowed between each pair and at the ends, with 18 in. for each bicycle, how many bicycles will be taken by a shed 61 ft. long ? (12 in. = 1 ft.)

38. How many yards has a man travelled after taking 288 paces of 30 in. each ? (36 in. = 1 yd.)

39. How many paces each of 30 in. does a boy take in walking 200 yd.? (1 yd. = 36 in.)

40. A roller turns 42 times in traversing 714 ft. What is its circumference ?

41. 6374 books are to be packed in parcels each containing 12 books. How many parcels will there be ? The parcels are put in rows of 8. How many rows will there be, and how many parcels will be left over ? How many of the original books will not be placed in the rows at all ?

If 6374 is divided by 96, what is the remainder ?

42. 4375 lb. of sugar is made up into 3-lb. packets. How many will there be ? The packets are placed in boxes each containing 7. How many boxes are required, and how many lb. of sugar are left unpacked in the boxes ? What is the remainder when 4375 is divided by 21 ?

Remainder. Short and Long Division

Example.—Divide 4391 *by* 24 *by short division* (see IId, 41, 42).

$$\begin{array}{r|l} 4 & \underline{4391} \\ 6 & \underline{1097}-3 \\ & \underline{182}-5 \end{array}\Big\} = 5 \times 4 + 3 = 23$$

Quotient 182.
Remainder 23.

Example.—Divide 28,465 *by* 317.

$$\begin{array}{r|r} & 89 \\ \cline{2-2} 317 & 28,465 \\ & \underline{25,36} \\ & 3,105 \\ & \underline{2,853} \\ & \underline{\underline{252}} \end{array}$$

Quotient 89.
Remainder 252.

Division by 4 gives 1097 groups of 4 and 3 units over. Division by 6 gives 182 groups of 24 and 5 groups of 4 over ; ∴ the total remainder is 3 units + 5 groups of 4.

The next example illustrates the Italian (shortened) method of division, which involves doing the multiplication and subtraction in a single operation.

Example.—Divide 28,465 *by* 317.

In the following explanation the figures to be written down are underlined, the rest is merely said.

$8\times7=56$, add $\underline{0}$ making 56, carry 5.
$8\times1=8$, add $\underline{5}$ making 13 add $\underline{1}$ making 14, carry 1.
$8\times3=24$, add 1 making 25, add $\underline{3}$ making 28.
$9\times7=63$, add $\underline{2}$ making 65, carry 6.
$9\times1=9$, add 6 making 15, add $\underline{5}$ making 20, carry 2.
$9\times3=27$, add 2 making 29, add $\underline{2}$ making 31.

$$\begin{array}{r|r} & 89 \\ \hline 317 & 28,465 \\ & 3,105 \\ & 252 \end{array}$$

Other Short Methods

(i) Multiplication by 25. Use the fact $25=\frac{100}{4}$.
Thus $623\times25=\frac{62\,300}{4}=15.575$.

(ii) Multiplication by 125. Use the fact $125=\frac{1000}{8}$.
Thus $623\times125=\frac{623\,000}{8}=77,875$.

(iii) Multiplication by 98. Use the fact $98=100-2$.
Thus $643\times98=64,300-1286=63,014$.
Similarly, $643\times998=643,000-1286=641,714$.

(iv) Division by 25. Multiply by 4 and divide by 100.
Thus $\frac{18\,775}{25}=\frac{75\,100}{100}=751$.

(v) Division by 125. Multiply by 8 and divide by 1000.
Thus $\frac{65\,250}{125}=\frac{522\,000}{1000}=522$.

N.B.—It is important to notice that in some questions the result of division is to give a **quantity** and in other cases to give a **number**.

E.g. (i) How many rolls of cloth, each 20 yd. long, can be cut from a length of 100 yd.?

The answer is $\frac{100 \text{ yd.}}{20 \text{ yd.}}=5$. (5 rolls, **not** 5 yards.)

20 yd. is contained in 100 yd. 5 times.

(ii) If a roll of 100 yd. of cloth is cut up into 20 equal lengths, how long is each piece?

The answer is $\frac{100 \text{ yd.}}{20}=5$ **yards.**

EXERCISE II (e)

Find the quotient and remainder when

1. 63,284 is divided by 48, 54, 128.

2. 41,786 by 90, 121, 49. **3.** 13,284 by 1200, 567, 863.

4. 7,875 by 25, 125, 250. 5. 42,386 by 213, 357, 469.
6. 863,194 ,, 375, 528, 3762. 7. 764,289 ,, 89, 1987, 2768.

8. How many motors, each carrying 34 people, will be required or a party of 578 ?

9. There are 112 lb. in 1 cwt. How many cwt. in 26,473 lb., and ow much is left over ?

10. There are 1760 yd. in 1 mile. How many miles in 834,629 yd., nd how much is left over ?

11. How many pounds will 644,160 farthings be worth ? 960 rthings in 1 pound.

12. How many sacks are required for 3472 lb. of potatoes if each ack takes 112 lb.?

13. If £5564 is divided equally among 214 men, how much will ach get ?

14. How many pieces of paper, each 14 in. long, can be cut from strip 216 in. long ? How much is left over ?

15. A steamer goes on an average 18 sea miles per hour. How far a voyage lasting 11 days 4 hours ?

16. A boy runs 880 ft. in 1 minute. How long will he take to run ,120 ft. ?

17. One million matches are to be packed in boxes, each taking 1 oss (144). How many boxes are required, and how many matches e unpacked ?

18. A man's income is £73,000 a year. How much is that per day ? year =365 days.)

19. A man earns 48s. a week. How much is that in a year of weeks ?

20. If a man spends £740 a year, how long will it take him to spend 0,720 ?

21. How many pence in £1, 14s. 2d.? How many in £410 ? If a an earns £1, 14s. 2d. a wk., how long will it take him to earn £410 ?

22. Piers to support a bridge are sunk in a river at intervals of yd. How many piers must be sunk if the bridge is 2475 yd. long ?

23. A surveyor puts a peg in the ground every 110 yd. along a d for a distance of 4 miles. How many pegs does he use, including e at each end ?

24. A register of names has 213 pages and contains 42,648 names. ch complete page has the same number of names. If the first ge of names is page 3, what is the number of the last page ?

25. If telegraph poles are placed 66 yd. apart, how many will be required on a stretch of road 3 miles long, excluding the end poles ?

26. If the value of 86,493 cwt. of lard imported into a country is £198,617, how much per cwt. is it worth to the nearest £ ? Show that you could get the same answer by dividing £198 by 86. Can you explain this ?

27. A board 47 in. long is ruled with lines 3 in. apart, beginning 1 in. from the top and ending 1 in. from the bottom. How many lines are there ?

28. A board has 120 lines ruled on it 2 in. apart, beginning 2 in. from the top and ending 1 in. from the bottom. What is the length of the board ?

MISCELLANEOUS EXAMPLES ON THE FIRST FOUR RULES

EXERCISE M (I)

1. A university started the year with 54,263 students ; during the year 14,263 came and 13,624 left. How many were there at the end of the year ?

2. A car travelling 23 miles an hour took 7 hours to go from A to B. How far was A from B ?

3. How long would it take a train to go 629 miles if it travelled 37 miles an hour ?

4. A boy went to school in September 1894 and stayed 5 years ; he then went to college for 3 years and to business until September 1914. How long was he at the business ?

5. A merchant receives 126 cases each containing 112 lb. of tea and makes the tea up into 7-lb. packages. How many packages will there be ?

6. How many days are there from mid-day June 5 to mid-day July 21 ?

7. How many years elapse between Jan. 1, 1624, and Jan. 1, 1922 ?

8. Eggs are sold at 12 for 20d. How much must you pay for 132 ?

9. A horse is 14 hands high. What is this in inches ? (1 hand = 4 in.)

10. Each knot tied in a cord reduces its length by 7 cm. ; a cord containing 17 knots is 1232 cm. long. What is its length when the knots are all untied ? What would be its length when 30 knots were tied in it ?

11. The outside dimensions of a box without a lid are 30 cm. long, 24 cm. wide, 12 cm. high ; the wood is 1 cm. thick. What are the internal dimensions ?

12. Two cyclists start at the same time from two towns 56 miles apart and ride towards each other, one at 12 miles an hour and the other at 13 miles an hour. How far apart are they after 2 hours ?

13. A man says that he works 40 hours a week and earns a penny a minute. What does he earn each year if he takes a fortnight's holiday ?

14. The product of three numbers is 1001 ; two of them are 7 and 13. What is the other ?

15. The mean distances of the Earth and Neptune from the Sun are 93,000,000 miles and 2,800,000,000 miles. How many times is Neptune as far from the Sun as the Earth is ?

16. For a certain journey a passenger is allowed 120 lb. of luggage free and is charged one farthing per lb. for the rest. What is the charge for 200 lb. of luggage ?

17. A coal-cellar holds 14 tons of coal and the coal is burnt at the rate of 8 cwt. a week. How long will the coal last after the cellar has been filled ? (1 ton = 20 cwt.)

18. One pile of coins consists of 40 florins and 48 half-crowns ; a second pile contains 48 florins and 40 half-crowns. What is the difference in value between the two piles ?

19. Ninety-seven years in every 400 contain 366 days and the rest have 365. How many days are there in 400 years ?

20. A man buys 640 bundles of wood each containing 144 pieces. How many bundles can he make from them, each containing 24 pieces ?

21. A traveller starts from a place 340 ft. high ; he ascends 620 ft., then descends 840 ft., climbs 432 ft., and then descends 860 ft. At what height above sea-level does he finish ?

22. The total number of children in a mixed school is 324. There were 36 more girls than boys. How many of each were there ?

23. What number divided by 214 has a quotient of 342 and a remainder of 75 ?

24. A sheet of paper 14 in. long is ruled with lines $\frac{1}{2}$ in. apart, beginning $\frac{1}{2}$ in. from the top. How many lines are there ?

25. The maps in an atlas begin on page 4 and end on page 112. How many pages of maps are there if they are only printed on one side of the page ?

26. A man plants 4 potatoes to every square yard, and 8 potatoes weigh a pound. How many pounds of potatoes does he need to plant an acre ? (1 acre = 4840 sq. yd.)

27. A climber is 4500 ft. above a village X on a mountain side; he descends 2700 ft. an hour. What is his height above X after two hours?

28. Thirty-three fields, each of area 17 acres, are taken out of an estate of area 1 sq. mile. What is the area of the ground that is left? (1 sq. mile = 640 acres.)

29. A rectangular field is 173 yd. long and 97 yd. wide. What length of fence is required to enclose it?

30. On January 1, 1923, high water at London Bridge was at 12.52 p.m. Each day it was 41 minutes later. What was the time of high water on the afternoon of January 5?

31. An estate of value £26,850 is left to A, B, and C. What does C receive (i) if the shares are equal, (ii) if A gets £5624 and B gets £9778?

32. At an election one candidate received 17,239 votes and the other 14,982 votes; 57 voting papers were spoilt. The number of electors was 41,207. How many electors did not vote?

33. The rungs of a ladder are 18 in. apart, and the top and bottom rungs are 1 ft. from the ends. What is the total length of a 15-rung ladder? (1 ft. = 12 in.)

34. A 10-acre field is to be cut up into allotments, each of area 1500 sq. yd. How many allotments can be formed, and what area is left unused? (1 acre = 4840 sq. yd.)

35. A goods train consists of 46 trucks, each of which is 16 ft. long: allowing 2 ft. between each truck, find the length of the train without the engine.

36. A soldier's marching pace is 30 in. How many paces does he make in marching 4 miles?

37. A garden is 50 ft. wide and 100 ft. long. It contains cabbages 1 ft. apart in rows also 1 ft. apart. If the end cabbages are 1 ft. from the edges of the garden, how many are there in it?

38. In 1901 the population of Wales was 1,760,609, of whom 278,892 spoke Welsh only, 615,242 spoke both Welsh and English. How many spoke English only?

39. A pail full of water weighs 36 lb.; half full it weighs 21 lb. What is the weight of the pail?

40. The average untrained soldier should excavate in ordinary easy soil the following volumes: 1st hour, 30 cu. ft.; 2nd hour, 25 cu. ft.; 3rd hour, 15 cu. ft.; 4th hour and after up to 8 hours, 10 cu. ft. How much earth will a party of 12 men excavate in 7 hours? How much will this earth weigh if 1 cu. ft. weighs 120 lb.?

41. An extension ladder is formed of 8 separate parts, each of length 9 ft., and the overlap at each junction is 1 ft. What is the length of the ladder when fully extended ?

42. In 1914 the number of gold coins struck was 18,752,241, as against 30,633,962 in 1913. The number of sovereigns was 11,501,170, which was 13,082,619 fewer than in 1913. Find the change in the number of half-sovereigns from 1913 to 1914.

43. A owes B £10 and owes C £15 ; B owes C £20 and owes D £15 ; C owes D £35 ; D owes A £20. What is the simplest way of settling the debts ?

44. The charge for an advertisement on the front page of the *Times* is one guinea for 3 lines and 5 shillings for each additional line. How many lines can be inserted for £2, 16s. ?

45. On January 1 the sun rises at 8.8 a.m. and sets at 3.59 p.m. ; on June 1 it rises at 3.51 a.m. and sets at 8.4 p.m. What is the difference in the length of daylight for these two days ?

46. A man paces one side of a field 144 yd. long, his length of pace being 32 in. How many more paces will he take if he reduces his stride by 5 in.?

47. An extract from Bradshaw reads as follows :

Waterloo, dep. 9.30 a.m. Woking arr. 10.7 dep. 10.12 Basingstoke arr. 10.42 dep. 10.48
Winchester arr. 11.14 dep. 11.19 Southampton, arr. 11.40 a.m.

For how many minutes is the train actually in motion between Waterloo and Southampton ?

48. A man starts with a salary of £300 a year and receives an increase of £40 a year after each 2 years of service. What is his salary (i) in his 19th year, (ii) in his 20th year ?

49. A chain is formed of 100 equal circular links each of which is of external diameter 5 in., and is made of material 1 in. thick. What s the total length of the chain when taut ?

50. A class of boys is told to work out examples numbered 2, 5, 8, 11, etc., in a set containing 60 questions in all. How many are they required to do ?

51. A sentry starts from A and walks to B, 90 yd. away, and then back again to A, and so on to and fro. How far is he from A after he has walked (i) 1300 yd., (ii) 1720 yd.?

52. A moving staircase rises at the rate of 20 steps a minute and a man walks up it at the rate of 1 step every 2 seconds. How many steps are there if he takes 36 seconds to get from the bottom to the top ?

53. Complete the following magic squares in which the sum of each row is equal to the sum of each column and the sum of each diagonal:

(i)

11	31	—
—	23	—
—	—	35

(ii)

—	—	44
—	37	—
30	—	58

CHAPTER III

COMPOUND QUANTITIES AND AVERAGES

Money

4 farthings = 1 penny (d.).
12 pence = 1 shilling (s.).
20 shillings = 1 pound or sovereign (£).

a florin = 2s.
a half-crown = 2s. 6d.
a crown = 5s.
a guinea = £1, 1s.

The symbols £, s. d. are the first letters of the Latin words *Libra*, *Solidus*, *Denarius*.

A farthing (Anglo-Saxon *fearthing*) was originally obtained by breaking a penny into four parts, as was done also with the shilling.

A penny was originally an Anglo-Saxon **silver** coin (*penig*). Copper was first used for making pennies in 1797 and bronze in 1860.

A shilling comes from the Anglo-Saxon **scylling**, a coin worth 5½d. It was marked with a cross, along the lines of which it could be broken into four pieces.

A pound was originally the value of 1 pound weight of silver. The term "pound sterling" is applied to the gold sovereign, the word sterling being probably derived from **Easterling**, a name for the Hanseatic merchants.

A guinea, so called because it was first coined from gold brought from Guinea (Africa), ceased to be minted in 1817.

Reduction

When a quantity is expressed in terms of more than one unit of the same kind, it is called a **Compound Quantity.** The process of expressing it in terms of a single unit is called **Reduction.**

Example.—Reduce £3, 17s. 4d. to pence.

£ s. d.
3 17 4
77 4
928 pence.

Example.—Express 68,432 pence as a compound quantity.

d.
68,432
5702s. 8d.
£285, 2s. 8d.

EXERCISE III (a) (Oral)

1. Reduce to pence 1s. 4d. ; 2s. 3d. ; 3s. 9d. ; 5s. 2d. ; 4s. 6d. ; s. ; 8s. 3d. ; 2s. 5d. ; 9s. 3d. ; 10s. 4d.

2. Reduce to shillings £2 ; £3, 10s. ; £4, 5s. ; 3 guineas.

3. Express in £, s. d., 14d. ; 18d. ; 26d. ; 100d. ; 25s. ; 32s. ; 50s. ; 250d. ; 310d. ; 500d.

4. How many half-crowns in £1 ; £2, 10s. ; £3, 2s. 6d.?

5. How many sixpences in £1, 2s. ; £2, 3s. 6d. ; 17s. 6d.?

6. Add together 3 half-crowns, 4 florins, 3 shillings.

7. Subtract 5 half-crowns from a guinea.

8. Add 100 pence to 100 shillings and subtract the result from £10.

9. What is required to make each of the following amounts up o one shilling : 3¼d., 7½d., 4¾d., 9½d., 5¼d.?

10. What change do you receive out of five shillings when paying or an article whose cost is (i) 1s. 1½d. ; (ii) 3s. 7d. ; (iii) 2s. 11½d. ; iv) 11¾d. ; (v) 3s. 10½d.?

11. What is required to make the following amounts up to £1 : i) 7s. 11d. ; (ii) 13s. 9½d. ; (iii) 10s. 5d. ; (iv) 17s. 2¼d. ; (v) 4s. 10¾d.?

12. Three thousand pennies are packed up in paper bags each ontaining five shillings' worth. How many bags are needed ? What s the value of 3000d.?

EXERCISE III (b)

1. Reduce to pence (i) £1, 3s. 6d. ; (ii) £2, 4s. 8d. ; (iii) £3, 7s. 10d. ; iv) £12, 2s. 5d. ; (v) £14, 7s. 9d. ; (vi) £15, 13s. 10d.

2. Express as compound quantities (i) 1526d. ; (ii) 6248d. ; iii) 7319d. ; (iv) 8497d. ; (v) 23,618d.

3. How many halfpence in (i) £6, 13s. 2d. ; (ii) £8, 14s. 1½d. ; iii) £12, 13s. 10d. ; (iv) £15, 17s. 7½d. ; (v) £27, 19s. 8½d.?

4. How many sixpences in (i) £30, 14s. 0d. ; (ii) £26, 12s. 6d.?

5. How many sixpences in £1, 2s. 6d.? How many in £23, 12s. 6d.? Iow many times is £1, 2s. 6d. contained in £23, 12s. 6d.?

6. How many tickets costing 3s. 4d. can be bought for £7 ?

7. Express one million pennies in £, s. d.

8. £20 is distributed in sixpences. How many recipients were there ?

9. A tram carried 2463 people in a week for a fare of 1½d. What vere the receipts ?

10. In one year the Mint issued 13,558,800 pennies. Express this in £, s. d.

11. In 1921–2 the Post Office delivered 3,400,000,000 letters. If the postage of each is taken as 1½d., find what revenue this represents.

12. A bus fare being 2d., what would be the receipts from 643,284 passengers?

13. How many halfpenny stamps can be bought for £4, 10s. 6d.?

14. If a man buys 10,000 articles at 1¼d. each, and sells them for 2d., what profit does he make?

15. If a manufacturer increases the cost of 1 oz. of tobacco from 5d. to 10d. when the duty is increased by 3½d. an ounce, what profit does he make on selling 500,000 lb.? (16 oz. = 1 lb.)

16. Do many people live a million hours?

17. A certain hospital advertises that the cost of its upkeep is 1d. per second. Calculate the cost for a year. (1 year = 365 days.)

18. How many people can go for an outing costing 6s. 8d. if £20 is provided?

19. How many times is £2, 11s. 4d. contained in £28, 4s. 8d.?

20. If a manufacturer reduces the cost of making an article by ¼d., how much does he increase the profit in a year on selling 120,000 of them a week?

Length

12 inches (12″) (in.) = 1 foot (1′) (ft.).
3 feet = 1 yard (1$^{\times}$) (yd.).
22 yards = 1 chain (ch.).
10 chains = 1 furlong (fur.).
8 furlongs = 1 mile (m.).
3 miles = 1 league.

220 yd. = 1 furlong.
1760 yd. = 1 mile.

100 links = 1 chain.

5½ yd. = 1 pole or rod or perch.
4 poles = 1 chain.
40 poles = 1 furlong.

6 feet = 1 fathom.
100 fathoms = 1 cable.

Inch.—Anglo-Saxon *ince*, Latin *uncia* = a twelfth part.
Foot.—The length of a man's foot.
Yard.—Old English *yerde* = a rod. Its length is said to have been fixed by the length of Henry II's arm, but it can be traced back to the Egyptians and Hebrews.
Chain.—It is made of 100 links with labels on each tenth link, and is the length of a cricket pitch.
Pole.—The length of the pole used by the ploughman.
Furlong.—A furrow long.

Mile.—Latin *mille passus* = a thousand paces.

Fathom.—Used by sailors, taken from the length of the outstretched arms when measuring rope.

A nautical or geographical mile is $\frac{1}{60}$th of the length of a degree of latitude; it is 6050 ft. at the Equator and 6110 ft. at the poles. It is usually taken as 6080 ft. (the Admiralty standard).

Weight

Avoirdupois

16 ounces (oz.) = 1 pound (lb.).
14 pounds = 1 stone.
28 pounds = 1 quarter (qr.).
4 quarters = 1 hundredweight (cwt.).
20 hundredweight = 1 ton (t.).

112 lb. = 1 cwt.
2240 lb. = 1 ton.

7000 grains (gr.) = 1 lb.

100 lb. = 1 cental.

1 cu. ft. of water weighs 62·3 lb. approx.; it is often taken as 1000 ounces.

Troy (for precious metals)

24 grains (gr.) = 1 pennyweight (dwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound (lb.).

1 grain (avoirdupois) = 1 grain (troy).
5760 grains = 1 lb.

Note.—The ounce and the pound (avoirdupois) are different from the ounce and the pound (troy). The troy pound was abolished by Act of Parliament, 1902.

Ounce.—Latin *uncia* = a twelfth part, 1 oz. (troy) = $\frac{1}{12}$ lb. (troy).

Pound.—Latin *pondus* = a weight.

Hundredweight.—Originally 100 lb.; it was changed to 112 lb. by Edward III in order to contain 8 stone of 14 lb. each.

Capacity

Dry Measure

2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).
2 gallons = 1 peck (pk.).
4 pecks = 1 bushel (bus.).
8 bushels = 1 quarter (qr.).

Liquid Measure

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.

1 gallon of water weighs 10 lb., or "a pint of water weighs a pound and a quarter."

Quart (L. *quartus* = $\frac{1}{4}$), a quarter of a gallon.
Gallon (O. Fr. *jalon*).
Bushel (O. Fr. *boissl*).

Metric System

Length

1 millimetre (mm.) = $\frac{1}{1000}$ metre (m.).
1 centimetre (cm.) = $\frac{1}{100}$ metre.
1 decimetre (dm.) = $\frac{1}{10}$ metre.
1 decametre (Dm.) = 10 metres.
1 hectometre (Hm.) = 100 metres.
1 kilometre (Km.) = 1000 metres.

Weight

1 milligram (mg.) = $\frac{1}{1000}$ gram (gm.).
1 centigram (cg.) = $\frac{1}{100}$ gram.
1 decigram (dg.) = $\frac{1}{10}$ gram.
1 decagram (Dg.) = 10 grams.
1 hectogram (Hg.) = 100 grams.
1 kilogram (Kg.) = 1000 grams.

Capacity

1 centilitre (cl.) = $\frac{1}{100}$ litre.
1 decilitre (dl.) = $\frac{1}{10}$ litre.
1 decalitre (Dl.) = 10 litres.
1 hectolitre (Hl.) = 100 litres.
1 kilolitre (Kl.) = 1000 litres.

Reduction of Lengths and Weights

As it is desirable whenever possible to work in terms of the **highest unit**, too much emphasis should not be laid on processes of reduction to the lowest unit.

Example.—Reduce 12 tons 14 cwt. 3 qr. 12 lb. to lb.

tons	cwt.	qr.	lb.
12	14	3	12
	254	3	12
		1019	12
		28,544 lb.	

$$\begin{array}{r} 1,019\\ \underline{7}\\ 7,133\\ \underline{4}\\ \underline{\underline{28,532}} \end{array}$$

Example.—Express 146,283 oz. as a Compound Quantity.

				oz. 146,283
			lb. 9142	11
		qr. 326	14	11
	cwt. 81	2	14	11
tons 4	1	2	14	11

$$\begin{array}{r|l} 16\begin{cases}4\\4\end{cases} & \begin{array}{l}\underline{146,283}\\ \left.\begin{array}{l}\underline{36,570}-3\\ \underline{9,142}-2\end{array}\right\}11\end{array}\\ 28\begin{cases}4\\7\end{cases} & \left.\begin{array}{l}\underline{2,285}-2\\ \underline{\underline{326}}-3\end{array}\right\}14 \end{array}$$

Example.—Reduce 13 m. 4 fur. 26 yd. to yards.

	m.	fur.	yd.
	13	4	26
Since 1760 yd. = 1 m.		108	26
∴ 220 yd. = 1 fur.		**23,786 yd.**	

$$\begin{array}{r}108\\220\\\hline 216\\216\\\hline 23{,}760\end{array}$$

EXERCISE III (c)

1. Reduce to lb. (i) 2 t. 3 cwt. 4 lb.; (ii) 13 t. 6 cwt. 1 qr. 8 lb.

2. Reduce to oz. (i) 3 cwt. 12 lb. 6 oz.; (ii) 4 qr. 12 lb. 5 oz.

3. Reduce to yards (i) 1 m. 614 yd.; (ii) 1 m. 3 fur. 72 yd.; (iii) 4 m. 2 fur. 60 yd.

4. Reduce to inches (i) 73 yd. 2 ft. 8 in.; (ii) 1 m. 62 yd. 5 in.; (iii) 3 fur. 80 yd. 1 ft.

5. Express in tons, cwt., etc. (i) 54,936 lb.; (ii) 64,328 oz.; (iii) 1,216,432 oz.

6. Express in m. yd. ft. in. (i) 261,432 ft.; (ii) 81,643 yd.; (iii) 2,619,834 in.

7. Reduce to pints (i) 18 gal. 2 qt. 3 pt.; (ii) 14 gal. 3 qt. 2 pt.

8. Express as a compound quantity (i) 1629 pt.; (ii) 8375 pt.; (iii) 21,643 pt.

9. Write in pence (i) £x; y shillings; £$x+y$ shillings.
,, pounds (ii) xd.; y shillings; x pence $+ y$ shillings.
,, lb. (iii) x ton; y cwt.; y ton $+ x$ cwt.

10. A map is drawn on a scale of 1 to 1,000,000, *i.e.* actual distances are 1,000,000 times the distances on the map. How far apart are two places which are 1 in. apart on the map? Give answer to nearest mile.

11. How long does an insect $1\frac{1}{2}$ in. long appear when magnified 1000 times?

12. The greatest depth sounded in the Atlantic is 4500 fathoms. Express this in miles and yards.

13. If 5 halfpennies weigh 1 oz., what was the weight in tons of the 7,903,200 halfpennies coined in one year?

14. How many 7-lb. packets can be made from 6 cwt.?

15. A bicycle wheel is 5 ft. 6 in. in circumference. How many times will it turn in going 1 mile?

16. A truck is loaded with 1400 packages each weighing 42 lb. What is the total load in tons, etc.?

17. An explorer employs 118 negroes, each of whom carries 60 lb. How many carts, each carrying 12 cwt., would have sufficed ?

18. A sluice discharges 52 tons of water per minute. How much is that per day of 24 hours ?

19. A man walks for 10 miles. If his stride is 2 ft. 6 in. and he makes 120 strides a minute, how long does his journey take him ?

20. How far apart on a map will two places be marked if they are 12 miles apart and the scale of the map is $\frac{1}{50,000}$? (To nearest inch.)

21. How many times can a glass holding ½ a pint be filled from a tank of water which holds 10 gallons ?

22. A cable has to be supported every 4 ft. How many points of support (including the ends) are required for a length of 4 m. 420 yd.?

Addition and Subtraction of Compound Quantities

Example.—Add together £62, 14s. 8½d., £135, 12s. 10d., £27, 5s. 8¾d.

£	s.	d.
62	14	8½
135	12	10
27	5	8¾
£225	13	3¼

Example.—Subtract £4, 6s. 9d. from £5, 3s. 2d.

£	s.	d.
5	3	2
4	6	9
	16	5

9d. + 3d. = 1s. ;
3d. + 2d. = 5d.

7s. + 13s. = £1 ;
13s. + 3s. = 16s.

EXERCISE III (d)

Add

	£	s.	d.		£	s.	d.		£	s.	d.
1.	15	3	7	2.	11	2	8	3.	25	13	9¼
	4	16	2		0	19	7		12	17	6
	3	11	4		4	16	5		4	15	8½

4.	23	17	8	5.	34	16	10	6.	112	16	4¾
	46	15	9½		68	17	9½		37	18	9½
	15	13	10¼		52	15	10¾		72	15	8¾

7.	513	14	8½	8.	642	13	7	9.	386	12	9
	269	17	5¾		586	14	9		721	13	8
	183	15	8½		263	11	2		462	16	11½
	341	12	7		758	17	11		218	7	9¾

10. £862, 13s. 4d., £917, 17s. 5d., £832, 11s. 4d., £469, 12s. 5d.
11. £263, 17s. 8½d., £369, 13s. 2¼d., £521, 11s. 5d., £483, 9s. 4½d.
12. £186, 13s. 2d., £784, 18s. 8½d., £621, 11s. 9½d., £839, 19s. 9d.

Subtract

	£	s.	d.
13.	16	14	8
	12	7	5

	£	s.	d.
14.	14	13	8
	12	10	9

	£	s.	d.
15.	14	12	6
	11	19	8

16.	15	13	7½
	13	16	10¾

17.	23	18	6½
	10	19	10

18.	624	18	8
	596	19	11½

Add

19.	yd.	ft.	in.
	3	2	7
		2	10
	4	1	8
	1	0	7

20.	tons	cwt.	qr.
	2	14	1
	3	13	2
	1	19	3
	1	17	1

21.	gal.	qt.	pt.
	2	3	1
	4	2	0
	3	1	1
	4	2	0

22.	tons	cwt.	qr.	lb.
	5	2	2	14
	1	14	1	7
	3	6	1	15

23.	hr.	min.	sec.
	16	49	32
	14	27	15
	12	15	10

24.	m.	fur.	yd.
	3	4	17
	2	2	62
	1	5	70

Subtract

25.	tons	cwt.	qr.
	3	12	1
	2	17	2

26.	gal.	qt.	pt.
	4	2	1
	3	3	0

27.	m.	fur.	yd.
	3	5	72
	2	6	83

28.	tons	cwt.	qr.	lb.
	4	12	3	16
	3	17	2	21

29.	yd.	ft.	in.
	16	2	6
	14	1	10

30.	m.	yd.	ft.
	1	1214	2
		1632	1

31. I go shopping with £10, and spend £1, 2s. 6d., 17s. 8d., 14s. 3½d., £2, 13s. 5d., £3, 7s. 4d. How much do I bring home ?

32. What must be added to £624, 13s. 2d. to make £719, 19s. 5d.?

33. What difference in the balance at the bank is made by paying in cheques for £17, 6s. 4d., £21, 15s. 2d., £126, 13s. 8d., and drawing cheques for £52, 13s. 8d., £7, 11s. 2d., and £63, 13s. 9d.?

34. How much must I add to my account to make my balance up to £500 if it is now £367, 19s. 7d.?

35. A railway truck loaded weighs 5 tons 3 qr. 14 lb., and the weight of the truck is 1 ton 1 cwt. 3 qr. What is the weight of the load ?

36. An experiment begins at 14 min. 8 sec. past 1 and ends at 11 min. 16 sec. past 3. How long did it last ?

37. A milk-can full contains 10 gallons. How much is left after drawing off 1 gal. 2 pt. ; 2 gal. 3 qt. ; and 3 gal. 2 qt. 1 pt.?

38. The wall of a room is 24 ft. 6 in. long. It has two doors, one of 3 ft. 6 in. width and the other 3 ft. 9 in. What is the length of skirting board ?

39. The sides of a field are 163 yd. 4 ft. ; 128 yd. 2 ft. ; 72 yd. 1 ft. ; 89 yd. 2 ft. ; 102 yd. 1 ft. Find its perimeter.

40. A traveller goes 142 miles by train, 27 m. 500 yd. by motor, and walks 3 m. 4 fur. 80 yd. What is the total length of his journey ?

41. What is the duration of night when the sun sets at 5.20 p.m. and rises at 6.10 the next morning ?

42. A stack of coal contains 50 tons. What weight is left when the following loads have been withdrawn : 5 tons 4 cwt. ; 3 tons 12 cwt. 2 qr. ; 12 tons 2 cwt. ; 7 tons 14 cwt. 1 qr.?

43. A road 432 yd. 2 ft. long has, on one side, a row of houses for 52 yd. 1 ft., a garden wall for 46 yd. 2 ft. 6 in., the entrance of two roads each 12 yd. wide, and the rest is fenced. Find the length of fence.

44. An experiment, timed to begin at 4 h. 32 min. on Tuesday afternoon, finished at 10 h. 6 min. 14 sec. on Wednesday morning. How long did it last ?

45. A merchant gains £4, 16s. 6d. by selling a truck of coal for twenty-five guineas. What must he sell it for to gain six guineas ?

When a high standard of accuracy has been obtained, rapidity may be acquired by taking together small numbers and by picking out near pairs which make 10.

10, 18, 28, 35 = 2s. 11d.
20, 28, 38, 48, 68,
78, 98, 105 = £5, 5s.
11, 21, 31, 41, etc.

```
       £     s.   d.
  32,647   16┐  1┐
 121,483    7│  7│
  19,225   14┘  9┘
 312,862   13┐  4
  91,513   17┘  1
   2,416   12   3
 143,524    6   3┐
  18,236   18   7┘
£741,911    5  11
```

In order to balance two columns it is not necessary to add the columns and then subtract. The method is indicated below.

	Dr.			*Cr.*		
	£	s.	d.	£	s.	d.
	15,413	12	8	14,216	13	2
	6,197	16	4	5,783	14	9
	2,138	18	2	615	2	7
	63,214	13	5	83,914	11	5
Balance				6,217	9	3
	£110,747	11	2	£110,747	11	2

Cast up the Cr. side and write the total beneath both sides.

Cast up the Dr. side.

5, 15, 19 = 1s. 7d. add **7d.** = 2s. 2d., put down **7**, carry 2s.

15, 25, 45, 61 = £3, 1s., add **10s.** = £3, 11s., put down **10s.**, carry £3.

The balance is £23,782, 10s. 7d.

EXERCISE III (e)

Cast up the following Accounts and find the balance.

1.

Dr.			*Cr.*		
£	s.	d.	£	s.	d.
314,218	3	7	512,618	11	2
73,529	12	8	23,426	6	5
212,172	15	3	312,128	19	8
6,148	11	9	1,816	13	7
2,376	19	7	2,512	11	6
614	13	6	28,614	14	3
Bal.			2,136	12	4

2.

Dr.			*Cr.*		
£	s.	d.	£	s.	d.
742,613	13	2	538,914	17	2
216,187	17	7	413,268	3	9
72,186	11	8	105,413	13	8
1,823	13	4	92,614	10	0
2,614	9	9	1,238	17	4
426	12	6	614	7	10
Bal.			2,635	13	6

3.

Dr.			*Cr.*		
£	s.	d.	£	s.	d.
518,719	4	10	612,834	8	7
63,925	16	8	73,547	11	5
9,263	8	10	123,017	0	4
21,486	17	4	21,632	13	5
247	13	7	528	18	6
1,368	17	6	2,735	4	8
236	12	3	326	11	4
Bal.			27	17	2

4.

Dr.			*Cr.*		
£	s.	d.	£	s.	d.
162,143	13	8	154,219	11	10
1,893	16	7	3,684	5	7
12,213	8	9	64,796	13	5
836	12	3	2,713	8	6
2,419	17	1	862	17	5
642	12	7	349	13	8
Bal.			126	19	2

5.	Dr.			Cr.		
	£	s.	d.	£	s.	d.
	26,742	17	9	31,284	11	6
	12,412	13	4	21,146	18	7
	2,016	0	0	1,467	13	4
	1,619	18	1	963	12	5
	684	19	4	382	5	6
	3,142	11	9	1,216	13	5
	864	13	6	723	14	4
Bal.				163	18	3

6.	Dr.			Cr.		
	£	s.	d.	£	s.	d.
	162,413	14	8	214,643	17	5
	86,936	15	3	74,218	16	8
	7,264	9	0	8,614	18	3
	6,279	16	7	4,963	11	2
	638	14	8	527	13	5
	263	17	9	863	14	8
	63	13	2	49	16	7
Bal.				83	12	5

Multiplication

Example.—Multiply £2, 13s. 7½d. by 5.

£	s.	d.
2	13	7½
		5
£13	8	1½

Example.—Multiply £1, 14s. 2d. by 27.

Since 27 = 3 × 9, we multiply by 3 and then multiply the result by 9.

Example.—Multiply £2, 12s. 4½d. by 127.

This is most rapidly done by Practice, but it can be performed by multiplying by 10, then again by 10, and adding in twice the product by 10 and 7 times the original amount.

£	s.	d.	
2	12	4½	×7
		10	
26	3	9	×2
		10	
261	17	6	
52	7	6	
18	6	7½	
£332	11	7½	

Multiplication by small numbers is conveniently done as follows :—

Example.—Multiply £124, 12s. 5½d. by 17.

£	s.	d.
124	12	5½
17	17	17
124	204	93½
868	↙	↙
2108		
10	4	
	7	9½
£2118	11	9½

EXERCISE III (f)

Multiply

1. £14, 3s. 7d. by 5, 7, 9, 12. **2.** £13, 18s. 4d. by 14, 21, 36.

3. £17, 5s. 9½d. by 6, 54, 100. **4.** £23, 13s. 10¼d. by 13, 17, 23.

5. £126, 11s. 8½d. by 19, 29, 31. **6.** 5 yd. 2 ft. 7 in. by 4, 9, 12.

7. 12 yd. 2 ft. 10 in. by 14, 21, 42. **8.** 161 yd. 1 ft. 8 in. by 17, 19, 31.

9. 2 tons 16 cwt. 1 qr. 14 lb. by 5, 7, 11.

10. 3 tons 14 cwt. 2 qr. 15 lb. by 16, 24, 32.

11. 14 tons 6 cwt. 1 qr. 12 lb. by 13, 17, 19.

12. 6 days 12 hr. 40 min. by 6, 14, 24.

13. 3 gal. 2 qt. 1 pt. by 8, 32, 45.

14. Find the total cost of 5 tons at £3, 14s. 2d. a ton and 12 cwt. at £1, 17s. 5d. a cwt.

15. Find the value of 12 bicycles each worth £12, 15s. 6d.

16. I give a cheque for £25 in payment of 2 watches each costing £3, 4s. 6d., 3 pins at 18s. 9d. each, and 5 rings at £2, 12s. 4d. each. What change do I receive?

17. Find the total weight of 6 boxes each 16 lb. 4 oz.; 4 cases each 1 cwt. 12 lb.; 12 parcels each 2 lb. 3 oz.

18. What should I pay for 18 lb. of tea at 2s. 2½d. per lb., 12 lb. of coffee at 10½d. per lb., and 14 lb. of cocoa at 1s. 10½d. per lb.?

19. A heap of coins contains 34 crowns, 30 florins, 210 shillings, and 315 pence. What is the total weight if 1 crown weighs 1 oz., 5 florins weigh 2 oz., 5 shillings weigh 1 oz., and 3 pennies weigh an ounce?

20. What is the cost of 17 cwt. at £1, 3s. 4d. per cwt.?

21. When coal costs £2, 13s. 4d. a ton, what will be charged for 12 tons if £1, 2s. 6d. is charged for freight?

22. A trolley load being 1 ton 12 cwt. 14 lb., what load can be carried by 14 of them?

23. If the price of wheat falls from £2, 7s. 4d. a sack to £2, 3s. 9d., what difference is there in the cost of 100 sacks?

24. What is the perimeter of a field which can be fenced by 246 hurdles each 2 ft. 10 in. long?

25. A milkman delivers 2 qt. 1 pt. at each of 4 houses, 3 qt. 2 pt. at each of 5, 1½ pt. at 24, 1 qt. 2 pt. at 16. What is the total amount delivered?

26. If a lorry runs 12 miles to a gallon of petrol, find the cost of a journey of 720 miles when petrol costs 1s. 10d. a gallon.

27. In the course of a year a man makes 284 journeys, the fare being 2s. 4d. What would he have saved by taking a season ticket costing £25?

28. How far does a bus travel in the day if it starts from the same place every hour from 7 a.m. till 10 p.m. inclusive and runs to and from a terminus 3 m. 6 fur. away ?

29. A person whose income is £640 a year spends on the average £1, 5s. 4d. a day. What does he save in a year of 365 days ?

30. In a motor-car test from Land's End to John o' Groat's a competitor left at 5.30 on Tuesday evening and arrived at 7.30 on Friday morning—he had 15 hours' rest altogether on the way. For how long was he driving ?

Division

Example.—Divide £2, 14*s.* 7*d.* by 14. Use factors if possible.

$$\begin{array}{r|rrr}
 & £ & \text{s.} & \text{d.} \\
2 & 2 & 14 & 7 \\ \hline
7 & 1 & 7 & 3 \\ \hline
 & & 3 & 10
\end{array}
\begin{array}{l}
\\ \\ \left.\begin{array}{l}\text{rem. } 1\\ \text{rem. } 5\end{array}\right\} 11
\end{array}$$

Answer, 3s. 10d. and 11d. over.

Example.—*Divide* £13, 6*s*. 2*d*. *by* 113.

$$\begin{array}{r|rr}
 & £ \quad \text{s.} & \text{d.} \\
 & \mathbf{2} & \mathbf{4} \\ \hline
113 & 13 \quad 6 & 2 \\
 & 266 & 482 \\
 & \underline{226} & \underline{452} \\
 & \text{s. } 40 & \text{d. } 30
\end{array}$$

Answer, 2s. 4d. and 30d. over.

Example.—*Divide* 14 *cwt.* 1 *qr.* 16 *lb. by* 34.

14 cwt. 1 qr. = 57 qr.
23 qr. 16 lb. = 660 lb

$$\begin{array}{r}
23 \\
\underline{4} \\
92 \\
\underline{7} \\
\underline{644}
\end{array}
\qquad\qquad
\begin{array}{r|rrr}
 & \text{cwt.} & \text{qr.} & \text{lb.} \\
 & & \mathbf{1} & \mathbf{19} \\ \hline
34 & 14 & 1 & 16 \\
 & & 57 & 660 \\
 & & \underline{34} & \underline{34} \\
 & & \text{qr. } 23 & 320 \\
 & & & \underline{306} \\
 & & \text{lb.} & \underline{14}
\end{array}$$

Answer, 1 qr. 19 lb. and 14 lb. over.

EXERCISE III (g)

Find the quotient and remainder in the following :
Divide

1. £26, 12s. 6d. by 6, 10, 12, 27. 2. £75, 14s. 8d. by 8, 32, 35.
3. £214, 6s. 11d. by 14, 36, 42. 4. £613, 14s. 8d. by 17, 31, 47.
5. £346, 12s. 5d. by 50, 113, 321.
6. 5 m. 3 fur. 11 yd. by 3, 7, 9, 12.

7. 15 m. 3 fur. 64 yd. by 24, 36, 42.

8. 26 m. 640 yd. by 97, 124.

9. 5 yd. 3 ft. 9 in. by 17, 23, 37.

10. 2 tons 13 cwt. 2 qr. 12 lb. by 4, 8, 14, 20.

11. 12 tons 6 cwt. 1 qr. 16 lb. by 24, 42, 45.

12. 14 tons 4 cwt. 36 lb. by 13, 17, 19.

13. Find the values of (i) $\frac{1}{5}$ of £13, 10s.; (ii) $\frac{1}{6}$ of £14, 12s. 6d.; iii) $\frac{1}{20}$ of £620, 10s.; (iv) $\frac{1}{12}$ of 2 tons 4 cwt. 4 lb.

14. A man buys 56 yd. of cloth for £14, 2s. 4d. How much per ard did he pay ?

15. A clerk receives £180 a year. How much is that per day to he nearest penny ?

16. If 142 tons of coal cost £259, 1s., what is that per ton ?

17. Baggage weighing 12 cwt. 16 lb. is divided up among 34 porters. ow much did each carry ?

18. A street 120 yd. 4 ft. long has a terrace of 26 houses on one ide. What is the average frontage for each house ?

19. 1 cwt. of sugar is made up into 7-lb. packets. How many will here be ?

20. A distance of 5 m. 80 yd. is spaced out with posts 60 yd. part. How many posts are required ?

21. How much is left over after dividing £5 among 7 people if no ne receives a coin smaller than 1d.?

22. A road has an average gradient of 1 in 20 (it rises 1 ft. in every 0 ft.). How many feet does it rise in 5 miles ?

23. An incline whose gradient is $\frac{1}{15}$ rises in all 48 ft. What is the ength of the incline ?

24. A tradesman spends £20,643 a year. How much is that per ay on the average to nearest shilling ?

25. I buy Osram lamps at 3s. 6d. each, and pay for them with a 5 note. What is the greatest number I can get, and what change hould I receive ?

26. How many articles, each worth 2s. 4d., can be bought for £4, 18s.?

27. How many rails 42 ft. long are required for a single-track ailway line 21 miles in length ?

28. Thirty-three telegraph posts, equally spaced out, cover a distance of 1 mile. Find their distance apart.

29. For a payment of 2s. $3\frac{1}{2}$d. a parcel sent abroad may be insured or £120. What is the least number of such parcels that must be nsured so that the Post Office may not lose if one be lost ?

30. How many bicycles can I buy for £100 if each costs £9, 12s.? By how much must the price be reduced if I can get one more for £100, the price being an exact number of shillings?

Averages

If the sum of a number of quantities is divided by the number, the quotient is called the **average** of the given quantities.

Example.—A cricketer makes the following scores (being out each time) 17, 5, 9, 23, 31, 16, 29, 45, 0, 11, 33, 9. *What is his average?*

He has played 12 complete innings, and the total number of runs he has scored is

$$17+5+9+23+31+16+29+45+0+11+33+9=228.$$

$$\therefore \text{ his average} = \tfrac{228}{12} = \underline{\mathbf{19.}}$$

Note.—(i) The average of any set of numbers must lie between the greatest and least number of the set.

(ii) If numbers are involved which do not differ much amongst themselves, it often saves time to guess an average and then calculate the average error, as in the next Example. But it is of course unnecessary to set out the work so fully as is shown there.

Example.—The ages of the boys in a class are respectively 14 *yr.* 7 *mo.*, 15 *yr.* 2 *mo.*, 15 *yr.* 5 *mo.*, 14 *yr.* 9 *mo.*, 14 *yr.* 3 *mo.*, 15 *yr.* 1 *mo.*, 15 *yr.* 2 *mo.*, 14 *yr.* 9 *mo.*, 14 *yr.* 7 *mo.*, 14 *yr.* 0 *mo.*, 15 *yr.* 5 *mo.* *What is the average age of the class?*

There are 11 boys, and it is clear that the average age is about 15 years.

Tabulate the data as follows:

Age.		*Difference from 15 Years.*	
Years.	*Months.*	*Months.*	*Months.*
14	7		− 5
15	2	+ 2	
15	5	+ 5	
14	9		− 3
14	3		− 9
15	1	+ 1	
15	2	+ 2	
14	9		− 3
14	7		− 5
14	0		−12
15	5	+ 5	
	Total	+15	−37

∴ total difference
$=15-37=-22$ months.
∴ average difference
$=-\frac{22}{11}=-2$ months.
∴ average age
$=15$ years -2 months.
$=$ **14 years 10 months.**

Example.—A man's hotel bill is 15 shillings a day for the first five days and 12 shillings a day for the next seven days. What is his average daily bill?

$$\begin{aligned} &5 \text{ days at 15s. a day costs } 5\times15 &&=75\text{s.}\\ &7 \text{ days at 12s. a day costs } 7\times12 &&=84\text{s.}\\ &\therefore \text{ the total bill for 12 days is } 75+84 &&=159\text{s.}\\ &\qquad\therefore \text{ average daily bill} = \tfrac{159}{12}\text{s.} &&=\underline{\underline{13\text{s. }3\text{d.}}} \end{aligned}$$

EXERCISE III (h)

1. Find the average of the following numbers: (i) 63, 104, 91, 82; (ii) 162, 16, 203, 71, 149, 83.

2. A man on a week's tour walks the following distances on successive days: 15, 22, 19, 26, 9, 19, 23 miles. What is his average daily walk?

3. A shopkeeper's weekly profits in September are £12, 14s. 4d., £15, 7s. 3d., £11, 8s. 7d., £10, 3s. 10d. What is his average weekly profit for the month?

4. Six boys estimate the length of a field to be respectively 210, 219, 203, 196, 217, 209 yards. What is the average of their estimates?

5. The weights of a crew of an eight are respectively 11 st. 7 lb., 11 st. 12 lb., 12 st. 5 lb., 13 st. 1 lb., 12 st. 9 lb., 12 st. 8 lb., 12 st. 2 lb., 11 st. 8 lb. What is the average weight per man?

6. There are five houses in a row; each end house is rented at £45, and the others at £35. What is the average rent per house?

7. A cricketer has scored 18, 33, 27, 14, 19 in five successive completed innings. How much must he score in the next innings (if he gets out) to make his average 25?

8. The average age of four men is 51, and the average age of three f them is 47. What is the age of the other man?

9. A man walks on an average 19 miles a day for the first eleven ays of a holiday and walks 260 miles altogether in the first fourteen ays. What was his daily average for the last three days?

10. The ages of five brothers on May 1, 1924, were 18 yr. 3 mo., 7 yr. 1 mo., 15 yr. 8 mo., 13 yr. 5 mo., 11 yr. 10 mo. What was their verage on December 1, 1923?

11. A man travels for 2 hours at 15 m.p.h., then for 3 hours at 21 m.p.h., then for 1 hour at 27 m.p.h. What is his average speed?

12. A man travels 24 miles at 6 m.p.h., then 24 miles at 8 m.p.h., hen 24 miles at 12 m.p.h. What is his average speed for the whole journey?

13. A grocer mixes 4 lb. of tea at 2s. 4½d. per lb. with 5 lb. at 2s. 9d. per lb. and with 9 lb. at 3s. 4d. per lb. What is the mixture worth per lb.?

14. A team of six enter for a gymnastic competition. The best gymnast scores 57 points. If he had scored 65 points the average score per man would have been 49 points. How many points did the team obtain between them ?

15. The heights of a number of boys are measured and tabulated as follows :

Height in cm. .	160	161	162	163	164	165	166	167	168	169
Number of boys	1	7	11	17	28	33	30	24	14	2

What is the average height ?

16. A man wishes to estimate the number of words in an article. It is spread over 17 pages of manuscript, and each page contains 22 lines. He counts the number of words in 8 lines at random, and finds there are 12, 7, 14, 9, 5, 10, 11, 12 words. What is roughly the number of words in the article ?

17. In 1905 there were 140,000 separate accounts in the Post Office Savings Bank, and the total sum deposited was £17,880,000. What is the average size of an account, neglecting fractions of a shilling ?

18. The average annual profit from a business for the last three years is £864. For the first of these years there was a loss of £328. What was the average annual profit for the last two years ?

19. What is the average of all the whole numbers *between* 20 and 50 ? What is their sum ?

20. During a year a hotel manager buys 30 tons of coal at 28 shillings per ton, 20 tons at 35 shillings per ton, and 20 tons at 42 shillings per ton. What is the average price per ton for the year ?

21. What is the average cost in pence of x articles at 2 shillings each and $2x$ articles at half a crown each ?

22. For his first x complete innings a cricketer has an average of p runs. In his next innings he makes 0. What does his average become ?

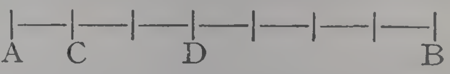
23. What is the average weight of a number of parcels if they consist of x parcels each weighing p lb. and y parcels each weighing q lb.?

24. A and B have each taken 8 wickets for 28. A then takes 1 for 35 and B 4 for 56. Which has then the better average ?

CHAPTER IV

FRACTIONAL PARTS AND PRACTICE

A whole number is called an **Integer** (Lat. *integer* = whole). A part of a whole is called a **Fraction** (Lat. *fractus* = broken). For instance, a halfpenny is $\frac{1}{2}$ penny. $\frac{1}{2}$ is a fraction. If the line AB is divided into 7 equal parts, each part, such as AC, is called $\frac{1}{7}$ (one-seventh) of the line AB. The number 7 is called the **Denominator** (Lat. *nomen* = a name), since it gives the name of the fraction.



If we take three of these divisions, such as AD, then AD is called $\frac{3}{7}$ (three-sevenths) of the line AB. 3 is the **Numerator** (Lat. *numerus* = number) since it states the number of the sevenths which have been taken.

EXERCISE IV (a) (Oral)

1. How many inches in $\frac{1}{12}$ ft., $\frac{1}{3}$ ft., $\frac{1}{4}$ ft., $\frac{1}{6}$ ft., $\frac{2}{3}$ ft., $\frac{3}{4}$ ft., $\frac{5}{6}$ ft.?

2. How many pence in $\frac{1}{12}$s., $\frac{1}{6}$s., $\frac{1}{4}$s., $\frac{1}{3}$s., $\frac{2}{3}$s., $\frac{3}{4}$s.?

3. How many shillings in $\frac{1}{4}$£, $\frac{1}{5}$£, $\frac{1}{8}$£, $\frac{3}{8}$£ ?

4. How many yards in $\frac{1}{2}$ mile, $\frac{1}{4}$ mile, $\frac{1}{8}$ mile, $\frac{1}{2}$ chain ?

5. How many lb. in $\frac{1}{2}$ cwt., $\frac{1}{2}$ ton, $\frac{1}{8}$ ton, $\frac{3}{4}$ ton ?

6. What fraction is 4d. of 1s. ; 8d. of 1s. ; 2s. 6d. of £1 ; 4s. of £1 ; 5s. of 15s. ; 1d. of £1 ; 10s. of £5 ; 10 sec. of 1 min. ; 7 in. of 1 yd. ; 2 ft. of 5 yd. ; 3 miles of 5 miles ; 6 in. of 2 ft. ; 1 sec. of 1 min.?

7. What is the value of $\frac{2}{3}$ of 1s. ; $\frac{3}{4}$ of £1 ; $\frac{3}{8}$ of £1 ; $\frac{3}{4}$ of 1 lb.; $\frac{2}{5}$ of 1 ton ; $\frac{3}{4}$ of 1 hr. ; $\frac{5}{6}$ of £3 ; $\frac{3}{4}$ of £5 ; $\frac{2}{3}$ of 2 hr. ; $\frac{5}{12}$ of 1 yd.?

Example.—Find the value of $\frac{7}{8}$ *of £2, 16s.*

$\frac{1}{8}$ of £2, 16s. = $\frac{1}{8}$ of 56s. = 7s.

∴ $\frac{7}{8}$ of £2, 16s. = 49s. = **£2, 9s.**

EXERCISE IV (b)

Find the value of **1.** $\frac{2}{3}$ of £1, 1s. **2.** $\frac{5}{6}$ of £3, 12s. **3.** $\frac{7}{8}$ of £6.

4. $\frac{7}{12}$ of £1, 16s. **5.** $\frac{5}{9}$ of 1 yd. **6.** $\frac{2}{3}$ of 15s. 6d.

7. $\frac{3}{4}$ of 1 m. (in yd.). **8.** $\frac{4}{7}$ of 3 yd. 1 ft. 6 in. **9.** $\frac{4}{9}$ of 1 gal. 1 pt.

10. $\frac{5}{8}$ of 2 qr. 11 lb. 8 oz.

What fraction of (11) 10 is 9 ? (12) 12 is 8 ? (13) 15 is 7 ?

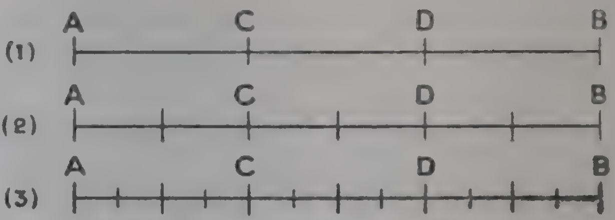
Simplification of Fractions

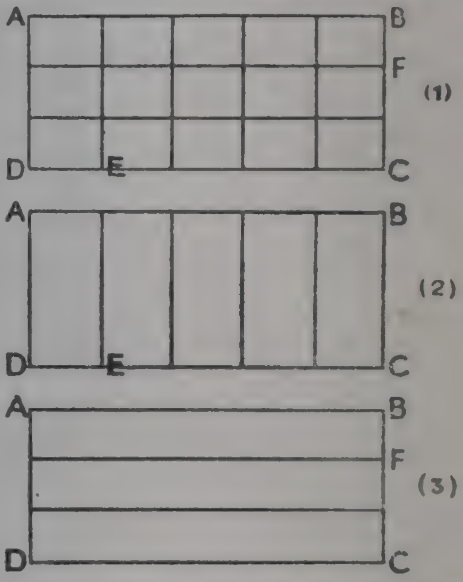
Since 6d. = $\frac{1}{40}$ of £1, ∴ 20 sixpences = £$\frac{20}{40}$.
1s. = $\frac{1}{20}$ of £1, ∴ 10 shillings = £$\frac{10}{20}$.
1 florin = $\frac{1}{10}$ of £1, ∴ 5 florins = £$\frac{5}{10}$.
But 20 sixpences = 10 shillings = 5 florins = £$\frac{1}{2}$.

$$\therefore \frac{20}{40} = \frac{10}{20} = \frac{5}{10} = \frac{1}{2}.$$

The line AB in (1) is divided into thirds, in (2) into sixths, and in (3) into twelfths.

The part AC in (i) is $\frac{1}{3}$ of AB, in (ii) is $\frac{2}{6}$ of AB, in (iii) is $\frac{4}{12}$ of AB. $\therefore \frac{1}{3}=\frac{2}{6}=\frac{4}{12}$.





The rectangle ABCD is divided in (1) into 15 equal parts, in (2) into 5 equal parts, in (3) into 3 equal parts.

What fractions can be shown equal by comparing AE in (1) and (2), and also by comparing AF in (1) and (3)?

When a line is divided into 12 equal parts and we take four of them, we are taking $\frac{4}{12}$ of the line.

If the line were only divided into 3 equal parts, each part would be 4 times as large as before, so that we should only need to take one of them to get the same length as by taking four of the twelfths, *i.e.* $\frac{4}{12}=\frac{1}{3}$.

The simplification of $\frac{4}{12}$ to the form $\frac{1}{3}$ is called "**reducing the fraction to its lowest terms.**"

When both numerator and denominator have a common factor, we may divide them both by this factor without altering the value of the fraction thus: $\frac{4}{12}=\frac{\cancel{4}\times 1}{\cancel{4}\times 3}=\frac{1}{3}$.

The fraction will not be in its *lowest* terms unless we divide by the *highest* common factor thus: $\frac{20}{40}=\frac{2\times 10}{2\times 20}=\frac{10}{20}$. But $\frac{20}{40}=\frac{20\times 1}{20\times 2}=\frac{1}{2}$.

It is also clear that we do not alter the value of a fraction by multiplying both numerator and denominator by the same number, for $\frac{1}{2}=\frac{10}{20}=\frac{20}{40}$.

EXERCISE IV (c)

Reduce to lowest terms: 1. $\frac{3}{6}$. 2. $\frac{5}{10}$. 3. $\frac{10}{25}$. 4. $\frac{12}{14}$. 5. $\frac{18}{24}$. 6. $\frac{12}{15}$. 7. $\frac{121}{132}$. 8. $\frac{169}{182}$. 9. $\frac{144}{180}$. 10. $\frac{242}{385}$.

Find x if—11. $\frac{2}{3}=\frac{x}{6}$. 12. $\frac{x}{4}=\frac{1}{2}$. 13. $\frac{5}{12}=\frac{x}{60}$. 14. $\frac{7}{15}=\frac{21}{x}$. 15. $\frac{6}{13}=\frac{78}{x}$. 16. $\frac{x}{15}=\frac{44}{165}$.

17. (i) Express the following fractions with 12 as their denominator and then arrange them in descending order of magnitude : $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{2}$, $\frac{2}{3}$.

(ii) Treat in a similar way:
(48 as denominator) $\frac{3}{8}$, $\frac{5}{12}$, $\frac{7}{16}$, $\frac{11}{24}$.

(iii) (60 as denominator) $\frac{5}{12}$, $\frac{13}{30}$, $\frac{7}{15}$, $\frac{17}{20}$.

18. Reduce to their lowest terms:

(1) $\frac{xa}{xb}$. (2) $\frac{5ab}{10a^2}$. (3) $\frac{a^2b}{ab^2}$. (4) $\frac{6a^2b}{12ab^3}$. (5) $\frac{12a^2x^3}{18ax^4}$.

19. Write the following fractions with $6x^2y^2$ as their denominator:

$$\frac{3}{xy}, \frac{1}{2x^2y}, \frac{xy}{6x^2}, \frac{y^2}{6xy^2}.$$

20. Write with $12\,a^2b^2c^2$ as their denominator: $\frac{3}{4ab^2}$, $\frac{4a}{3b^2c}$, $\frac{b}{6a^2c^2}$.

Simple Practice

EXERCISE IV (d) (Oral)

1. What fraction of £1 is 1s., 2s., 2s. 6d., 5s., 6s. 8d., 7s. 6d., 13s. 4d., 15s., 4s. ?

2. What fraction of 1s. is 6d., 4d., 3d., 2d., $1\frac{1}{2}$d.?

3. What fraction of 2s. 6d. is 3d., 6d., 5d., $2\frac{1}{2}$d.?

4. What fraction of 4s. is 6d., 8d., 1s. 4d., 4d.?

Example.—Find the cost of $168\frac{1}{2}$ tons at £2, 15s. 7d. per ton.
Rough check 160 tons at £3 = £480.

	£	s.	d.			
	168	10	0	cost @	£1 per ton.	
	337	0	0	,,	£2	,,
10s. = $\frac{1}{2}$ of £1.	84	5	0	,,	10s.	,,
5s. = $\frac{1}{2}$,, 10s.	42	2	6	,,	5s.	,,
6d. = $\frac{1}{10}$,, 5s.	4	4	3	,,	6d.	,,
1d. = $\frac{1}{6}$,, 6d.		14	$0\frac{1}{2}$	,,	1d.	,,
	£468	5	$9\frac{1}{2}$			

Note.—The working on the left-hand side explains the reasoning; it may be omitted as soon as the process is understood.

Example.—Find the cost of 126 *yd. at* £1, 14*s.* 2*d. per yard.*

Rough check 130 yd. at £1, 10s. = £195.

	£	s.	d.		
	126	0	0	cost @	£1.
10s. = $\frac{1}{2}$.	63	0	0	,,	10s.
4s. = $\frac{1}{5}$ of £1.	25	4	0	,,	4s. (£6, 6s. = cost @ 1s.).
2d. = $\frac{1}{6}$ of 1s.	1	1	0	,,	2d.
	£215	5	0		

Example.—Find the weight of 362 *cases each weighing* 8 *cwt* 2 *qr.* 12 *lb.*

Rough check 360 times 8 cwt. = 2880 cwt.

	cwt.	qr.	lb.		
	362	0	0	wt. @	1 cwt. each.
	2896	0	0	,,	8 cwt. ,,
2 qr. = $\frac{1}{2}$ of 1 cwt.	181	0	0	,,	2 qr. ,,
8 lb. = $\frac{1}{7}$,, 2 qr.	25	3	12	,,	8 lb. ,,
4 lb. = $\frac{1}{2}$,, 8 lb.	12	3	20	,,	4 lb. ,,
	3115	3	4		

155 tons 15 cwt. 3 qr. 4 lb.

Note the method used in the following example. Find the cost of 168 *articles at* £3, 18*s.* 4*d. each.*

Rough check 168 at £4 = £672.

	£		
	168	cost @	£1.
	672	,,	£4.
1s. 8d. = $\frac{1}{12}$ of £1.	14	,,	1s. 8d.
	£658	(by subtraction).	

EXERCISE IV (e)

Find by Practice the cost of

1. 46 articles at £2, 15s. each.
2. 124 at £3, 6s. each.
3. 276 at £1, 7s. 6d. each.
4. 88 at £5, 14s. 3d. each.
5. 220 at £7, 13s. 8d. each.
6. 217 at £1, 15s. 3d. each.
7. 348 at £2, 18s. 9d. each.
8. 165 at £2, 15s. 4½d. each.
9. 73 at £4, 17s. 8½d. each.
10. A gross at 11s. 7½d. each.
11. 159 at £3, 8s. 10½d. each.
12. 113 at £1, 12s. 2¼d. each.

Find by Practice the weight of

13. 426 cars at 1 ton 4 cwt. 1 qr. 4 lb. each.
14. 512 loads at 12 cwt. 2 qr. 21 lb. each.
15. 127 trucks at 3 tons 2 cwt. 16 lb. each.

Find by Practice the tax on

16. £624 at 2s. 4d. in the £.
17. £1226 at 4s. 9d. in the £.
18. £358 at 2s. 10d. in the £.

Find by Practice the dividend paid on

19. £648, 10s. at 9s. 6d. in the £.
20. £1050, 5s. at 12s. 6d. in the £.
21. £236, 10s. at 3s. 10d. in the £.

Find the cost of

22.. 134 at 19s. $8\frac{1}{2}$d.
23. 136 at £1, 7s. 11d.
24. 428 at 2s. $10\frac{1}{2}$d.
25. 164 at 9s. $11\frac{1}{2}$d.
26. 172 at £2, 18s. 4d.
27. 92 at 16s. 8d.

Compound Practice

When both the given quantities are compound quantities it is usually best to employ the method given below, which is called **Compound Practice.**

Example.—Find the cost of $3\frac{1}{2}$ yd. at 2s. $4\frac{1}{2}$d. per yard.

s.	d.		
2	$4\frac{1}{2}$	cost of 1 yd.	
7	$1\frac{1}{2}$	,,	3 ,,
1	$2\frac{1}{4}$	,,	$\frac{1}{2}$,,
8	$3\frac{3}{4}$		

Example.—Find the cost of 3 cwt. 1 qr. 18 lb. at £23, 16s. per cwt.

	£	s.	d.		
	23	16	0	cost of 1 cwt.	
	71	8	0	,,	3 ,,
1 qr. = $\frac{1}{4}$ of 1 cwt.	5	19	0	,,	1 qr.
14 lb. = $\frac{1}{2}$ of 1 qr.	2	19	6	,,	14 lb.
4 ,, = $\frac{1}{7}$ of 1 qr. line	0	17	0	,,	4 ,,
	£81	3	6		

N.B.—Practice should be given in treating rapidly by this method such questions as $5\frac{1}{2}$ yd. at $2\frac{1}{2}$d.; $3\frac{1}{2}$ yd. at $4\frac{1}{2}$d., etc.

EXERCISE IV (f)

Find by Practice the cost of

1. $5\frac{1}{2}$ yd. at. 3s. 7d. a yard.
2. $3\frac{2}{3}$ yd. at 2s. $10\frac{1}{2}$d. a yard.
3. $7\frac{3}{4}$ yd. at 5s. 10d. a yard.
4. $3\frac{1}{4}$ yd. at £1, 11s. a yard.
5. $8\frac{2}{5}$ yd. at 13s. 9d. a yard.
6. $5\frac{1}{2}$ yd. at 2s. 10d. a foot.
7. 7 cwt. 2 qr. 21 lb. at £8, 12s. 8d. per cwt.
8. 9 cwt. 1 qr. 16 lb. at £7, 4s. 8d. per cwt.

9. 2 tons 12 cwt. 2 qr. 14 lb. at £76, 13s. 4d. per ton.

10. 8 yd. 2 ft. 8 in. at 19s. 6d. per yard.

11. 14 yd. 2 ft. 7 in. at £4, 3s. 3d. per yard.

12. 7 yd. 2 ft. 10½ in. at 16s. 6d. per yard.

13. 3 ac. 2 ro. 10 per. at £264, 10s. per acre. (See p. xi for Table.)

14. 45 ac. 2 ro. 16 per. at £146, 15s. per acre.

15. 18 gal. 2 qt. 1½ pt. at 14s. 8d. per gallon. (See p. xii for Table.)

16. Find the tithe rent on 30 ac. 1 ro. 12 per. at 3s. 4d. per acre.

17. A coal shipper contracted to deliver 4530 tons of coal abroad at £1, 16s. 8d. per ton. The cost was £1, 1s. 7d. per ton, and the freight 11s. 4½d. per ton. What profit did he make?

18. Find the value of a casting weighing 24 tons 16 cwt. 2 qr. at £5, 11s. 8d. per ton.

19. Loads of stone each weighing 15 cwt., and sufficient for repairing 40 ft. of roadway, are dumped along the side of a road. What weight of stone is required to repair 6 fur. 120 yd. of road?

20. Find the income tax to be paid on £235 at 2s. 9d. in the £, except for £120 on which no tax is paid.

21. Find the cost of 3127 tons of coke at £1, 12s. 10½d. per ton.

22. How far will a train go in 3 hr. 52½ min. at 48½ miles per hour?

23. A mine yields 26,438 oz. of gold per year. What is its value at £4, 11s. 9½d. per oz.?

24. The rateable value of a town is £264,328. How much will a rate of 3s. 7½d. in the £ produce?

25. A man owing £436, 12s. 6d. paid only 9s. 4d. in the £. How much were his assets?

Rapid Methods of Calculation

Example.—To find the cost of 12 articles.

The price of each in pence = cost of 12 in shillings.

thus

Price of each	4d.	6½d.	1s. 4½d. (16½d.)
Cost of 12	4s.	6s. 6d.	16s. 6d.

Example.—Find the cost of 47 articles at 1s. 7½d. each (19½d.).

Cost of 4 doz. − 1 = 4 (19s. 6d.) − 1s. 7½d. = £3, 16s. 4½d.

Example.—To find the cost of 20 articles.

The price of each in shillings = cost of 20 in £.

Price of each	3s.	4s. 3d. (4¼s.)	£1, 6s. 4d. (26⅓s.)
Cost of 20	£3	£4, 5s.	£26, 6s. 8d.

Example.—Find the cost of 138 *articles at* 2*s.* 4*d. each.*

Cost of 140 − 2 = 7 (£2, 6s. 8d.) − 4s. 8d. = **£16, 2s.**

Invoices

When goods are bought, the seller sends with them a statement of the goods with their cost, so that they may be checked by the purchaser.

120 Eastcheap, London, E.C.

Mr A. Scholar. *May* 4, 1922.

To The General Trading Company.

	£	s.	d.
54 yd. @ 3s. 7d. per yd.	9	13	6
46 ,, @ 2s. 4½d. ,,	5	9	3
62 ,, @ 1s. 5d. ,,	4	7	10
32 ,, @ 3s. 4d. ,,	5	6	8
	£24	17	3

Working—

1. 4 doz. + ½ doz. @ 43d. = £8, 12s. + £1, 1s. 6d. = £9, 13s. 6d.
2. 4 ,, − 2 @ 28½d. = £5, 14s. − 4s. 9d. = £5, 9s. 3d.
3. 5 ,, + 2 @ 17d. = £4, 5s. + 2s. 10d. = £4, 7s. 10d.
4. 32 @ 3s. = £4, 16s., 32 @ 4d. = 10s. 8d., = £5, 6s. 8d.

EXERCISE IV (g)

Make out the following invoices :—

1. 36 yd. @ 3s. 9d. per yd.
14 ,, @ 4s. 6½d. ,,
25 ,, @ 5s. 3d. ,,
18 ,, @ 4s. 9d. ,,

2. 136 yd. @ 4s. 3d. per yd.
212 ,, @ 2s. 8d. ,,
140 ,, @ 2s. 4d. ,,
162 ,, @ 4s. 6d. ,,

3. 12½ lb. @ 4½d. per lb.
62 ,, @ 8½d. ,,
35 ,, @ 6½d. ,,
21 ,, @ 9d. ,,

4. 18 lb. @ 9½d. per lb.
27 ,, @ 10¾d. ,,
24 ,, @ 1s. 3½d. ,,
17½ ,, @ 8½d. ,,

5. 64 cycles @ £12, 10s. 0d. each.
50 ,, @ £10, 17s. 6d. ,,
46 ,, @ £8, 14s. 9d. ,,

6. 14 t. 2 cwt. @ £2, 4s. 2d. per ton.
8 t. 3 cwt. @ £1, 18s. 4d. ,,
5 t. 2 cwt. 1 qr. @ £3, 6s. 8d. ,,

7. 3 lb. coffee @ 1s. 7d. per lb.
4½ ,, tea @ 2s. 3½d. ,,
21 ,, sugar @ 8½d. ,,
9 ,, currants @ 7½d. ,,

8. 24½ yd. @ 2s. 11d. per yd.
17 ,, @ 2s. 4¾d. ,,
13 pairs @ 4s. 8d. per pair.
35½ yd. @ 3s. 11d. per yd.

9. 78 yd. @ 3s. 9½d. per yd.
50 ,, @ 4s. 2½d. ,,
15 ,, @ 5s. 7d. ,,
81 ,, @ 5s. 4¾d. ,,

10. 27 lb. @ 1s. 3d. per lb.
35 ,, @ 2s. 8d. ,,
74 ,, @ 3s. 4d. ,,
52 ,, @ 2s. 10d. ,,

11. 6½ lb. @ 1s. 2d. per lb.
7 tins @ 9½d. per tin.
2 doz. eggs @ 1s. 10d. per doz.
12½ lb. @ 4½d. per lb.
7 ,, @ 2s. 9d. ,,

12. 5½ lb. @ 1s. 3½d. per lb.
7¼ ,, @ 2s. 5d. ,,
4¾ ,, @ 1s. 9d. ,,
6¼ ,, @ 2s. 1d. ,,
15 ,, @ 7½d. ,,

13. 4½ yd. @ 4½d. per yd.
7½ ,, @ 1s. 0½d. ,,
15 ,, @ 1s. 2½d. ,,
13 ,, @ 11½d. ,,
17 ,, @ 1s. 10d. ,,

14. 420 yd. @ 4s. 3d. per yd.
140 ,, @ 3s. 9d. ,,
210 ,, @ 5s. 3d. ,,
160 ,, @ 4s. 8d. ,,
180 ,, @ 2s. 10d. ,,

MISCELLANEOUS EXAMPLES ON THE COMPOUND RULES

EXERCISE M (II)

1. How many payments of £5 each would be required to pay off a debt of £125 ?

2. How many half-crowns are there in £1, 2s. 6d. and in £23, 12s. 6d.? How many payments of £1, 2s. 6d. would be required to wipe off a debt of £23, 12s. 6d.?

3. A train travels at 60 miles an hour. How many feet does it travel in (1) 1 hr., (2) 1 sec.?

4. Mt. Everest is 29,000 ft. high. Express this in miles and yards.

5. A train is made up of an engine weighing 56½ tons, a tender weighing 26 tons, 6 carriages each weighing 9 tons 2 cwt., and a brake van of 8 tons 3 cwt. What is the total weight of the train ?

6. Find the total cost of 16 lb. of tea at 2s. 4½d. a lb. ; 24 lb. of coffee at 1s. 11½d. a lb. ; 17 lb. of cocoa at 2s. 4½d. a lb.

7. How far apart to the nearest furlong are two places which on a map of scale $\frac{1}{50000}$ are 3½ in. from one another ?

8. Two trees are 178 chains apart. What is this in miles and yards ?

9. Find the cost of 264 tons at £34, 17s. 4½d. a ton.

10. A man earns 7s. 8d. a day, and his son 5s. 4d. What is their combined wage for a year of 306 working days ?

11. A man has an income of £1261 a year. How much is that per week ?

12. A book of stamps contains 6 at ½d., 6 at 1d., and the rest at 1½d. It costs 3s. How many 1½d. stamps are there ?

13. My railway ticket costs 2s. 3d. a day for 5 days a week except 3 weeks in the year. How much should I save by taking a season for £18, 10s.?

14. What is the total length of 340 rails, each 2 yd. 2 ft. 8 in. long ?

15. One cart is loaded with 7 cwt. 2 qr. 26 lb. and another with 3 cwt. 1 qr. 4 lb. How much should be transferred from the first to the second to make the loads equal ?

16. One boy has £42, 12s. in the bank and his elder brother has twice as much. How much must be transferred from the younger's account to let the elder have three times as much as his brother ?

17. A wheel makes 350 revolutions when rolling 1 mile. What is its circumference, to the nearest inch ?

18. The heights of A, B, C above sea-level are : A, 150 ft. ; B, 562 ft. ; C, 724 ft. If from A to B is 1½ miles, and from B to C 6 fur. 180 yd., find the gradient from A to B and from B to C (say 1 in x). Which is the steeper ascent ?

19. The rates of a town are calculated on a sum of £234,612 (its rateable value). How much will a rate of 3s. 6½d. in the £ produce ?

20. A clock loses 5 seconds in every 45 minutes. If it is 12 minutes fast at mid-day on Monday, when will it be correct ?

21. A bucket holds 20 lb. of coal. How often can it be filled from a stack of 1½ ton ?

22. How many times will a cricket ball turn round in rolling 22 yd. if its circumference is 9 in.?

23. A man drinks a half-pint glass of cider three times a day. How long will a barrel holding 6 gallons last him ?

24. A man smoked 4 oz. of tobacco a week when the cost was 6d. an oz. If the price goes up to 10s. 8d. a lb., how much may he smoke per week without increasing his expenditure ?

25. Find the cost of 126 pheasants at 11s. 6d. a brace.

26. When the rouble was worth 6½d., Lenin received a grant of 20 million roubles to reorganise the Russian army. Evaluate this n £, s. d.

27. A sum of £64, 2s. 6d. is raised to be distributed among 29 families. What will each receive, excluding sums less than 1d., and how much will be left?

28. It was stated in 1918 that Red Cross work cost £60,000 a week, and that £8,123,782, 6s. 10½d. had been collected. How many weeks' work does this represent, to the nearest week?

29. The war indemnity demanded from France in 1871 was 5000 million francs. What is this in English money if a franc was then worth 9⅗ pence?

30. The commander of a cadet corps sells 120 belts which originally cost 7s. 6d. each, and 200 haversacks which cost 3s. 6d. He sells each haversack for 1s. 6d. What price must the belts fetch if the total price is half the original cost?

31. A workman is paid 1s. 10d. per hour. He works 9 hours on 5 days of the week and 6 hours on Saturday. How much does he earn per week? If by working 7 hours' overtime he makes his weekly earnings up to £5, 9s. 3d., at what rate per hour is he paid for overtime?

32. On a certain telegraph line the wire used weighed 450 lb. per mile. If 3 tons 2 cwt. of wire were used, what was the length of the line in miles and yards to the nearest 100 yd.?

33. How many steps of 1 yd. must a man take in a minute to walk at the rate of 4½ miles per hour?

34. A length intended to be 75 yd. was measured with a rod which was supposed to be 2 ft. 6 in. long, but was afterwards found to be ¾ in. short of that. What was the length marked out?

35. It is stated that the labour required to keep tidy the Royal parks in London costs £400 a month. If 32 men are employed on the work, find what each is paid per day, taking a month as 30 days?

36. The import duty on coffee is £1, 8s. per cwt., on tea is 8d. per lb., and sugar 15s. 6d. per cwt. Find the total duty on ½ ton of coffee, 4 cwt. of tea, and 1 ton of sugar.

37. The duty on cigars is 15s. 7d. per lb., and on cigarettes is 12s. 7d. per lb. Find the total duty on a consignment of 100 lb. of cigars and 1 cwt. of cigarettes.

38. A standard barrel of pork contains 224 lb. How many barrels are equivalent to 8 tons?

39. A standard bag of black pepper contains 316 lb. What is the shortness of weight if 7 bags are taken as holding a ton?

40. An (old) English ell was 45 in., and an (old) Irish mile was 2240 yd. How many ells were there in an Irish mile?

41. The record distance run in one hour is 11 m. 1286 yd., and in wo hours is 21 m. 1100 yd. What is the average number of yards un per minute in each of these trials?

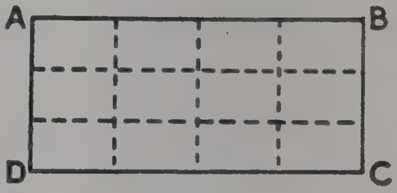
42. A man enlisted on August 20, 1914, and was demobilised on ecember 17, 1918. For how many weeks did he serve?

CHAPTER V

INTRODUCTION TO AREAS AND VOLUMES
(Whole Numbers only)

The length of a straight line is found by comparing the line with some ne of standard length, *e.g.* a foot-rule. In the same way the area of a at surface is found by comparing it with some surface of standard area, *g.* a square of side 1 in. (area 1 sq. in.) or a square of side 1 cm. (area sq. cm.).

Suppose ABCD is a rectangle 4 cm. long, 3 cm. wide: it can be divided to 12 compartments each of which is a square side 1 cm., and its area is therefore 12 sq. cm.



For any rectangle we have the following le: the number of centimetres in its length the number of centimetres in its breadth the number of square centimetres in its ea.

Any other unit can, of course, be chosen, it we must have the same unit for measuring both length and breadth. the length is 3 ft. and the breadth 24 in., the area is 3 × 2 sq. ft. or × 24 sq. in. We state the result more briefly as follows:

For a rectangle, length × breadth = area; **or** area ÷ length = breadth; **or** ea ÷ breadth = length.

The **perimeter** of any figure is the length of the boundary enclosing it.

The perimeter of a rectangle is therefore equal to twice the sum of its ngth and breadth.

Example.—The cover of a book is 5 in. wide, 7 in. high; find its area.

Area of rectangle = length × breadth.
∴ its area = 7 × 5 sq. in.
= 35 sq. in.

Example.—The area of a tennis court is 312 sq. yd.; it is 12 yd. ide; find its length and perimeter.

s length = area ÷ breadth
= 312 ÷ 12 yd. = 26 yd.

Its perimeter = twice the sum of its length and breadth
= 2(26 + 12) yd.
= 2 × 38 yd. = 76 yd.

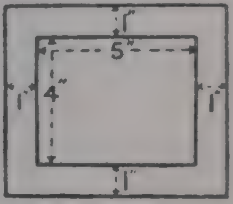
Example.—A photograph 5 in. by 4 in. is mounted on a card, leaving a margin 1 in. wide all the way round. What is the area of the margin?

Since the margin is 1 in. wide, the length of the card is $1+5+1=7$ in., and the breadth of the card is $1+4+1=6$ in.

$\therefore$ the total area of the card $=7\times6=42$ sq. in.

But the area of the photograph $=5\times4=20$ sq. in.

$\therefore$ the area of the margin $=42-20=$ 22 sq. in.



EXERCISE V (a)

1. Draw freehand a rectangle 5 in. long, 3 in. wide, and show how to divide it into 1-inch squares. What is its area?

2. Draw freehand a square of side 3 in., and show how to divide it into 1-inch squares. What is its area?

3. How can you show how many square millimetres there are in 1 sq. cm.? What is the number?

4. Draw a figure to show how many square inches there are in 1 sq. ft.

Find the area and perimeter of the rectangles in Ex. 5–10.

5. A sheet of paper 6 in. long, 5 in. wide.

6. A courtyard 10 yd. long, 6 yd. wide.

7. A bath mat 4 ft. long, 2 ft. wide.

8. A football ground 110 yd. long, 75 yd. wide.

9. A blackboard 5 ft. long, 3 ft. wide.

10. A strip of cloth 1 yd. wide, 7 yd. long.

11. How many stamps are there in a sheet which contains 24 rows and 15 stamps in each row?

12. A rectangular sheet of stamps contains 240; there are 16 rows. How many stamps are there in each row?

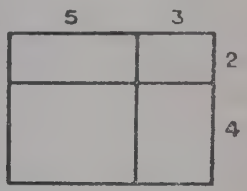
13. The area of a bath towel is 48 sq. ft.; it is 8 ft. long. What is its width and perimeter?

14. What (if any) is the difference in area between (i) 3 sq. ft. and 3 ft. square; (ii) 1 sq. ft. and 1 ft. square?

15. The floor of a room is 12 ft. long, 6 ft. wide, and is covered with tiles 1 ft. square. How many tiles are needed?

16. A closed box is 4 ft. long, 3 ft. wide, 5 ft. high. What is the total area of the outside surface of the box ?

17. A chessboard consists of 64 1-in. squares, with a margin of 1 in. all the way round. What is the total area of the board ? What is the area of the margin ?



18. The dimensions, given in the figure, are in centimetres. What are the areas of the four parts of the figure ? What is the total length of the lines in the figure ?

19. From a sheet of cardboard three squares are cut out whose sides are 3 in., 4 in., 5 in. How does the weight of the large square compare with the combined weights of the other two ?

20. From a sheet of cardboard two squares are cut out whose sides are 12 in., 13 in., and are put in opposite scale-pans of a weighing machine. What size square cut from the same sheet must be used to balance the scale-pans ?

21. A photograph 8 cm. by 9 cm. is printed on a piece of paper 9 cm. by 10 cm. What area of the paper is unused ?

22. A chain = 22 yd. ; an acre = 4840 sq. yd. How many square chains are there in 1 acre ?

23. How many dish-cloths, 2 ft. by 3 ft., can be made from a sheet of cloth 12 ft. long, 8 ft. wide ?

24. A tennis court is 26 yd. by 12 yd. If it is netted in by a surround which allows 7 yd. run back at each end and 4 yd. along each side, what extra area of ground is required ?

25. A rectangular 5-acre field is 100 yd. wide. What is its length ? (1 acre = 4840 sq. yd.)

26. The perimeter of a rectangular plot of grass is 120 yd., and its ength is 36 yd. What is its area ?

27. A rectangular sheet of cardboard is 20 in. long and 12 in. wide ; t is cut down so that the length is reduced by 4 in. and the width by 3 in. What area of cardboard is cut away ?

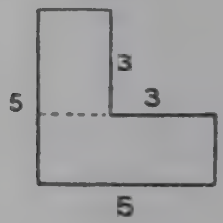
28. A rectangle is $3x$ ft. long and $2x$ ft. wide. What is (i) its area, (ii) its perimeter ?

29. The area of a rectangle is x sq. in. ; it is x in. long. What is i) its width, (ii) its perimeter ?

30. The perimeter of a square is $8x$ ft. What is its area ?

Find the areas of the figures in Ex. 31–36, the dimensions being given in inches and the angles being all right angles.

31.



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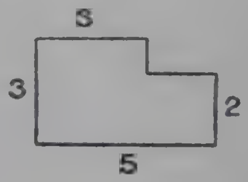

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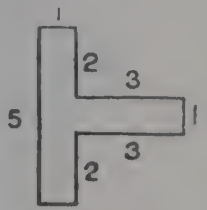

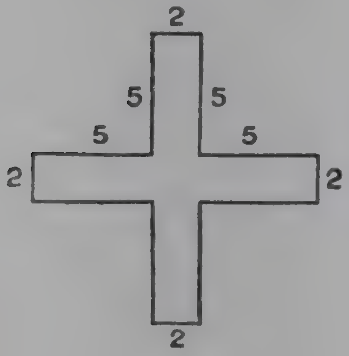

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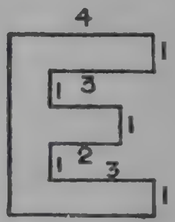
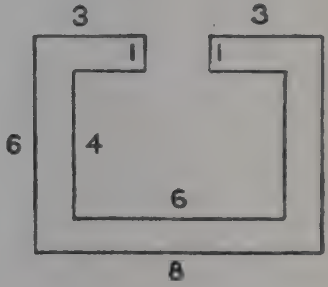


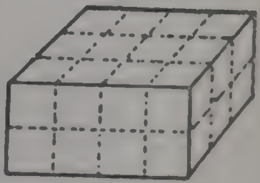
35.



36.

Volumes

The diagram represents a block of wood 4 in. long, 3 in. wide, and 2 in. high.



By sawing it up into smaller blocks by cuts along the dotted lines, we get $4 \times 3 \times 2$ blocks, each of which is 1 in. each way, *i.e.* is of volume 1 cu. in. Therefore the volume of the original block is $4 \times 3 \times 2 = 24$ cu. in.

Hence we see that the **volume of a rectangular block** (sometimes called a cuboid) is found by the rule

$$\text{length} \times \text{breadth} \times \text{height} = \text{volume},$$

or

$$\text{area of base} \times \text{height} = \text{volume}.$$

This may also be written

$$\frac{\text{volume}}{\text{height}} = \text{area of base, or } \frac{\text{volume}}{\text{area of base}} = \text{height}.$$

If cubic-inch blocks of wood (or cubic-centimetre blocks) are available, some experimental work should be done in building up cuboids of various shapes and making them the basis for oral work on the lines of Exercise V (b), question 1. It will also be found useful to have at hand the following models:

(i) A block of wood, 1 cu. dm., with a movable layer 1 cm. thick, and 10 movable cubic-centimetre blocks.
(ii) A tin just big enough to hold the block, to illustrate a litre.
(iii) A pint tin.

Example.—How many cubic feet of water are required to fill a swimming bath 40 ft. long, 30 ft. wide, to a depth of 8 ft.?

Volume of water = 40 × 30 × 8 cu. ft. = **9600 cu. ft.**

Example.—A cistern is 6 ft. long, 3 ft. wide. What is the depth of water when it contains 36 cu. ft. of water?

Area of base = 6 × 3 = 18 sq. ft.
Depth = volume ÷ area of base = 36 ÷ 18 ft. = **2 ft.**

EXERCISE V (b)

1. An open box is 5 in. long, 3 in. wide, and 4 in. deep. It is to be filled with 1-inch cubes. How many of these blocks can be placed on the floor of the box? How many layers can be placed in the box? How many of these blocks will the box hold? What is the volume of the box?

2. A cellar is 12 ft. long, 10 ft. wide, 8 ft. high. It is used for storing boxes, each of which is a 1-foot cube. How many boxes can rest touching one of the long walls? How many similar slabs of boxes can be put into the cellar? How many boxes does the cellar hold when full? What is the volume of the cellar?

3. A board of wood 1 in. thick, 6 in. long, 5 in. wide is marked out in 1-inch squares, and a 1-inch cube is placed on each square. What is the volume of the solid, including the board, thus obtained? Three more equal layers of 1-inch cubes are placed on the first layer. What is the volume of the new solid, and what is its length, breadth, and height?

4. A block of wood is in the shape of a cubic decimetre: it is cut into ten equal layers. What are the dimensions of each layer? Each layer is now cut into ten equal bars. What are the dimensions of each bar, and how many bars are there altogether? Each bar is now cut into ten equal blocks. What are the dimensions of each block, and how many are there altogether? How many cubic centimetres are there in 1 cu. dm.?

5. Find the volumes of the following rectangular blocks:

(i) 4 in long, 3 in. broad, 5 in. high.
(ii) 8 cm. long, 5 cm. broad, 1 cm. high.
(iii) 4 ft. long, 1 ft. wide, 1 ft. high.
(iv) base area 9 sq. in., height 4 in.

6. Find the volume of air in a room 15 ft. long, 12 ft. wide, 8 ft. high.

7. How much water is required to fill a dock 500 ft. long, 200 ft. broad, 24 ft. deep?

8. A packing-case is 5 ft. long, 3 ft. wide. How high must it be if it is required to hold 120 boxes each of which is a 1-foot cube?

9. A room is 20 ft. long, 15 ft. wide. What must be its height if it is to contain 2400 cu. ft. of air-space?

10. A tank is 6 ft. long, 4 ft. wide. How much will the water-level rise if 72 cu. ft. of water is poured into it?

11. 1 cu. dm. contains 1 litre: how many cubic centimetres are there in 1 litre? How many litres does 1 cu. m. contain? (1 m. = 10 dm. = 100 cm.)

12. A pan is 10 in. long, 6 in. wide, 3 in. deep, and contains water to a depth of 1 in. How many metal cubic-inch blocks can be placed in the pan before the water overflows?

13. A beam is 12 ft. long and 1 ft. square. What does it weigh if the timber weighs 15 lb. per cubic foot?

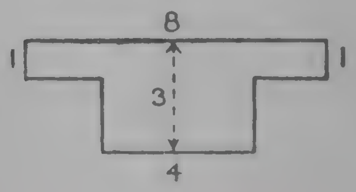
14. A trench 15 ft. long, 3 ft. wide, 4 ft. deep is made, and the soil which has been removed is spread out evenly to a depth of 1 ft. What area of ground will it cover, if the broken-up soil occupies twice as much space as it did when undisturbed?

15. Cubes of wood of edge 2 in. are cut from a cubical block of edge 7 in. How much wood is unused?

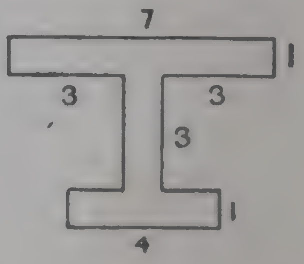
16. A stream of water is flowing at 2 ft. per second along a channel 8 ft. wide and 3 ft. deep. How many cubic feet of water pass under a bridge in 15 seconds?

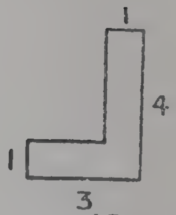
Find the volume of a 6-in. run of a girder whose cross-section is as under (measurements in inches).

17.

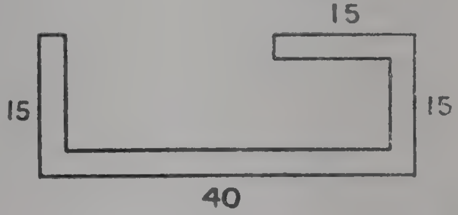


18.





19. The diagram represents the section of an angle iron (measurements in inches): its length is 8 in. What is its weight if 4 cu. in. of iron weigh 1 lb.?



20. The diagram represents the ground-plan of a stone parapet (dimensions in feet) which is 2 ft. high and 2 ft. broad all the way round. Find its volume.

21. A closed box is x in. high, $2x$ in. wide, $3x$ in. long. What is (i) its volume, (ii) the area of its surface?

22. From a cube whose edge is $3x$ in., a cube of edge $2x$ in. is cut out. What is the volume of the remainder?

23. A tank is x ft. long, y ft. wide. How much water must be poured into it to raise the water level (i) 1 ft., (ii) z ft.?

24. The area of the base of a tank is $12x^2$ sq. ft., and it contains $24x^2y$ cu. ft. of water. What is the depth of the water?

REVISION PAPERS

(R 1–10)

R 1

1. Multiply 6247 by 998.

2. What number multiplied by 273 will make 95,277?

3. What sum of money is necessary to pay 24 people 2s. $8\frac{1}{2}$d. each?

4. How many pence are there in £18, 12s. 7d.?

5. How many pieces of string, each 18 in. long, can be cut from 10 yd.? How many cuts will be required?

6. Simplify $162-(118-53)$. What is the difference between this expression and $162-118+53$. By how much is it greater than $162-(118+53)$?

R 2

1. Find the value of
   (i) $607+7086+90+1254+733+1010$.
   (ii) $863-999-645+1273+1000-99$.

2. Find the values of (i) $a-(b-c)$, (ii) $a-b-c$, (iii) $a-b+c$, (iv) $a-(b+c)$, given that $a=16$, $b=10$, $c=1$. Find also their values if $a=6$, $b=11$, $c=3$.

3. Multiply 784 by 999; subtract the result from 784,000.

4. The circumference of a bicycle wheel is 7 ft. How many revolutions will it make in a mile?

5. What is the difference between £5, 3s. 4½d. and £2, 17s. 11¾d.?

6. A prize of £100 is divided equally between 13 competitors. How much will each receive if amounts less than a penny are disregarded?

R 3

1. Divide 88,200 by 630. Write down the values of $\frac{8820}{630}$, $\frac{882000}{6300}$, $\frac{88200}{63}$.

2. A tramway company during a year takes 3,826,496 passengers at 1d. fares and 2,164,315 at 1½d. State the total receipts in £, s. d.

3. What number divided by 198 will give 273 as a quotient?

4. Sixty-two feet is divided into lengths each of which is $\frac{1}{100}$ ft. How many lengths are there?

5. Find the total weight of 126 cases each weighing 3 cwt. 2 qr. 12 lb.

6. Express 10,000 yd. in miles, furlongs, yards.

R 4

1. Add the following:

$$\begin{array}{r} 6{,}893 \\ 2{,}078 \\ 31{,}294 \\ 703 \\ 61 \\ 7{,}070 \\ \hline \\ \hline \end{array} \qquad \begin{array}{rrr} £ & s. & d. \\ 62 & 11 & 2 \\ 13 & 18 & 4\frac{1}{2} \\ 3 & 3 & 9\frac{1}{4} \\ & 17 & 7\frac{3}{4} \\ \hline \\ \hline \end{array}$$

2. A freezing mixture has a temperature of −17° C. It is melted and warmed to 19° C. How many degrees has its temperature risen?

3. Divide the sum of 17^3 and 37^3 by 21^3.

4. A man with a barrel-organ makes on an average twopence every 5 minutes. How much will he earn in a week if he works 8 hours a day and takes a rest on Sundays?

5. Posts are placed at intervals of 2½ ft. along a course of one-quarter of a mile. There is one at the beginning and one at the end. How many posts are required?

6. What is the cost of 131 tons of coke if the price is £2, 9s. 6d. per ton?

R 5

1. What would be the result of writing down 26,428 twenty times and finding the sum of these numbers?

2. Find the total contents of 9 sacks each containing 6 qr. 2 bush. 2 pk.

3. Divide 217 tons 13 cwt. 42 lb. by 141.

4. I bought 112 oranges at 7 a shilling. If I had spent the same amount on apples I should have got 128. How many apples were sold for 1s.?

5. A liquid X is 27° above the temperature of the room. The temperature of another liquid T is −12°, which is 54° below X. What is the temperature of the room?

6. Find the average age of a class of boys, two of whom are 12 yr. 4 mo.; three are 12 yr. 7 mo.; three are 13 yr. 1 mo.; and the others are 12 yr. 2 mo., 12 yr. 5 mo., 12 yr. 8 mo., 12 yr. 10 mo., 13 yr. 2 mo., 13 yr. 4 mo., 13 yr. 0 mo.

R 6

1. (i) What number must be added to 789 to give 1213? (ii) What number must be subtracted from 712 to give 378?

2. What change is left from £1 after paying two bills, one of 7s. 8½d. and the other of 9s. 9d.?

3. Multiply (i) 1001 by 749; (ii) $2x+3y$ by $2x-3y$.

4. Telegraph poles are placed along a road at intervals of 80 yd.; how many are there each mile? How many more are required if the interval is reduced by 25 yd.?

5. A barrel contains 4½ gallons of methylated spirit which is sold at 6½d. per pint. Find the sum of money realised, allowing half a pint for waste.

6. Two boy scouts are given a chain and told to mark out a length of 400 yd. How many complete chain-lengths do they measure out, and, if they regard a link as equivalent to 8 in., how many links must they then count out along the chain to get the full distance? (1 chain = 22 yards.)

R 7

1. When 28,478 is divided by 63 the quotient is 452 and the remainder 2. State without division the quotient and remainder when 284,780 is divided by 630.

2. Find the cost of 143 acres at £12, 16s. 8d. per acre.

3. Simplify $623-17(821-793)$. What is the difference between this expression and $(623-17)(821-793)$?

4. Divide £2786, 6s. 8d. by 643.

5. How many bundles, each containing 30 sticks, can be made from 62,483 sticks? How many sacks can be completely filled from these bundles if each sack contains 20? How many sticks will remain not packed in sacks?

Without further work state the remainder when 62,483 is divided by 600.

6. How many inches are there in 26 yd. 2 ft. 8 in.?

R 8

1. A cricketer makes the following scores: 3, 75, 38, 119, 27, 0, 4, 63, 45. How far short is he of a total of 500 runs?

2. The Continental areas are as follows:

Europe, 3,870,000 sq. miles; Asia, 17,000,000 sq. miles; Africa, 11,500,000 sq. miles; America, 16,700,000 sq. miles; Australasia, 3,280,000 sq. miles. What is the total area?

3. Find the quotient and remainder when (i) 87,900 is divided by 3700; (ii) 8790 is divided by 370; (iii) 879 is divided by 37.

4. How many equal payments of £1, 6s. 9d. can be made out of £50, and what sum will be left over?

5. A side of bacon weighs 72 lb. and costs 1s. 2½d. per lb. What is its total cost?

6. Find the weight of steel rails required to lay 3 miles of a single-track light railway if each yard length of a rail weighs 30 lb. [Answer in tons, cwt., lb.]

R 9

1. What number, multiplied by 171, gives the same product as 576 multiplied by 323?

2. Fifty bicycles are placed in stands side by side, the distance between the centre lines of adjacent machines being 2 ft. 9 in. What is the distance from the first to the last bicycle?

3. If the scale of a map is $\frac{1}{100,000}$, how far apart are two towns whose distance apart on the map is 4½ in.?

4. Calculate the value of a^2-b^2 if $a=64$ and $b=56$. Calculate also the values of $a+b$, $a-b$, and $(a+b)(a-b)$.

5. A man walks 12 miles at 3 miles an hour and then cycles 30 miles at 10 miles an hour. Find the average rate for the whole journey.

6. A man buys £50 worth of francs when they are at 88 to the £ and sells them when they are at 80 to the £. What is his gain or loss ?

R 10

1. A man starts with £784 in the bank and draws out the following sums : £37, £219, £69, £87, £106. What balance has he left ?

2. The population of Ireland has changed as follows :

1881.	1891.	1901.	1911.
5,174,836	4,704,750	4,458,775	4,381,951

Calculate the decrease of population in each period of 10 years, and the total decrease from 1881 to 1911.

3. Find a number such that when it is divided by 349 the quotient is 27 and the remainder is 104.

4. A bath holds 15 gallons of water. How long does it take to fill it if the water runs in at the rate of half a pint per second ?

5. How many $2\frac{1}{2}$-lb. packets of tea can be made from 1 cwt. of tea, and how much is left over ?

6. The wages paid to the servants at a school boarding-house are as follows : gardener, 35s. per week ; cook, £48 a year ; housemaid, £36 a year ; two dormitory-maids, £24 a year each ; kitchen-maid, 8s. per week. Find the total wage-bill for the month of February 1923, taking the month as $\frac{1}{12}$ of the year for yearly wages.

PROBLEM PAPERS

(P 1–10)

P 1

1. A number is divided by 52 and the remainder is 45. What would be the remainder if the number were divided by 13 ?

2. A roller takes 6 turns when rolling a cricket-pitch. What is the circumference of the roller ?

3. A man whose debts are £1643 can pay 12s. 6d. in the £ with his assets. Find his assets.

4. If I had bicycled 12 miles an hour instead of 10 miles an hour I should have travelled 14 miles further in the same time. How far did I actually go ?

5. The first number of a paper published every week-day appeared on Tuesday, March 1. Find the number of the copy dated Wednesday, June 29, in the same year.

P 2

1. A family going to the seaside require 3 whole tickets and 5 half tickets. A whole single ticket costs £1, 5s. 4d. What did the single rail fare for the whole family cost?

2. A man left England on June 1, 1910, and returned after an absence of 5000 days. In what year did he return?

3. An electric light company charges 7d. per unit. What amount does a man pay whose house is supplied by two meters which read as follows:

	1st meter.	2nd meter.
Beginning of period .	3179	2575
End of period . .	3241	2610

4. A field is 180 yd. long and 60 yd. wide, and is fenced with wire supported on posts at intervals of 5 yd., there being a post at each corner. How many posts are needed for the fence? How many would be needed if one of the short sides is left unfenced?

5. A jar full of water weighs 30 lb.; when half full it weighs 22 lb. What does it weigh when empty?

If the jar is filled with sand instead of water, and if a cubic inch of sand weighs four times as much as a cubic inch of water, find the weight of the jar full of sand.

P 3

1. Find the number which, when divided by 643, gives a quotient 78 and a remainder 7.

2. A man's income is £842 a year. How much does he spend in a year if he spends £440 more than he saves?

3. By increasing the rates 2½d. in the £, a town council expected to raise £2460 more revenue. On how much money was the tax levied?

4. Two posts are erected 2 miles apart and other posts inserted between them at intervals of 220 yd. How many posts were there altogether?

5. Two men start at the same time from the same place to go round a park in opposite directions. One bicycles at 10 miles an hour and the other walks at 4 miles an hour. After half an hour they meet. How much longer will it be before the cyclist reaches his starting-place?

P 4

1. Chairs cost a dealer 22s. 6d. each and he charges 1s. 3d. per afternoon for hiring them out: his expenses for sending them out

and collecting them amount to 4d. per time. How many separate afternoons must he hire them out before he makes any profit ?

2. A motorist recorded his holiday as follows :

	Monday, Birmingham to Rhyl.	Tuesday, on to Aberystwyth.	Wednesday, on to Abergavenny.	Thursday, back to Birmingham.
Distances	114 miles.	133 miles.	85 miles.	81 miles.
Times .	11.10 a.m. to 6 p.m.	9.30 a.m. to 7.15 p.m.	10.25 a.m. to 5.5 p.m.	2.5 p.m. to 5.30 p.m.
Halts .	$1\frac{1}{2}$ hours.	$3\frac{1}{2}$ hours.	$2\frac{1}{2}$ hours.	. .

What was the total distance he went, and how long did the actual driving take ?

3. A report on work in the engineering trade in 1913 gives the average weekly wage as 27s. 5d. and the average annual earnings as £68, 10s. What is the discrepancy per year between these two statements, assuming the man works for 52 weeks ?

4. Simplify (i) 397^2-394^2, (ii) 300 times 6s. 8d.

5. A man walks at 8 ft. per second for the first 5 minutes, then 7 ft. per second for the next 10 minutes, then 6 ft. per second for the next 10 minutes, and after that at 5 ft. per second. How many yards does he walk in the first hour ?

P 5

1. When 628,437 is subtracted from a number the difference is the same as if the number had been subtracted from 817,439. Find the number.

2. If a debtor's assets were increased by £86, he could pay 6d. in the £ more to his creditors. How much did he owe ?

3. A gentleman divided £4, 18s. among 150 school children, giving the girls a shilling each and the boys sixpence. How many boys and girls were there ?

4. A man puts up 2 fur. 66 yd. of fencing at a cost of 6d. a yard, and spends the same amount on another fence of 3 furlongs. How much a mile did the second fence cost ?

5. The lowest score made in an innings was 12. If this batsman had made 15 the average of the eleven for the innings would have been 31 exactly. What score did the side make ?

P 6

1. In 1841 the total number of houses in Ireland was 1,328,839, of which 491,278 contained only one room. In 1901 the corresponding

figures were 851,158 and 9872. What was the change in the number of houses containing more than one room between these dates ?

2. A workman is paid £5, 5s. per week and is fined 2s. 4d. for each hour he is absent of his ordinary time, and receives an extra 2s. 9d. for each hour's work overtime. What does he receive for 12 weeks' work if he was absent 15 ordinary hours and put in 7 hours' overtime ?

3. A cubic foot of coal weighs 90 lb. How many cubic feet of cellar-space is needed to store 9 tons of coal ?

4. When a certain number is divided by 264 the quotient is 197 and the remainder is 183. What is the quotient and remainder when the number is divided by 132 ?

5. In 1595 Bristol was threatened with a grain famine, and the aldermen commissioned John Whitson to purchase rye in Dantzic. He bought 3000 qr. at 28s. per quarter. When it arrived at Bristol the aldermen gave 500 qr. to the poor and sold the rest at 46s. per quarter, and thereby made a profit of £774. How much profit did Whitson make ?

P 7

1. Find a number which, when divided into 62, gives a remainder 2, and when divided into 57 gives a remainder 12.

2. There are four stations on a branch line. How many different kinds of single third-class tickets must be supplied for use on the line ?

3. A body falls $16t^2$ ft. in t seconds. How far does it fall in the 3rd second ?

4. January 1, 1909, fell on a Friday. On what day of the week will January 1, 1969, fall ?

5. By reducing his expenditure from £4 a week to £3 a week a man could increase his holiday by 2 weeks. How long did he expect to have originally ?

P 8

1. An offertory contained the following coins :

Half-crowns.	Florins.	Shillings.	Sixpences.	Three-pennies.	Pence.	Half-pence.
14	21	69	137	78	119	34

What was the collection worth ?

2. Newspaper wrappers with ½d. stamps can be bought in packets of 22 for 1s. and with 1d. stamps in packets of 23 for 2s. A firm uses 60 packets of the former and 40 packets of the latter. How

much more would it have cost if the wrappers had been bought unstamped at 20 for 1d. and stamped afterwards?

3. Telegraph wire weighs 450 lb. per mile. Find in tons, cwt., lb. the total weight of wire required for a line 12 miles long.

4. A sign-post on the main road from London to Portsmouth says $27\frac{1}{4}$ miles to Portsmouth, and another sign-post further along says $8\frac{3}{4}$ miles to Portsmouth. How many milestones are there between these two sign-posts?

5. A householder bought 16 tons of coal at 45s. per ton in July and 5 tons of coal at £2, 11s. 6d. per ton in March. What did he spend on coal altogether?

P 9

1. When a certain number is multiplied by 327 the result is 35,469 greater than when the number is multiplied by 264. Find the number.

2. When the price of tea goes up from 2s. 6d. to 2s. 10d. a lb. a shopkeeper takes £2, 12s. more a week on the sale of tea. How many lb. does he sell if it is the same amount in each case?

3. How many numbers are there between 300 and 800 which are multiples of 9?

4. Divide £350, 14s. into 2 shares, one of which is six times as large as the other.

5. One truck has a load three times as heavy as another. If the difference between the loads is 2 cwt. 3 qr. 14 lb., find both of them.

P 10

1. The price of silver per standard ounce varied as under:

	1902.	1904.	1906.	1908.	1910.
Price in pence	24	26	31	24	25

State the *increase* during each period of 2 years and the total increase from 1902 to 1910.

2. A hospital states that it burns on an average 1 lb. of coal each minute. Express in tons, cwt., lb. the weight of coal consumed in a year (365 days).

3. A taxi travels at 12 miles an hour. The charge for the journey is at the rate of 1s. per mile travelled and 6d. for every 5 minutes it has to wait. What is the charge for a journey of 3 miles which took, including halts, 25 minutes?

4. A knot is a speed of 6080 ft. per hour. An aeroplane flying at 80 miles per hour flies over a destroyer steaming in the same direction at 35 knots. Show that 3 minutes later the aeroplane will be very nearly 2 miles ahead of the destroyer.

5. What is the smallest sum of money that must be added to £34, 17s. 8d. to make an exact multiple of £3, 19s. 5d.?

PART II

CHAPTER VI

DECIMALS

Historical Note.—The introduction of Decimal Fractions is due to Stevinus, a Dutchman who was born at Bruges in 1548. In 1585 he published a tract on Decimal Arithmetic in which he calls tenths, hundredths, and thousandths by the names firsts, seconds, and thirds, and either puts dashes or a number inside a small circle over the decimals. Thus he writes 94·1305 in the forms

$$9\overset{(0)}{4}\ \overset{(1)}{1}\ \overset{(2)}{3}\ \overset{(3)}{0}\ \overset{(4)}{5} \quad \text{or} \quad 94\ \overset{\prime}{1}\ \overset{\prime\prime}{3}\ \overset{\prime\prime\prime}{0}\ \overset{\prime\prime\prime\prime}{5}.$$

John Napier, the inventor of logarithms, points out that there is the same facility in working with these fractions as with whole numbers, and that in a division sum by whole numbers, when the remainder has been obtained, we can add a comma to the dividend and then bring down a zero and proceed as before. Napier used the form 94·1305, but Briggs, who is also famous for his tables of logarithms, although he sometimes used the decimal point, also wrote $94^{\underline{1305}}$, meaning $94\frac{1305}{10,000}$, and it is a surprising fact that it was not until the eighteenth century, long after Napier's time, that the use of the decimal point became general.

Easy Decimals

A decimal is a fraction in which the denominator is 10 or a power of 10.

In the number 333 the left-hand 3 represents 3 hundreds = 300,
the next 3 represents 3 tens = 30,
the last 3 represents 3 units = 3.

Each 3 therefore represents a value which is **one-tenth** of the value it would have if it were moved one place to the **left**. We can extend this notation in the following way :

Thousands. 1000	Hundreds. 100	Tens. 10	Units. 1	Tenths. $\frac{1}{10}$	Hundredths. $\frac{1}{100}$	Thousandths. $\frac{1}{1000}$	Etc.
			6	2			
		3	..	4			
			1	..	5		
		7	8	9	1		
	2	3	4	..	..	5	
				6	7		
				..	8		
				..	..	9	
1	..	2	..	3	..	4	
5	..	..	6	..	2	8	
		2	8	..	2	3	

This table shows what value is attached to the various positions of the digits. Positions to the right of the thick line are called **decimal places.** When writing down a number, the position of the thick line is indicated by a dot, called the **decimal point.** If a blank occurs in a column this is indicated by inserting a 0. The numbers given in the table are as follows: 6·2, 30·4, 1·05, 78·91, 234·005, 0·67, 0·08, 0·009, 1020·304, 5006·028, 28·023.

The table makes it clear that 0·20 has the same value as 0·2; the number ·20 should be read "point two nought"; the phrase "point twenty" must never be used.

Note.—When there is no whole number, a zero is often written on the left of the decimal point; thus $\cdot 27 = 0\cdot 27$ and $\cdot 0847 = 0\cdot 0847$. The object is to avoid the decimal point being overlooked.

Example.—Express $\frac{7}{100}$ *and* $28\frac{23}{1000}$ *in decimal form.*

$\frac{7}{100}$ is represented by the digit 7 in the *hundredth* column,

$$\therefore \ \frac{7}{100} = 0\cdot 07.$$

$$28\frac{23}{1000} = 28 + \frac{20}{1000} + \frac{3}{1000} = 20 + 8 + \frac{2}{100} + \frac{3}{1000}.$$

It is therefore represented by the final number printed in the table above, $\therefore \ 28\frac{23}{1000} = 28\cdot 023.$

Example.—Express 78·91 *as a mixed number and* ·009 *as a fraction.* (A mixed number is the sum of a whole number and a fraction.)

From the table we see that

$$78\cdot 91 = 70 + 8 + \frac{9}{10} + \frac{1}{100} = 78 + \frac{91}{100} = 78\frac{91}{100}, \text{ and } 0\cdot 009 = \frac{9}{1000}.$$

Example.—Express as a decimal sixty-two tenths.

Sixty-two tenths $= 62 \times \frac{1}{10} = \frac{62}{10} = 6\frac{2}{10} = 6\cdot 2.$

It can be obtained at once from the table by writing 2 in the "tenths" column and 6 one place to the left; see the first number in the table above.

EXERCISE VI (a)

1. Make a table as on p. 65 and write in the following numbers: $\frac{3}{10}$, $\frac{2}{100}$, $\frac{7}{1000}$, $\frac{23}{100}$, $\frac{37}{1000}$, $\frac{137}{10}$, $\frac{2489}{100}$, $\frac{201}{1000}$, $\frac{303}{10}$, $5\frac{61}{1000}$, $203\frac{9}{100}$, $300\frac{3}{10}$.

2. Write the numbers in Ex. 1 in decimal form.

3. Write in decimal form the following: (i) $2 + \frac{3}{10} + \frac{7}{100}$; (ii) $20 + 4 + \frac{5}{10} + \frac{1}{1000}$; (iii) $100 + 30 + \frac{3}{10} + \frac{7}{100}$; (iv) $\frac{4}{10} + \frac{5}{1000}$; (v) $\frac{7}{100} + \frac{8}{1000}$; (vi) $10 + \frac{7}{100}$.

4. Write in decimal form the following: (i) twenty-six tenths, (ii) one hundred and twelve tenths, (iii) sixty-seven hundredths, (iv) one hundred and five hundredths, (v) three tenths + four thousandths, (vi) thirteen ten-thousandths.

5. Express as fractions or mixed numbers: (i) 0·3, (ii) 0·07, (iii) 2·005, (iv) 0·17, (v) 10·04, (vi) 4·637, (vii) 3·90, (viii) 3·09, (ix) 0·0300, (x) 41·8603.

6. Write down the first four numbers in the table on p. 65, with each figure moved one place to the left. What is the effect of this?

7. Repeat Ex. 6, moving each figure two places to the right. What is the effect of this? What would you do in order to multiply each number by a thousand? What would you do in order to divide by 10?

8. Write down the values of (i) 0·2 × 10, (ii) 0·23 × 100, (iii) 2·3 × 10, (iv) 0·24 ÷ 10, (v) 12·4 ÷ 10, (vi) 0·2 ÷ 10, (vii) 0·2 ÷ 1000, (viii) 10·4 ÷ 10, (ix) 73 ÷ 1000, (x) 9 ÷ 100.

9. From a foot-rule state how many tenths of an inch there are in 4, 6·2, 5·3, 0·7, 11·3 inches.

10. How many hundredths of an inch are there in 3, 4·2, 2·16, 11·37, 0·08, 5·03, 0·7 inches?

11. (i) Divide 6·24 by 10, 100, 1000, 10,000; (ii) multiply 6·24 by 10, 100, 1000, 10,000.

12. (i) Divide 0·52 by 10, 100, 1000; (ii) multiply 0·52 by 10, 100, 1000.

13. Express as fractions in their lowest terms: (i) 0·5, (ii) 0·25, (iii) 0·75, (iv) 0·05, (v) 0·075, (vi) 0·6, (vii) 0·24, (viii) 0·0075, (ix) 0·125, (x) 0·375.

14. What is the value of (i) 51 ÷ 10, (ii) 51 ÷ 5·1, (iii) 3·7 × 100, (iv) 370 ÷ 3·7?

15. By what must 0·038 be multiplied to give 3·8?

16. By what must 730 be divided to give 7·3?

17. Express as decimals, (i) $\frac{63}{10^2}$, (ii) $\frac{7}{10^3}$, (iii) $\frac{783}{10}$, (iv) $\frac{1924}{10^2}$, (v) $\frac{13}{10^4}$.

18. Which is the greater, and by how much: (i) 0·10 or 0·2, (ii) 0·5 or 0·45, (iii) 0·0249 or 0·025?

Significant Figures and Places of Decimals

Answers are often required to be given in an approximate form. If you are measuring the length of the side of a triangle drawn on the paper, your ruler will give you its length to the nearest tenth of an inch, and by more refined methods it is possible for you to measure it to the nearest hundredth of an inch; but each of these measurements is probably only an approximation to the true length. If, for example, the equal sides of an isosceles right-angled triangle are each 4 in., the length of the hypotenuse is (i) rather less than 6 in., (ii) rather more than 5·6 in., (iii) rather less than 5·66 in. Measurement with a ruler could not, however, give you any closer accuracy, but calculation gives further approximations as 5·657, 5·6569, 5·65686, etc. The conditions of the question show what degree of approximation is useful or necessary. Thus if in the above the length of the hypotenuse was asked for in inches **correct to 2 places of decimals,** the answer would be 5·66 in., because 5·66 is nearer to 5·6568 . . . than 5·65 is. If, however, the length was required **correct to 2 significant figures,** the answer would be 5·7 in., because this contains 2 figures, and 5·7 is nearer to 5·6568 than 5·6 is. If, however, the number is a decimal less than 1 with zeros at the **beginning,** these do not count as significant figures. **The first significant figure of any number is the first figure which is not a 0.** Thus 0·0030284 correct to 4 significant figures is 0·003028, and correct to 3 significant figures it is 0·00303, for this is nearer the given number than 0·00302 is. If asked for 2 significant figures we write it 0·0030, **not** 0·003, as it is necessary to indicate that the second significant figure is 0; but correct to 1 significant figure it would be 0·003.

Example.—Express correct to 3 *significant figures and to* 2 *significant figures* (i) 0·6248, (ii) 1794.

(i) 0·6248 correct to 3 significant figures is 0·625, but correct to 2 significant figures it is 0·62, because the given number is nearer to 0·62 than to 0·63.

(ii) 1794 correct to 3 significant figures is 1790: the final zero does not count as a significant figure.

1794 correct to 2 significant figures is 1800, because the given number is nearer 1800 than 1700, and the two final zeros do not count as significant figures.

Example.—Express 39·30374 *correct to* 3 *places of decimals and to* 2 *places of decimals.*

39·30374 correct to 3 places of decimals is 39·304, because it is nearer to 39·304 than to 39·303.

39·30374 correct to 2 places of decimals is 39·30: the zero in the second place must be written down to show that this second place has been taken into account.

EXERCISE VI (b)

Write down correct to 1 significant figure:

1. 7·342. **2.** 7·542. **3.** 0·824. **4.** 0·874.
5. 0·0821. **6.** 0·0851. **7.** 0·000924. **8.** 0·000964.
9. 9342. **10.** 8729. **11.** 19·67. **12.** 99·91.

Write down correct to 3 significant figures:

13. 3·141592. **14.** 10·08674. **15.** 86,792. **16.** 1·73504.
17. 0·8348. **18.** 24,997. **19.** 0·06789. **20.** 0·0054999.

Write down correct to 3 places of decimals and correct to 2 places of decimals:

21. 18·6372. **22.** 0·04839. **23.** 0·04555. **24.** 0·00504.

25. What is the value of 89721 correct to (i) the nearest hundred, (ii) the nearest thousand?

26. What is the value of 8·9749 correct to (i) the nearest tenth, (ii) the nearest hundredth?

27. What would be a reasonable degree of approximation if you were asked for (i) your height, (ii) your weight, (iii) the distance from London to Dover, (iv) the distance of the Earth from the Sun, (v) the time of a 100 yards' race, (vi) the length of the summer holidays?

28. The length of a line is given as 3·47 in. correct to 3 significant figures. Between what limits must its true length lie?

The Metric System

The metric system was introduced in France at the time of the French Revolution. In 1790 the French Government appointed a commission of scientists to frame a new system of measures and to determine a unit of length. They decided that this should be $\frac{1}{10,000,000}$ of a quadrant of the meridian. This length, based on a measurement made between Dunkerque and Barcelona, was called a **metre**; its British equivalent is 39·3708 in. But it has since been found that the measurement of the meridian was not accurate, and so the metre is now defined as the length of a standard bar kept at Paris. The system became law in France in 1801 and was included in the Code Napoleon. The system has obvious advantages, but there are various reasons why it has not been adopted in this country. It would involve great expense to manufacturers, as it would mean the construction of new plant to fit the new standards of length, and it introduces difficulties in sale prices; for instance, "the exact price of half a yard of cloth at 1·25 fr. a yard cannot be paid, and the buyer suffers; if sugar is sold at 25 centimes per lb., a buyer of $\frac{1}{2}$ lb. must pay 4 per cent. more than the proper price" (*Chambers's Encyclopædia*). The application to time is so inconvenient that it has never been tried.

The tables for length, weight, capacity, money have already been given (see p. 26); milli-, centi-, deci- are **Latin** prefixes, and kilo-, hecto-, deca- are **Greek** prefixes. Millimetres and centimetres are the graduations on a metre rule; distances on sign-posts are marked in kilometres.

The following approximations are often used:

1 metre $=39·4$ inches. 1 kilogram $=2\frac{1}{5}$ lb.
1 kilometre $=\frac{5}{8}$ mile. 1 litre $=1\frac{3}{4}$ pints.

By definition, 1 cu. cm. of water weighs 1 gram (exact).

1 litre $=$ 1 cu. dm. $=$ 1000 cu. cm. (exact).

For other constants see p. xiv.

The class-room should contain measures of (i) a metre and a yard; (ii) a litre and a pint and a cubic decimetre; (iii) a kilogram and a pound.

Lengths measured in metric units can be entered in a table (similar to that on p. 65) as follows:

	Km.	Hm.	Dm.	m.	dm.	cm.	mm.
(i)						5	7
(ii)				3	1	2	
(iii)				6	..	4	
(iv)	8	7	1	2			
(v)					5	7	2
(vi)			1	9	..	3	4
(vii)	2	..	4	..	..	3	
(viii)		3	..	5	..	..	7

(i) may be read off as either 5 cm. 7 mm. or 57 mm. or 5·7 cm. or ·57 dm. or ·057 m., etc.

(ii) may be read as either 3 m. 1 dm. 2 cm. or 312 cm. or 31·2 dm. or 3·12 m. or ·00312 km., etc.

NOTE.—In practice lengths would never be expressed in terms of more than 2 units, and usually only in terms of 1 unit; e.g. you would not write 3 Km. 7 Hm. 4 Dm. 9 m., but either 3·749 Km. or 3749 m. or 3 Km. 749 m.

EXERCISE VI (c)

In the table above, express

1. (i) in Dm., also in Km. **2.** (ii) in mm.

3. (iii) in m., also in cm., also in Dm., also in Km.

4. (iv) in Km., also in m.

5. (v) in mm., also in cm., also in m., also in Km.
6. (vi) in m., also in cm., also in Km.
7. (vii) in Km., also in Dm., also in m.
8. (viii) in Km., also in m., also in cm.

Make a table like the table on p. 70, and enter in it

9. 2 m. 34 cm. **10.** 3·7 m. **11.** 13·02 m. **12.** 1 Km. 73 m.
13. 4·823 m. **14.** ·704 m. **15.** ·60 dm. **16.** 11 m. 24 mm.
17. 18 m. 3·7 cm. **18.** 9·302 Hm. **19.** 704 mm. **20.** ·0061 Km.

Express in metres

21. 4 dm. **22.** 3 cm. **23.** 12 mm. **24.** 137 cm.
25. 30 cm. **26.** 11·2 cm. **27.** 13·4 mm. **28.** 6·437 Dm.
29. 1 m. 3 cm. **30.** 2 Dm. 14 cm. **31.** 2 Km. 17 mm.
32. 3 dm. 27 mm. **33.** 7 Hm. 46 cm. **34.** 2 Hm. 3 Dm. 5 cm.
35. 19 Dm. 27 cm. **36.** 23 Hm. 37 dm. **37.** 40 dm.
38. 8 Km. 6 Hm. **39.** 4 Km. 3 Hm. 2 m. 8 dm. **40.** 812 mm.
41. 5 Hm. 7 Dm. 2 cm. **42.** 1 Dm. 42 cm. 6 mm.

Express in kilometres, also in centimetres,

43. 6 Km. 8 Dm. 5 m. **44.** 5 Km. 4 Hm. 8 m. **45.** ·816 Hm.
46. 4·32 Dm. **47.** ·056 m. **48.** ·0032 m.

Write out in full in Km., Hm., Dm., m., dm., cm., mm., the following :

49. 5·264 Km. **50.** 13·02 Km. **51.** 1·803 Dm. **52.** ·08403 Km.
53. 874·02 m. **54.** 10·3012 Hm. **55.** 701·34 dm. **56.** 1005 cm.
57. 29·071 Dm.

State (i) what fraction, (ii) what decimal is

58. 1 mm. of 1 cm. **59.** 1 cm. of 1 m. **60.** 1 m. of 1 Km.
61. 4 cm. of 1 dm. **62.** 3 cm. of 1 Dm. **63.** 24 m. of 1 Km.
64. 16 m. of 1 Hm. **65.** 5 m. 6 dm. of 10 m. **66.** 37 cm. of 1 Km.
67. 500 m. of 1 Km. **68.** 2 Dm. of 1 Km. **69.** 1 cm. of 1 Hm.
70. 27 mm. of 1 Dm. **71.** 80 m. of 1 Km. **72.** 30 dm. of 1 Dm.
73. How many cm. are there in 3 Dm., 3·2 Dm., ·04 Dm.?
74. How many mm. are there in 27 m., 2·71 m., 19 Dm.?
75. How many dm. are there in ·03 Km., 28 Dm., 1·004 Hm.?

Addition and Subtraction of Decimals

Example.—Add together the following lengths: 3·07 *m.*, 19·23 *m.*, 94·015 *m.*, 0·073 *m.*

The lengths may be entered in a table as follows:

Hm.	Dm.	m.	dm.	cm.	mm.
		3	0	7	
	1	9	2	3	
	9	4	0	1	5
		0	0	7	3
1	1	6	3	8	8

Add up in the usual way and we see that the result is 116·388 m.

It is, of course, unnecessary to write out the table in full: if each length is in the same unit, we write down the numbers one below the other **with the decimal points straight down the page.** The work then appears as follows:

3·07
19·23
94·015
0·073
116·388

Example.—Subtract 26·37 *from* 34·9.

This could be done by using a table as above.

We say that 9 tenths = 8 tenths + 10 hundredths; then, as in ordinary subtraction,

7 hundredths + 3 hundredths = 10 hundredths.

Write down 3 and proceed as usual.

34·90
26·37
8·53

EXERCISE VI (d)

Add the following:

1.

m.	dm.	cm.	mm.
1	0	3	2
4	7	6	9
	3	0	3

Answer in metres.

2.

Km.	Hm.	Dm.	m.
	2	0	3
3	0	9	5
1	0	0	4

Answer in Km.

3.

Hm.	Dm.	m.	dm.	cm.
	3	8	0	7
1	2	4	9	
2	0	5	4	4
		3	2	5

Answer in metres

4.

Km.	Hm.	Dm.	m.	dm.
2	0	3	0	4
	1	7	9	5
	9	2	4	4
1	0	0	2	

Answer in Dm.

Simplify the following, using a table similar to that on p. 70:

5. 7 m. 5 cm. + 4 m. 3 dm. 7 cm. + 5 m. 5 dm. Answer in metres.

6. 2 Km. 3 Dm. + 3 Km. 1 Dm. 7 m. + 9 Hm. 7 Dm. 5 m. Answer in Km.

7. 4 Km. 385 m. + 7 Km. 94 m. + 41 Dm. Answer in Km.

8. 3 m. 527 mm. + 3 dm. 7 mm. + 5·4 cm. Answer in metres.

9. 77 m. 23 mm. + 194 m. 37 cm. + 4·83 dm. Answer in metres.

10. 8·23 Hm. + 1·93 Km. + 375 m. + 0·42 Km. Answer in Km.

11. 8 Hm. + 7 m. + 37 cm. + 40 dm. Answer in metres.

12. 4·37 Dm. + 19·07 m. + 500 m. Answer in metres.

Read off the results of the following additions:

13.	**14.**	**15.**	**16.**	**17.**	**18.**
0·7	1·6	·08	·056	1·01	2·7
0·3	2·4	·02	·044	3·09	12·3

19.	**20.**	**21.**	**22.**	**23.**	**24.**
·75	·6	·25	1·6	·025	1·01
·25	·04	·5	5·4	·025	·101
		·25	3·1	·05	·99

Add the following:

25. 23·74, 18·01, 10·3, 3·09, ·011.
26. 117·4, 103·55, 69·01, 7·66, 3·04.
27. 1·0107, ·1804, 2·007, 7·7, ·009.
28. ·0073, 12, ·94, ·905, 3·0602.
29. 1, 0·1, 0·01, 0·001, 10.
30. 873, 62·07, ·94, 1·03, 30·1, ·08.

Find the difference between the following pairs of numbers:

31. 0·8, 0·35.
32. 4·7, 0·8.
33. ·073, ·009.
34. 10, ·87.
35. 100, 16·4.
36. 11·1, 9·9.
37. ·30, ·5.
38. ·301, ·05.
39. 1000, 99·9.
40. 4·53, 3·42.
41. 7·387, ·38.
42. 3·08, ·001.
43. 1, ·01.
44. 15, 27·1.
45. 2·89, ·99.

Which is the greater, and by how much?

46. ·08 or ·076.
47. 1 or ·98.
48. ·9 or ·90.
49. ·007 or ·0064.
50. 1·3 or 1·288.
51. 4·37 or 4·4.
52. 0·1 or $(0{\cdot}1)^2$.
53. 0·3 or $(0{\cdot}3)^2$.

54. By how much does ·083 differ from ·08 and from ·09?

55. By how much does 2·08 differ from 2·1 and from 2·0?

Find the value of the following:

56. 17·4 − 32·8 + 30·01.
57. ·003 − ·049 − ·005 + ·5.
58. 10·03 + 7·48 − 3·91 − ·08.
59. 1000 − 73·9 − 104·2 − 81·04.
60. ·0032 − ·0101 − ·101 + ·18.
61. 1·01 + 9·09 − 7·7 − ·803.

62. Subtract 13·4 m. from 1 Hm. and give the answer in metres.

63. Subtract 9·4 dm. from 1 m. and give the answer in Dm.

64. Subtract 94 m. from 1 Km. and give the answer in Km.

65. Subtract 29 mm. from 3 dm. and give the answer in metres.

66. Subtract 9 Km. 99 m. from 10 Km. and give the answer in Km.

67. By how many dm. does 3 m. exceed 79 cm.?

68. What must be subtracted from 1 Km. to leave 0·8 cm.? Answer in metres.

69. How many metres remain if 37 cm. are taken from 2 Dm.?

70. Express in Km. the difference between 1 dm. and 1 cm.

Multiplication

Example.—Multiply 0·436 *by* 7.

$$\frac{6}{1000} \times 7 = \frac{42}{1000} = \frac{40}{1000} + \frac{2}{1000}$$

$$= 4 \text{ hundredths} + 2 \text{ thousandths.}$$

Put down 2 and carry 4 to the hundredths, and so on.

$$\begin{array}{r} \cdot 436 \\ 7 \\ \hline \underline{\underline{3 \cdot 052}} \end{array}$$

Example.—Multiply ·436 *by* ·4.

$\cdot 436 \times 4 = 1 \cdot 744$. $\therefore$ $\cdot 436 \times \cdot 4 =$ **0·1744**, since ·4 is $\frac{1}{10}$ of 4.

Example.—Multiply ·026 *by* ·03.

$\cdot 026 \times 3 = \cdot 078$. $\therefore$ $\cdot 026 \times \cdot 03 = \underline{\cdot 00078}.$

Or we may say $\cdot 026 \times \cdot 03 = \frac{26}{1000} \times \frac{3}{100} = \frac{78}{100000} = \underline{\cdot 00078}.$

This method of working shows that the number of figures to the right of the decimal point in the product is equal to the sum of the numbers of figures to the right of the decimal point in the factors. This rule gives a very useful check for testing the correct position of the decimal point in the product.

EXERCISE VI (e)

Find the values of

1. ·74 × 3.
2. ·86 × 6.
3. 1·24 × 5.
4. ·24 × 4.
5. ·036 × 4.
6. 1·27 × 7.
7. ·0083 × 3.
8. ·072 × 7.
9. 1·25 × 12.
10. ·0027 × 12.
11. 2·83 × 11.
12. 20·37 × 10.

Multiply

13. ·74 by 3, ·3, ·03, 30, ·003.
14. 1·26 by 4, ·04, 400, ·004, ·4.
15. 2·13 by ·05, 5, 50, ·0005, ·5.
16. 1·82 by ·003, 30, 3, ·03, ·3.
17. ·036 by ·04, 40, 4, ·004, ·4.
18. ·0027 by ·6, 60, 6, ·06, ·006.
19. 3·57 by 7, 70, ·07, ·0007.
20. 27 by ·7, ·007, ·0007.
21. 3700 by 4, ·004, ·0004, ·4.
22. 30·09 by 90, ·9, ·009, ·09.

Example.—Multiply 62·34 *by* ·0048.

Rough check.—The product is approximately 60 ×·005.

$$60\times5=300.\quad\therefore\ 60\times{\cdot}005={\cdot}3$$

$$62{\cdot}34\times{\cdot}0048=\frac{6234}{100}\times\frac{48}{10{,}000}$$

$$=\frac{6234\times48}{1{,}000{,}000}.$$

We therefore multiply 6234 ×48 and then insert the decimal point in the position indicated by the rough check.

$$\begin{array}{r}62{,}34\\ 48\\ \hline 249{,}36\\ 49{,}872\\ \hline \underline{\underline{299{,}232}}\end{array}$$

$$\therefore\ 62{\cdot}34\times{\cdot}0048=\underline{\underline{{\cdot}299232}}.$$

The position of the decimal point should also be checked as follows: 62·34 contains 2 figures after the decimal point and ·0048 contains 4 figures after the decimal point; therefore the product contains 2 + 4 = 6 figures after the decimal point, and this is the number of figures after the decimal point in ·299232. The proof of this check lies in the fact that the denominator of $\frac{6234\times48}{1{,}000{,}000}$ is 10^6.

Example.—Multiply ·076 *by* 18·5.

Rough check.—The product is approximately ·08 ×20.

$${\cdot}08\times2={\cdot}16.\quad\therefore\ {\cdot}08\times20=1{\cdot}6.$$

$$\therefore\ \text{the correct answer}=1{\cdot}4060=\underline{\underline{1{\cdot}406.}}$$

$$\begin{array}{r}185\\ 76\\ \hline 12{,}95\\ 1{,}110\\ \hline \underline{\underline{14{,}060}}\end{array}$$

Applying the other check, ·076 contains 3 and 18·5 contains 1 figure after the decimal point; ∴ the product contains 3 + 1 = 4 figures after the decimal point and is therefore 1·4060.

N.B.—The decimal point must be inserted before the final zero is crossed off.

EXERCISE VI (f)

Give approximate answers for

1. ·036 × 1·72.
2. ·036 × 172.
3. ·036 × ·00172.
4. 7·034 × 11·01.
5. 17·19 × ·0684.
6. ·074 × ·083.
7. 999 × ·099.
8. ·0073 × 81·74.
9. ·0017 × ·0017.
10. $({\cdot}373)^2$.
11. 837·2 × ·00634.
12. 973 × 1184.

Find the values of

13. 1·26 × ·84.
14. 1·26 × ·0084.
15. ·00126 × ·084.
16. 2·47 × 1·32.
17. ·0247 × 13·2.
18. ·0247 × ·0132.
19. 3·75 × ·046.
20. ·00375 × 46.
21. 37·5 × ·00046.
22. 0·213 × ·47.
23. 30·07 × 6·12.
24. 21·03 × ·0502.
25. 17·37 × 21.
26. ·00103 × 15·2.
27. ·00515 × ·044.

28. $(·083)^2$. **29.** $(·161)^2$. **30.** $40·04 \times ·035$.

31. $3000·7 \times ·11$. **32.** $·0102 \times ·305$. **33.** $4300 \times ·214$.

34. $·11 \times ·02 \times ·13$. **35.** $(·012)^3$. **36.** $140 \times ·05 \times ·72$.

37. $3·14 \times 5·1 \times 5·1$. **38.** $·2 \times ·4 \times ·6 \times ·8$. **39.** $1·01 \times ·101 \times ·11$.

40. $70·03 \times ·3 \times ·2$. **41.** $·148 \times 7·01 \times 5·5$. **42.** $·02 \times ·033 \times 5·5$.

Division

Example.—Divide ·064 *by* 4.

$$\begin{array}{l|l} 4 & \underline{·064} \\ & \underline{\underline{·016}} \end{array}$$

Example.—Divide ·064 *by* ·4.

$$\text{Since } \frac{·064}{4} = ·016, \therefore \frac{·064}{·4} = ·16.$$

Or we may proceed as follows :

$$\frac{·064}{·4} = \frac{·064 \times 10}{·4 \times 10} = \frac{·64}{4}. \qquad \begin{array}{l|l} 4 & \underline{·64} \\ & \underline{\underline{·16}} \end{array}$$

Example.—Divide 3 *by* 4.

$$\begin{aligned} 3 \text{ units} &= 30 \text{ tenths} \\ \text{and } \tfrac{30}{4} \text{ tenths} &= 7 \text{ tenths} + \frac{2 \text{ tenths}}{4} \\ &= 7 \text{ tenths} + \frac{20 \text{ hundredths}}{4} \\ &= 7 \text{ tenths} + 5 \text{ hundredths}. \end{aligned} \qquad \begin{array}{l|l} 4 & \underline{3·00} \\ & \underline{\underline{·75}} \end{array}$$

EXERCISE VI (g)

Evaluate

1. $·08 \div 4$; $·08 \div ·4$; $·08 \div ·04$; $·08 \div 40$.

2. $·24 \div 6$; $·24 \div 60$; $·24 \div ·06$; $·24 \div 600$.

3. $1·36 \div 8$; $1·36 \div ·8$; $1·36 \div 80$; $1·36 \div ·008$.

4. $·144 \div 12$; $14·4 \div 12$; $·00144 \div 12$; $·144 \div ·12$.

5. $·081 \div ·3$; $·81 \div ·03$; $8·1 \div ·3$; $·0081 \div 30$.

6. $4 \div 5$; $4 \div ·005$; $·04 \div 50$; $·40 \div ·05$.

Express as decimals

7. $\frac{7}{10}$. **8.** $\frac{3}{5}$. **9.** $\frac{7}{20}$. **10.** $\frac{18}{25}$. **11.** $\frac{3}{4}$.

12. $\frac{3}{400}$. **13.** $\frac{3}{8}$. **14.** $\frac{1}{25}$. **15.** $\frac{7}{800}$. **16.** $\frac{13}{8}$.

Example.—Divide ·0925 *by* 3·7.

Rough check.—The answer is about $\cdot 09 \div 4 \bumpeq \cdot\mathbf{02}$.

$$\begin{aligned} 925 \div 37 &= 25. \\ \therefore\ \cdot 0925 \div 37 &= \cdot 0025, \\ \therefore\ 0\cdot 0925 \div 3\cdot 7 &= \cdot 025. \end{aligned}$$

It is therefore clear that the significant figures are not affected by any operations which merely involve multiplication or division by powers of 10. We therefore ignore the decimal point in the actual division and afterwards use the rough check to insert it in the quotient.

$$\begin{array}{r} 25 \\ 37\,\overline{)\,925} \\ \underline{74} \\ 185 \\ \underline{185} \end{array}$$

$$\therefore\ \frac{\cdot 0925}{3\cdot 7} = \underline{\cdot 025}$$

Example.—Divide ·0012 *by* ·0375.

Rough check.—$\frac{\cdot 001}{\cdot 04} = \frac{1}{4} \bumpeq \cdot 02.$

$$\text{Now } \frac{1200}{375} = 32.$$

$$\therefore\ \text{from the rough check, } \frac{\cdot 0012}{\cdot 0375} = \underline{\cdot 032.}$$

$$\begin{array}{r} 32 \\ 375\,\overline{)\,1200} \\ \underline{1125} \\ 750 \\ \underline{750} \end{array}$$

The following is a convenient method for determining the position of the decimal point when dividing.

Example.—Divide ·00597 *by* 12·1.

The first significant figure in the quotient is placed over the position taken by the multiple of the **unit** figure in the divisor; *e.g.* the 4 in the quotient is placed over 8, the multiple of 2 in the divisor.

Zeros are then inserted in the quotient as far as the decimal point in the dividend.

$$\begin{array}{r} 4 \\ 12\cdot 1\,\overline{)\,\cdot 00597} \\ \underline{484} \end{array}$$

$$\begin{array}{r} \cdot 0004 \\ 12\cdot 1\,\overline{)\,\cdot 00597} \\ \underline{484} \\ 113 \end{array}$$

Example.—Divide ·000057 *by* ·0234.

In this case the position taken by the multiple of the unit figure in the divisor, if it existed, would be below the 3rd zero in the dividend.

$$\begin{array}{r} \cdot 002 \\ \cdot 0234\,\overline{)\,\cdot 000057} \\ \underline{{}_{\wedge\wedge}468} \end{array}$$

N.B.—**The decimal point in the quotient always comes immediately over the decimal point in the dividend.** The reason for this rule is clear if a divisor is taken with one significant figure in the unit place.

Another method is to move the decimal points until there is a convenient integer in the divisor thus:

Divide ·00597 *by* ·121.

$$\frac{\cdot 00597}{\cdot 121} = \frac{\cdot 00597 \times 10}{\cdot 121 \times 10} = \frac{\cdot 0597}{1\cdot 21}.$$

Hence the rough check is $\frac{\cdot 06}{1} = \cdot 05$, and the answer will begin with the figures ·04.

If the divisor is given with an integral part, the position of the decimal point can be seen by inspection thus:

Divide ·00597 *by* 123·4. Obviously 123 | ·00597

·00004 . . . .

EXERCISE VI (h)

Give *approximate* answers for

1. ·0873 ÷ 3·21. **2.** ·0645 ÷ 32·7. **3.** ·0418 ÷ ·234.
4. 41·78 ÷ 737. **5.** 693·8 ÷ ·416. **6.** 7839 ÷ ·0482.
7. ·00692 ÷ 39·43. **8.** ·8374 ÷ 27·32. **9.** 6·394 ÷ ·00528.

Find the values of

10. 8·058 ÷ 2·37. **11.** ·8058 ÷ 237. **12.** 8058 ÷ ·237.
13. 32·012 ÷ 60·4. **14.** ·032012 ÷ 6·04. **15.** ·32012 ÷ ·00604.
16. 26·452 ÷ ·34. **17.** ·26452 ÷ ·034. **18.** 2·6452 ÷ 340.
19. 10·836 ÷ 17·2. **20.** ·010836 ÷ 172. **21.** 1083·6 ÷ ·172.
22. ·155 ÷ 12·4. **23.** 155 ÷ ·0124. **24.** ·0155 ÷ 124.
25. 885·5 ÷ 3·22. **26.** ·08855 ÷ 27·5. **27.** 624 ÷ 192.
28. ·0624 ÷ ·0325. **29.** ·8568 ÷ 11·9. **30.** 85·68 ÷ ·0072.

Find the values of the following, correct to 3 significant figures:

31. 1 ÷ 3·14. **32.** ·01 ÷ 31·4. **33.** 100 ÷ ·73. **34.** ·86 ÷ ·062.
35. 17·34 ÷ 29. **36.** ·00745 ÷ ·602. **37.** 770 ÷ 89. **38.** ·606 ÷ ·084.
39. 3·07 ÷ 40·5. **40.** 1 ÷ ·0623. **41.** ·0746 ÷ 13·7. **42.** 69 ÷ 1300.
43. ·1011 ÷ 11·1. **44.** ·839 ÷ ·0909. **45.** 13¾ ÷ ·384.

46. Express in shillings, £0·6, £0·8, £·45.

47. Express in pence, 0·75s., 0·625s., 0·375s.

48. Express 0·4 ton in cwt.; 0·75 cwt. in qr.; 0·375 lb. in oz.; 0·125 m. in yd.

49. Express 16 cwt. as a decimal of 1 ton.
17s. ,, ,, £1.
21·6 in. ,, ,, 1 yd.

50. Express 3·65 tons in cwt.; 4·34 yd. in ft. £8·14 in s.; 5·28s. in pence.

The connection between British and Metric units should be illustrated by practical work and examples taken orally (see p. 70).

EXERCISE VI (j) (Oral and Practical)

1. On squared paper ruled in inches and tenths of an inch draw a line 10 cm. long. State its length in inches as accurately as you can, and express 1 cm. as a decimal of 1 in. *Calculate* how many cm. there are in 1 in.

2. Take two rulers, one graduated in inches and one in centimetres, and place them side by side and fill in the following blank spaces :

5 in. = ... cm. ; 15 cm. = ... in.
∴ 1 in. = ... cm. ; ∴ 1 cm. = ... in.

3. Estimate the length of the page of this book in inches and in cm.

4. Draw a straight line and divide it by eye (i) into inches, (ii) into cm.

5. State approximately your height in cm.

6. State approximately the number of inches in 1 dm.

7. Express roughly in km., 5 miles, 20 miles, 100 miles.

8. Express roughly in miles, 10 km., 50 km., 100 km.

9. Which is the longer, 100 metres or 110 yd. ?

10. On paper, squared in cm., draw a square of side 1 dm. How many (i) sq. cm., (ii) sq. mm. does it contain ? Mark off on it areas of (i) ·25 sq. dm., (ii) 600 sq. mm., (iii) ·0005 sq. m., (iv) 10 sq. cm.

11. On paper squared in inches and tenths of an inch draw a square of side 10 cm. Count the number of small squares it contains, and hence find the area in sq. inches. Use this (i) to express 1 sq. cm. in sq. inches, and (ii) to find the number of sq. cm. in 1 sq. inch.

Check your results by calculation, assuming that 1 in. = 2·54 cm., and 1 cm. = ·3937 in.

12. Estimate the size of this page in sq. cm.

13. Draw a square of area 16 sq. cm.

14. Estimate the area of the ceiling of the room in sq. metres.

15. How many acres are there approximately in 1 sq. km.?

16. Express roughly in litres, 5 pints, 2 quarts, 1 gallon.

17. Express in litres, 2000 cu. cm., 500 cu. cm., 1 cu. m.

18. Express roughly in pints, 10 litres, 200 cu. cm., 3 cu. dm.

19. How many cu. cm. approximately are there in 1 pint ?

20. A man weighs 70 Kg. What is this roughly in stones ?

21. Express roughly in Kg., 1 cwt., 1 stone, 1 ton.

22. What is the weight of (i) 1 litre of water, (ii) 1 cu. metre of water?

23. What volume of water (in c.c. and cu. ft.) weighs (i) 5 Dg., (ii) 125 lb.?

24. What is the weight of 4 pints of water ?

25. How many gallons of water weigh 50 lb.?

26. Given that any volume of olive oil weighs ·9 times as much as the same volume of water, state approximately the weights of (i) 1 litre of oil, (ii) 1 gal. of oil, (iii) 1 cu. dm. of oil, (iv) 1 cu. ft. of oil.

27. The area of a field is 100 hectares. What is this approximately in acres ?

28. A cubical jar holds 8 litres. What is the length of its edge in inches, approximately ?

29. A tank is 2 metres long, 1 metre wide, 1 metre deep. How many litres will it hold ?

30. A litre of mercury weighs 13·6 kg. What is the weight (in oz. roughly) of 1 cu. ft. of mercury ?

Decimalisation of Money

Learn carefully the following results :

$$\text{1s.} = \pounds\tfrac{1}{20} = \pounds\tfrac{5}{100} = \pounds\underline{\cdot 05.}$$
$$\tfrac{1}{4}\text{d.} = \pounds\tfrac{\cdot 05}{48} = \pounds\cdot 001\tfrac{2}{48} = \underline{\pounds\cdot 001\tfrac{1}{24}.}$$

Example.—Express 13s. 2½d. *as the decimal of £1 correct to 4 places.*

13s. $= \pounds\cdot 05 \times 13 = \pounds\cdot 65.$
2½d. $= 10$ farthings $= \pounds 10(\cdot 001\tfrac{1}{24}) = \cdot 010\tfrac{10}{24} = \cdot 010\tfrac{5}{12} = 10\tfrac{5}{12}$ thousandths of £1 $= 10\cdot 41\ldots$ thousandths of £1 $= \pounds\underline{\cdot 0104.}\ldots$

The work is done mentally : we write :

$$\begin{array}{rl} \text{13s.} = & \pounds\cdot 65 \\ 2\tfrac{1}{2}\text{d.} = & \underline{\cdot 010\tfrac{10}{24}} \\ & \pounds\underline{\cdot 6604} \end{array}$$

Example.—Express 16*s.* 7¼*d. as a decimal of £1.*

$$\begin{array}{rlrlll} \text{16s.} = & \pounds\cdot 80 & 7\tfrac{1}{4}\text{d.} = & 29\tfrac{29}{24} & \text{thousandths} & \text{of } \pounds 1. \\ 7\tfrac{1}{4}\text{d.} = \text{29 farthings} = & \underline{\cdot 029\tfrac{29}{24}} & = & 30\tfrac{5}{24} & ,, & ,, \\ & \underline{\cdot 8302} & = & 30\cdot 2 & ,, & ,, \end{array}$$

It is rarely necessary to express decimals of £1 in pence more accurately than to the nearest 1d. ; we may therefore take ¼d. = £·001, remembering that this is an under-statement.

Example.—Express £2·384 in £, s. d.

£·35 = 7s. ∴ £·384 = 7s. + £·034
= 7s. + 34 farthings (rather less) = 7s. 8d.
∴ £2·384 = £2, 7s. 8d. to nearest 1d.

Otherwise £2·384
shgs. 7·680 (multiplying by 20).
pence 8·160 (,, ,, 12).

Decimalisation of Compound Quantities

Example.—What decimal of 1 ton is 12 cwt. 3 qr. 10 lb. (correct to 3 places)?
Rough check, about $\frac{1}{2}$ = ·5.

28 { 4	10 lb.	Divide 10 lb. by 28 to
7	2·5	bring to quarters.
4	3·3571 qr.	(add the 3 qr.).
20	12·8393 cwt.	(add the 12 cwt.).
	·64196 ton.	

Answer—·642 ton (correct to 3 places).

Example.—Express 3·642 miles as a compound quantity.

3·642 miles.
8
5·136 furlongs.
220
272
272
29·92 yards.

Answer—3 m. 5 fur. 30 yd.

EXERCISE VI (k)

Express as decimals of £1 correct to 4 places:

. £2, 13s. 6d. **2.** £1, 17s. $4\frac{1}{2}$d. **3.** £1, 11s. $6\frac{1}{4}$d. **4.** £1, 0s. $2\frac{1}{2}$d.
. 18s. $3\frac{3}{4}$d. **6.** 17s. $10\frac{1}{2}$d. **7.** £1, 11s. $3\frac{1}{4}$d. **8.** £1, 19s. $9\frac{1}{2}$d.
. £3, 4s. $11\frac{1}{2}$d. **10.** £3, 6s. $11\frac{1}{4}$d. **11.** £1, 2s. $7\frac{1}{2}$d. **12.** £1, 13s. $5\frac{1}{4}$d.

Express in £, s. d. correct to the nearest 1d.:

3. £·0364. **14.** £·1258. **15.** £3·2167. **16.** £1·9298.
7. £1·0307. **18.** £2·0068. **19.** £3·2494. **20.** £·0086.
1. £1·0807. **22.** £1·7988. **23.** £1·5649. **24.** £2·0793.

Express (to 3 places) as a decimal of the highest unit given:

5. 2 t. 15 cwt. 3 qr. **26.** 1 t. 14 cwt. 2 qr. **27.** 14 cwt. 2 qr. 18 lb.
. 12 yd. 2 ft. 8 in. **29.** 15 yd. 2 ft. 7 in. **30.** 1 m. 620 yd.
. 16 ac. 2 ro. 32 po. **32.** 12 d. 14 hr. 32 min. **33.** 3 gal. 2 qt. 1 pt.
. 3 m. 5 fur. 140 yd.

Express as compound quantities to the nearest unit of the quantity in brackets:

35. 16·432 yd. (in.). **36.** 1·0482 tons (lb.). **37.** 3·248 tons (lb.).
38. 6·437 ac. (po.). **39.** 7·436 m. (mm.). **40.** ·6348 days (min.).
41. ·8764 cwt. (oz.). **42.** £·0587 (pence).

43. 15 million farthings weigh 42 tons. Find the weight of a farthing as a decimal of 1 oz. (1 place).

44. Express as the decimal of 1 acre the area of a rectangular field 350 ft. by 108 ft.

45. 1 gallon = 4·5435 litres. What decimal of 1 gallon is 1 litre?

46. 1 lb. = 0·4536 Kg. Find the value of 1 Kg. in lb.

47. 1 yard = 0·9144 m. Find the equivalent of 1 metre in yards.

48. What decimal of £5 is £2, 7s. 4d.?

49. What decimal of 1 ton is 14 cwt. 2 qr. 12 lb.?

50. Express £2, 10s. as a decimal of £5, and £2, 4s. 9d. as a decimal of £5, 12s. 8d. (3 places).

The method of Practice may usefully be employed with decimals.

Example.—Given that 1 *lb.* = 0·4536 *Kg., express* 9 *lb.* 13¼ *oz. in Kg.*

Kg.	
0·4536	1 lb.
9	
4·0824	9 lb.
·2268	8 oz.
·1134	4 oz.
·0283	1 oz.
·0071	¼ oz.
4·4580	

Answer—4·46 Kg. approximately.

EXERCISE VI (1)

1. Given that 1 mile = 1·609 Km., express 3 m. 5 fur. 120 yd. in Km.

2. Given that 1 gallon = 4·546 litres, express 2 gal. 3 qt. 1 pt. in litres.

3. Given that 1 cu. ft. = 6·232 gallons, express 4 cu. ft. 1000 cu. in. in gallons.

4. Given that 1 inch = 2·54 cm., express 4⅝ in. in cm.

5. Given that 1 yard = 0·9144 m., express 12 yd. 2 ft. 5 in. in metres.

6. Given that 1 lb. = 0·4536 Kg., express 12 lb. 5¾ oz. in Kg.

7. Given that 1 ton = 1·016 metric tons, express 2 tons 12 cwt. 1 qr. n metric tons.

8. Given that 1 cu. ft. of water weighs 62·4 lb., find the weight of 3 cu. ft. 1500 cu. in. of water. (Answer in lb.)

9. Given that 1 gallon = 0·1605 cu. ft., express 3 gal. 1 qt. 1 pt. in cubic feet.

10. Given that the circumference of a circle is 3·1416 × its diameter, xpress in feet the circumference of a circle of diameter 2 ft. 7$\frac{1}{2}$ in.

11. Evaluate £2, 14s. × 2$\frac{17}{24}$.

12. Find the value of £13, 1s. × 3$\frac{11}{16}$.

MISCELLANEOUS EXAMPLES ON DECIMALS

EXERCISE VI (m)

1. Express in pence £0·15 and £0·0375.

2. Subtract 0·01 from 0·1 and divide the result by ·3.

3. Find the difference between the squares of 3·1 and 2·1.

4. Express the difference between ·1 metre and ·03 metre (i) in nm., (ii) in km.

5. Five boys are given the dimensions of a triangle and told to draw t and measure the longest side; their results are 8·63, 8·67, 8·59, ·64, 8·60 cm. What is the average of these results?

6. The rainfall on successive days of a week in April was 0·36 in., ·08 in., 0·11 in., 0·41 in., 0·32 in., 0·05 in., 0·09 in. What was the otal rainfall for the week?

7. A metre is about 39·37 inches. Find in inches the difference etween 12 metres and 13 yards.

8. A kilometre is about ·62137 miles. Find in yards the difference etween 8 kilometres and 5 miles.

9. Express 5000 lb. in tons correct to 2 places of decimals.

10. What is the perimeter of a triangle whose sides measure 1·17 m., 8·39 cm., 7·78 cm.?

11. Express in metres the sum of 5 m. 8 cm., 62 dm., 17 cm., dm. 58 mm., 347 mm., 13 dm. 7 mm.

12. Find the cost of 2·4 metres of cloth at 6·20 fr. per metre.

13. A beaker weighs 48·42 gr. when empty, and 123·27 gr. when ll of water. Find the weight of the water and the capacity of the eaker.

14. A man walks 2 km. 410 m., then 3 km. 520 m., and then turns ack and walks 4 km. 810 m. along the same road. How far is he om his starting-place?

15. The outside diameter of a metal tube is 2·34 cm. Find the bore if the metal is 6·2 mm. thick.

16. A car travels 120 m. in 6 sec. Express this rate in km. per hour.

17. Find the cost of 12·3 Kg. of tobacco at 42·20 fr. per Kg.

18. The pre-war rate of exchange was 25 fr. 40 c. to the £. When it became 78 fr. 20 c., how many more francs did you get for £5 than before the war ?

19. A beaker weighs 24 gr. 5 cg. when empty ; it holds 1·5 litres. Find the total weight when it is full of water.

20. If the rainfall is 2·4 cm., find how many litres fall on 1 hectare (10,000 sq. m.).

21. If 250 grams of water leak from a vessel every minute, how many litres escape in 3 hours ?

22. The French " 75 " gun has a bore of 75 mm. Express this in inches.

23. What number is halfway between 13·87 and 17·33 ?

24. The heights of two people are 167·8 cm. and 181·3 cm. The shorter stands on a stool 23·9 cm. high, and the other stands on the ground. What is the difference in level of the tops of their heads ?

25. Taking 1 metre $=$ 39·37 in., express 1 cm. in inches and 1 inch in centimetres correct to 2 significant figures.

26. Taking 1 lb. as ·4536 Kg., express 1 cwt. in Kg. to 3 significant figures.

27. A train averages 48·4 miles per hour between London and Winchester (66 miles). How many minutes does the journey take (to the nearest minute) ?

28. £1 is worth 24·4 Swiss francs. How much British money does a man get for 500 Swiss francs ?

29. The circumference of a circle $= \pi \times$ the diameter where $\pi \overset{\frown}{=} 3{\cdot}1416$. Find, to 2 significant figures, (i) the circumference of a circle of diameter 9·2 in., (ii) the diameter of a circle of circumference 15·4 in.

30. The pressure in a cylinder is 163 lb. per sq. inch. What is the total pressure on a piston of area 3·28 sq. in., correct to nearest lb. ?

31. 1 sq. dm. of a sheet of cardboard weighs 8·71 gr. What is the area of a piece of similar cardboard of irregular shape which weighs 11·34 gr. ? (Answer to 3 significant figures.)

32. Express as a decimal to 2 significant figures the difference between $\frac{13}{23}$ and $\frac{8}{17}$.

33. The thickness of a book without the cover is 4·2 cm., and its pages are numbered from 1 to 716. Find in mm. the thickness of the paper used, correct to 2 significant figures.

34. The distance of Venus from the Sun is about ·77 times the distance of the Earth from the Sun. If both move in circles, and the distance of the Earth from the Sun is 93 million miles, what is approximately the greatest and least distances between Venus and the Earth?

35. A bar of iron 12 ft. long weighs 17·34 lb.; if a length of 18 in. is cut off, what will be the weight of the remainder?

36. $-\cdot 74 = -1 + \cdot 26$. Express in a similar form, that is a negative integer + a decimal, the following: (i) $-1\cdot 39$, (ii) $2\cdot 63 - 3\cdot 17$, (iii) $2(-2 + \cdot 58)$, (iv) $(-3 + \cdot 86) \div 2$, (v) $3\cdot 2(-2 + \cdot 71)$.

37. From a rod 35 in. long portions, each of length 2·7 in., are cut off. How many can be cut off, and what length remains over?

38. An empty flask weighs 157·37 gr., and when full of water weighs 193·13 gr. What will it weigh when half full of water?

39. Simplify $\left(\frac{7}{0\cdot 14} + \frac{3}{0\cdot 12}\right) \div \left(\frac{7}{0\cdot 14} - \frac{3}{0\cdot 12}\right)$.

40. The sides of a triangle are measured as 7·3, 6·9, 4·7 cm., and there may be an error of as much as ·05 cm. in each measurement. Between what limits must the perimeter of the triangle lie?

41. Given that 1 mile = 1·609 Km., express by Practice 1000 yd. in Km.

42. Given that 1 lb. = 0·4536 Kg., express 10 lb. $11\frac{3}{4}$ oz. in Kg.

CHAPTER VII

PRIME FACTORS: H.C.F. AND L.C.M.

If one number can be divided by a second number without remainder, the second number is said to be a **factor** of the first, and the first number is said to be a **multiple** of the second.

$48 \div 6 = 8$. ∴ 6 is a **factor** of 48; note also that 8 is a factor.

Further, 48 is a **multiple** of 6 and also a multiple of 8.

If a number has no factors except itself and 1, it is called a **prime number**, *e.g.* 2, 3, 5, 7, 11, 13, etc., are prime numbers.

Example.—What numbers are factors of 63, and which of them are prime?

$63 = 1 \times 63 = 3 \times 21 = 7 \times 9$.
∴ the factors are 1, 3, 7, 9, 21, 63,
and of these 1, 3, 7 are prime numbers

EXERCISE VII (a)

1. Which numbers between 20 and 30 are prime ?

2. What are the prime numbers less than 50 which end with 9 ?

3. Express as a product of two numbers in as many ways as possible (i) 28, (ii) 36.

4. Write down all the multiples of 6 between 20 and 40.

5. Which of the following numbers are multiples of 6 : 2, 3, 8, 18, 26, 48, 150 ?

6. Which of the following numbers are factors of 6 : 2, 3, 4, 12, 18 ?

7. What prime numbers are factors of 84 ?

8. Is 137 a prime number ? Why is it unnecessary to try any factor above 11 ?

9. If 221 is not a prime number, it must have a factor less than 15. Why is this ?

10. By what numbers must you divide 211 in order to find out whether it is a prime number or not ?

Indices

If a factor is a prime number, it is called a **prime factor,** *e.g.* 11 is a prime factor of 44.

The product of several equal numbers is called a **power** of that number, *e.g.* $6 \times 6 \times 6 \times 6$ is a power of 6 ; it is written 6^4 and read as " 6 to the power 4."

The number of times the factor is repeated is called the **index,** *e.g.* in 6^4 the index is 4.

A number raised to the power 2 is said to be **squared,** *e.g.* 6^2 is the square of 6 ; and a number raised to the power 3 is said to be **cubed,** *e.g.* 6^3 is the cube of 6.

Example.—Express in index form $2 \times 2 \times 3 \times 2 \times 3 \times 3 \times 2 \times 2$.

Rearranging the factors, the number $= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$$= \underline{\mathbf{2^5 \times 3^3}}.$$

Example.—Express in prime factors 1584.

$$1584 = 2 \times 792 = 2^2 \times 396 = 2^3 \times 198 = 2^4 \times 99 = 2^4 \times 3 \times 33$$

$$= \underline{\mathbf{2^4 \times 3^2 \times 11.}}$$

N.B.—Choose the factors in ascending order of magnitude ; first try 2 and repeat as often as possible, then 3, 5, 7, 11 etc., in turn (prime numbers only).

Example.—Simplify (i) $2^3 \times 2^2$, (ii) $(2^3)^2$.

$$2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) \qquad = \underline{2^5.}$$
$$(2^3)^2 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = \underline{2^6.}$$

N.B.—The fact that $2^3 \times 2^2$ is not the same as $(2^3)^2$ requires emphasis.

Example.—Express in prime factors the **square** *of* $2^4 \times 3^3 \times 11$.

$$(2^4 \times 3^3 \times 11) \times (2^4 \times 3^3 \times 11) = 2^4 \times 2^4 \times 3^3 \times 3^3 \times 11 \times 11$$
$$= \underline{2^8 \times 3^6 \times 11^2.}$$

The **square root** of a number is that number whose square is equal to the given number: *e.g.* $6^2 = 36$; $\therefore$ 6 is the square root of 36.

Note.—$(-6) \times (-6) = 36$; $\therefore$ -6 is also a square root of 36. A number has therefore two square roots, but in arithmetic we are concerned usually only with the positive square root.

The cube root of a number is that number whose cube is equal to the given number: *e.g.* $6^3 = 216$; $\therefore$ 6 is the cube root of 216.

Example.—Express in prime factors the square root of $2^8 \times 3^6 \times 11^2$.

The square of $2^4 \times 3^3 \times 11$ is $2^8 \times 3^6 \times 11^2$.
$\therefore$ the square root of $2^8 \times 3^6 \times 11^2$ is $2^4 \times 3^3 \times 11$.

Larger factors.—At a later stage the work may often be shortened by taking out a large factor (whether prime or not) which is obvious from inspection;

e.g. $864 = 8 \times 108 = 8 \times 9 \times 12 = 2^3 \times 3^2 \times 2^2 \times 3 = \underline{2^5 \times 3^3}$
or $171717 = 17 \times 10101 = 17 \times 3 \times 3367$
$= 17 \times 3 \times 7 \times 481 = 17 \times 3 \times 7 \times 13 \times 37$
$= \underline{3 \times 7 \times 13 \times 17 \times 37.}$

EXERCISE VII (b)

Write in index form with prime factors:

1. $2 \times 2 \times 2 \times 3 \times 3$.
2. $3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$.
3. $2 \times 3 \times 4 \times 5 \times 6$.
4. 4×8.
5. $3^2 \times 3^3$.
6. $2 \times 2^2 \times 2^3 \times 18$.
7. $2^5 \div 2^3$.
8. $3^{10} \div 3^5$.
9. $2^4 \times 2^2 \div 2$.
10. $3^2 \times 3^3 \div 3^4$.
11. $2^6 \times 3^4 \div (2^3 \times 3^2)$.
12. 10^3.
13. $6^4 \times 3^2$.
14. $6^4 \div 3^2$.
15. $2^4 \times 4^2$.

Simplify

16. $2^3 \times 5^3$.
17. $2^3 \times 5^2 \times 2^2 \times 5^3$.
18. $a^3 \times a^2$.
19. $(a^2)^3 \div (a^3)^2$.
20. $(a^4)^2 \div a^4$.
21. $(ab)^2 \div ab^2$.

Simplify, leaving the answer in factors,

22. $10^3 \div 5^3$.
23. $12^3 \div (2^2 \times 3^3)$.
24. $2^2 \times 3^3 \times 2^2 \times 3$.
25. $3^4 \times 5^3 \times 11 \div (3^2 \times 5^2)$.
26. $36^2 \div 3^2$.
27. $36^4 \div (4^3 \times 3^2)$.

Express the following in prime factors:

28. 36. **29.** 120. **30.** 144. **31.** 196. **32.** 54.
33. 65. **34.** 106. **35.** 180. **36.** 111. **37.** 216.
38. 252. **39.** 864. **40.** 315. **41.** 343. **42.** 1001.
43. 1430. **44.** 1584. **45.** 8736. **46.** 111,111. **47.** 4356.

48. Is 3^6 a perfect square? If so, of what is it the square?

49. Is 3^6 a perfect cube? If so, of what is it the cube?

50. Express in prime factors the square of

(i) $2^2 \times 3^2$, (ii) $2^3 \times 3^2$, (iii) $2^4 \times 3^2 \times 5$, (iv) 2^5.

51. Express in prime factors the cubes of

(i) $2^2 \times 3^2$, (ii) $2^3 \times 3$, (iii) 5^4, (iv) 12.

52. Express in prime factors a square root of

(i) $3^4 \times 5^2$, (ii) 2^{16}, (iii) $7^4 \times 11^2$, (iv) 9^5.

53. Express in prime factors a cube root of

(i) $2^3 \times 3^3$, (ii) 2^6, (iii) $5^6 \times 7^3 \times 11^3$, (iv) 2^{27}.

54. Find by factors a square root of

(i) 576, (ii) 441, (iii) 17,424, (iv) 22,500.

55. Find by factors a cube root of

(i) 1728, (ii) 27,000, (iii) 288×48, (iv) $36 \times 72 \times 144$.

56. What is the least number by which 540 must be multiplied to make the result (i) a perfect square, (ii) a perfect cube?

57. Express (i) 2^8 as a power of 4, (ii) 3^{27} as a power of 27, (iii) x^8y^8 as a power of x^2y^2.

58. What is the least expression by which $4ab^2c^6$ must be multiplied to make the result (i) a perfect square, (ii) a perfect cube?

59. What is (i) the cube root of x^{27}, (ii) the square root of x^{16}?

60. A bar of lead is 4 in. wide, 6 in. deep, and 6 ft. long; it is melted down and cast into the form of a cube. What is the length of its edge?

H.C.F. and L.C.M.

The **Highest Common Factor** (H.C.F.) of several numbers is the greatest number which is a factor of each of them.

Thus the H.C.F. of 32, 48, 56 is 8.

The **Lowest Common Multiple** (L.C.M.) of several numbers is the smallest number which is a multiple of each of them.

Thus the L.C.M. of 6, 8, 12 is 24.

Example.—Find the H.C.F. of $2^3 \times 3$ *and* $2^2 \times 3^2 \times 5$.

$$2^3 \times 3 = 2 \times 2 \times 2 \times 3.$$
$$2^2 \times 3^2 \times 5 = 2 \times 2 \times 3 \times 3 \times 5.$$
$$\therefore \text{ the H.C.F.} = 2 \times 2 \times 3 = \underline{2^2 \times 3}.$$

Example.—Find the H.C.F. of 30, 60, 105.

$$30 = 2 \times 15 = 2 \times 3 \times 5.$$
$$60 = 2 \times 30 = 2 \times 2 \times 15 = 2 \times 2 \times 3 \times 5.$$
$$105 = 3 \times 35 = 3 \times 5 \times 7.$$
$$\therefore \text{ the H.C.F.} = 3 \times 5 = \underline{15}.$$

Note.—It was unnecessary to factorise 60, because any factor of 30 must be a factor of 60.

Example.—Find the L.C.M. of 24, 36, 48, 75.

$$24 = 2 \times 12 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3.$$
$$36 = 2 \times 18 = 2 \times 2 \times 9 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2.$$
$$48 = 2 \times 24 = 2 \times 2 \times 12 = 2 \times 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3.$$
$$75 = 3 \times 25 = 3 \times 5 \times 5 = 3 \times 5^2.$$
$$\therefore \text{ the L.C.M.} = \underline{2^4 \times 3^2 \times 5^2}.$$

Note.—It was unnecessary to factorise 24, because any multiple of 48 must be a multiple of 24.

Example.—Find the H.C.F. and L.C.M. of $4abc$, $6a^2b^2c^2$, $10a^3b^3c^3$.

The H.C.F. is clearly $2abc$, since

$$4abc = 2abc(2).$$
$$6a^2b^2c^2 = 2abc(3abc).$$
$$10a^3b^3c^3 = 2abc(5a^2b^2c^2).$$

The L.C.M. is 60 $a^3b^3c^3$.

Division Method for H.C.F.

Example.—Find the H.C.F. of 2613, 10,868.

2613)10,868(4
10,452
416)2613(6
2496
117)416(3
351
65)117(1
65
52)65(1
52
13)52(4
52

$\therefore$ **H.C.F.** = **13** (the last divisor).

It saves time, however, to arrange the above work as follows:

6	2613	10,868	4
	2496	10,452	
1	117	416	3
	65	351	
4	52	65	1
	52	52	
		13	

Explanation.—Divide 2613 into 10,868, the remainder is 416; divide

416 into 2613, the remainder is 117; divide 117 into 416, the remainder is 65; and so on until there is no remainder: the last divisor is the H.C.F.

The proof of this method depends on the fact that any common factor of two numbers is a factor of their difference, *e.g.* 7 is a common factor of $7x$ and $7y$; their difference is $7x-7y$ or $7(x-y)$, so that 7 is a factor of their difference.

In the above example the H.C.F. of 2613 and 10,868 must be a common factor of $2613\times4=10,452$ and 10,868, and must $\therefore$ be a factor of $10,868-10,452=416$: this argument is repeated in each step of the division until there is no remainder.

EXERCISE VII (c)

1. Of which of the numbers 2, 2^3, $2^2\times3^2$, $2^2\times3\times5$, $2\times3^2\times5^2$ is $2^2\times3$ a factor?

2. Of which of the numbers 2, 2×3, $2^2\times3^2\times5$, 2×3^2, $2^3\times3^3$ is $2^2\times3^2$ a multiple?

3. Is 3×5^2 a common factor of $3^2\times5^2$ and $3\times5^2\times7$? Give reasons.

4. Is $3^2\times5^2$ a common multiple of 3^2 and 3×5^2? Give reasons.

5. Write down all the factors of 24 and all the factors of 36; write down all the common factors of 24 and 36; write down the highest common factor of 24 and 36.

6. Write down all the multiples of 18 less than 100; write down all the multiples of 24 less than 100; write down all the common multiples of 18 and 24 less than 100; write down the lowest common multiple of 18 and 24.

7. Write down all the common factors of 24 and 30. Which is the highest common factor?

8. Write down all the common multiples of 4 and 6 which are less than 50. What is the lowest common multiple?

9. Write down all the common factors of $2^3\times3^3$, $2^2\times3^2\times5$, $2\times3^2\times7$, $2^4\times3^4\times5^2$. What is the highest common factor?

10. Write down three common multiples of 2×3^2, $2^2\times3$, $3^2\times5$, including the lowest common multiple.

11. Write down all the common factors and three common multiples of $3^3\times5^2$, $3^2\times5\times7^2$, $2\times3\times5$. What is the highest common factor? What is the lowest common multiple?

Find by prime factors the H.C.F. and L.C.M. of the following: give the L.C.M. in prime factors:

12. 8, 12. **13.** 21, 35. **14.** 36, 90. **15.** 126, 350.
16. 96, 144. **17.** 45, 360. **18.** 91, 140. **19.** 720, 1200.
20. 128, 324. **21.** 495, 3150. **22.** 187, 1111. **23.** 780, 455.
24. 16, 24, 40. **25.** 56, 70, 98. **26.** 36, 126, 900. **27.** 75, 100, 125.

28. 96, 144, 240. 29. 14, 16, 32, 56. 30. 52, 78, 130, 182.

31. 69, 115, 161, 460.

Find the H.C.F. and L.C.M. of

32. $34a^6b^2c^5$, $48a^5b^4c^3$. 33. $3a^2b^2c$, $6a^3bc^2$, $9a^2b^3c^3$.

34. $2x^4y^2z$, $5x^3y^3$, $10x^2yz^2$. 35. $12x^2z^2$, $15xyz$, $30x^3y^3$.

36. $2^4\times3^2\times5^3$, $2^2\times3^3\times5\times7$, $2^5\times3^4\times5^2\times7^2$.

37. Find the L.C.M. of $2^3\times5^3\times11$, $2^2\times5\times7\times11^2$. How many times does it contain each term?

38. Find the H.C.F. of $2^3\times4\times6^2\times100$, $4^3\times9^2\times15$.

39. Find the L.C.M. of $3^2\times6^2\times8^2$, $4^2\times5^2\times7\times9$. How many times does it contain each term?

40. Is there a common factor of 5×11, 7×11, $7\times11-5\times11$?

41. If 175 and 161 have a common factor, show that it must be a factor of 14.

42. Why is it obvious that 663 and 674 have no common factor?

43. Is there a common factor of 1737 and 1759?

44. Write down the H.C.F. of 2169 and 2181.

Find by the division method the H.C.F. of

45. 8397, 9641. 46. 2147, 3390. 47. 2057, 3417.

48. 7752, 24814. 49. 7992, 10989. 50. 2295, 7020.

Problems on Factors, H.C.F., L.C.M.

EXERCISE VII (d)

1. What factor of 45 is a perfect square?

2. What is the least multiple of 24 that is a perfect square?

3. Find the smallest whole number by which 88 must be multiplied to give (i) a perfect square, (ii) a perfect cube.

4. A book weighs 28 oz. What is the least number of copies of it which together weigh a whole number of pounds?

5. What is the least sum of money that contains an exact number of half-crowns and florins?

6. What is the least length of a piece of wire that can be cut into equal strips either 2 ft. or 2 ft. 4 in. or 2 ft. 6 in. long?

7. Two measuring rods are respectively 15 in. and 24 in. long. What is the least length that can be measured exactly by either rod?

8. Two companies of 144 and 176 men respectively are told off into squads of equal strength. What is the least number of squads in each company?

9. A room 15 ft. 9 in. long, 12 ft. wide, is paved with square blocks. What is the largest size of block that can be used, if no block is to be cut ?

10. What is the least sum of money that contains an exact number of guineas or crowns or florins ?

11. What is the smallest number that is divisible by every number up to 15 ?

12. Lamp-posts on one side of a road are placed at intervals of 60 yd., and on the other side at intervals of 75 yd. What is the least distance between pairs of lamp-posts directly opposite each other ?

13. What is the greatest length of a measuring rod which could be used to measure exactly the sides of a rectangle 54 ft. long, 30 ft. wide ?

14. Three bells are tolling at intervals of 4, 6, 10 minutes : they all start at 12 o'clock. What is the time when they next toll all together ?

15. Two wheels in gear contain 24 and 30 teeth respectively. What is the least whole number of revolutions the first must make if in the same time the second also makes a whole number of revolutions ?

16. Find the H.C.F. of 882, 396, 1404.

17. Simplify (i) $5^2 \times 13 - 3 \times 5^2$, (ii) $7 \times 11^2 + 11^2 \times 13$.

18. What is the least number (other than 3) which leaves a remainder 3 when divided by 7 or by 11 ?

19. What is the greatest number such that, when 45 or 61 are divided by it, the remainder is 5 ?

20. What is the least number of four digits which, when divided by 17 or 41, leaves a remainder 5 ?

21. Find the prime factors of 111111111111, given that 9901 is a prime number.

22. Find the two least consecutive prime numbers whose sum is a perfect square.

23. A pile of books is such that when tied up in bundles of (i) 2 there is 1 over, (ii) 3 there are 2 over, (iii) 5 there are 4 over, (iv) 7 there are 6 over. What is the least number of books in the pile ?

24. The product of any two numbers is equal to the product of their H.C.F. and L.C.M. Verify this in the following cases : (i) $2^2 \times 3^3 \times 5$, $2 \times 3^4 \times 7$; (ii) 56, 24 ; (iii) ax, ay.

25. The H.C.F. of three numbers is 36, and their L.C.M. is $2^4 \times 3^4 \times 5^2 \times 7$. Two of the numbers are 360, 432. What is the third number ?

26. The H.C.F. of four numbers is 2^2, and their L.C.M. is $2^5 \times 3^2 \times 5^3$. Three of the numbers are 36, 48, 300. What is the fourth number ?

27. If the books on a shelf are tied together in bundles of six, there are five over. How many are over if they are put into bundles of three ?

28. The ascent of a tower is made by a corkscrew spiral staircase : each step is 5 in. high, and consists of a sector of a circle of angle 33°. The top step is exactly vertically above the bottom step. What is the least height of the tower ?

Tests of Divisibility

A number is divisible by **2,** if its last digit is divisible by 2.

A number is divisible by **4,** if the number formed by its last two digits is divisible by 4 ; *e.g.* 24 is divisible by 4, ∴ 31,724 is divisible by 4.

A number is divisible by **8,** if the number formed by its last three digits is divisible by 8 ; *e.g.* 264 is divisible by 8, ∴ 970,264 is divisible by 8.

A number is divisible by **5,** if its last digit is either 5 or 0.

A number is divisible by **10,** if its last digit is 0.

A number is divisible by **3,** if the sum of its digits is divisible by 3 ; *e.g.* $1+7+2+5$ is divisible by 3, ∴ 1725 is divisible by 3.

A number is divisible by **9,** if the sum of its digits is divisible by 9.

A number is divisible by **11** if, when the 1st, 3rd, 5th, 7th, etc., digits are added together and the 2nd, 4th, 6th, 8th, etc., digits are added together, the difference between the two sums is 0 or is divisible by 11, *e.g.* $(2+7+3)-(4+8)=0$, ∴ 24,783 is divisible by 11, and $(9+9+8)-(1+2+1)=26-4=22$, ∴ 192,918 is divisible by 11.

A number is divisible by **6,** if both 2 and 3 are factors.

A number is divisible by **12,** if both 4 and 3 are factors.

There is no *simple* test for divisibility by 7 or 13. (See Exercise VII (e), No. 12 .)

Example.—Find which of the numbers 3, 4, 8, 9, 11 *are factors of* 45,276.

$4+5+2+7+6=24$; 3 is a factor of 24 but 9 is not a factor of 24, ∴ 3 is a factor of 45,276 but 9 is not a factor. $76 \div 4 = 19$, ∴ 4 is a factor of 45,276. 8 is not a factor of 276, ∴ 8 is not a factor of 45,276. $(4+2+6)-(5+7)=12-12=0$, ∴ 11 is a factor of 45,276.

∴ 3, 4, 11 are factors of 45,276.

EXERCISE VII (e)

1. Which of the following are prime numbers : 27, 37, 57, 67, 117 ?

2. Which of the following numbers are divisible by 3 : 216, 732, 1649, 1924, 101,001, 862,583 ?

3. Which of the following numbers are divisible by 4 : 9316, 87,046, 999,000, 47,222, 79,068 ?

4. Which of the following numbers are divisible by 9: 8622, 10,803, 66,999, 73,701 ?

5. Which of the following numbers are divisible by 11: 5247, 80,102, 11,111, 10,120, 81,818 ?

6. Fill in the missing digit if 862*4 is divisible by (i) 9, (ii) 11, (iii) 8.

7. Fill in the unknown digits in the following:

(i) 8*35 if 11 is a factor. (ii) 16*29 if 9 is a factor.
(iii) 37,241* if 8 is a factor. (iv) 823*4 if 18 is a factor.

8. Is 499 a prime number ?

9. What is the least whole number which is neither prime nor the sum of two prime numbers ?

10. Arrange the digits 1, 2, 3, 9, 9 so as to form a number divisible by 264. Is there more than one answer ? Explain your method.

11. Arrange the digits 1, 3, 5, 6, 9 so that they form a number divisible by 1848.

12. Divisibility of any number less than a million by 7 or 13 may be tested as follows: take the number formed by the last three digits and the number formed by the remaining digits and find their difference: if this is divisible by 7 or 13, so is the original number; *e.g.* 876,512 is divisible by 7 or 13 if $876-512=364$ is divisible by 7 or 13; and 41,132 is divisible if $132-41=91$ is.

Use this rule to find whether 7 or 13 are factors of (i) 174,118, (ii) 33,059, (iii) 724,815, and verify by actual division.

CHAPTER VIII

FRACTIONS

Addition.—If the numerator of a fraction is less than the denominator, the fraction is called "**proper**," *e.g.* $\frac{5}{7}$ is a proper fraction.

If we add $\frac{3}{4}$d. to $\frac{1}{2}$d. we obtain $1\frac{1}{4}$d.; $1\frac{1}{4}$ consists partly of an integer and partly of a fraction, and is called a **mixed number.**

When the numerator of a fraction is greater than the denominator, the fraction is called an "**improper fraction**"; thus $\frac{9}{7}$ is an improper fraction.

Example.—Express $\frac{9}{7}$ as a mixed number.

$$\frac{9}{7}=\frac{7+2}{7}=\frac{7}{7}+\frac{2}{7}=1+\frac{2}{7}=1\frac{2}{7}.$$

Thus 9 days $=\frac{9}{7}$ of a week.

But 9 days $=$ 7 days $+$ 2 days $=$ 1 week $+\frac{2}{7}$ week $=1\frac{2}{7}$ weeks.

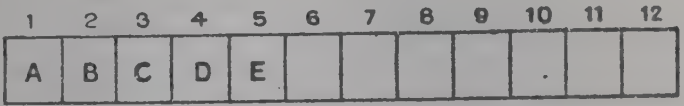
$\therefore$ $\frac{9}{7}$ of a week $=1\frac{2}{7}$ weeks.

EXERCISE VIII (a) (Oral)

1. Express as mixed numbers : $\frac{13}{5}$, $\frac{17}{10}$, $\frac{4}{3}$, $\frac{11}{3}$, $\frac{21}{4}$, $\frac{111}{16}$, $\frac{132}{17}$.

2. Express as improper fractions : $1\frac{2}{3}$, $2\frac{3}{4}$, $1\frac{1}{10}$, $7\frac{1}{6}$, $8\frac{5}{6}$, $11\frac{2}{3}$, $14\frac{3}{5}$, $23\frac{1}{20}$.

3. State from observation of the diagram what fraction of the whole is AB, in its lowest terms.

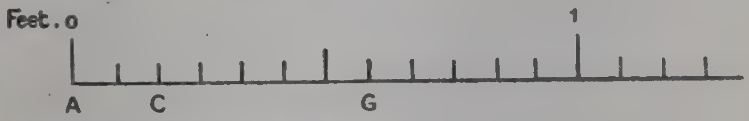


Also what fraction of the whole is CDE, in its lowest terms.

Also what fraction of the whole is ABCDE.

What do you deduce to be the value of $\frac{1}{6}+\frac{1}{4}$?

4. From your foot-rule, by adding AC and CG, find the value of $\frac{1}{6}+\frac{5}{12}$. Use it also to find $\frac{1}{4}+\frac{1}{3}$; $\frac{2}{3}+\frac{1}{4}$; $\frac{1}{3}+\frac{1}{4}+\frac{1}{12}$; $\frac{1}{6}+\frac{1}{2}+\frac{1}{4}$.



Historical Note.—The Egyptian papyrus of Ahmes (2000 B.C.) begins with fractions, but he restricts himself to fractions with unity as the numerator, with the exception of $\frac{2}{3}$, which has a special sign.

Ahmes wrote $\frac{1}{2}+\frac{1}{4}$ for $\frac{3}{4}$; $\frac{1}{6}+\frac{1}{18}$ for $\frac{2}{9}$. The splitting up of fractions was performed by the aid of a table which he compiled.

Thus $\frac{5}{21}=\frac{1+2+2}{21}$. His table gave $\frac{2}{21}=\frac{1}{14}+\frac{1}{42}$.

$$\therefore \frac{5}{21}=\frac{1}{21}+\left(\frac{1}{14}+\frac{1}{42}\right)+\left(\frac{1}{14}+\frac{1}{42}\right)$$
$$=\frac{1}{21}+\frac{2}{14}+\frac{2}{42}=\frac{1}{21}+\frac{1}{7}+\frac{1}{21}$$
$$=\frac{1}{7}+\frac{2}{21}=\frac{1}{7}+\frac{1}{14}+\frac{1}{42}.$$

In a manuscript written more than 1000 years later we find fractions still treated in this way.

The Greeks had a system of writing the numerator of a fraction on the line with the denominator *above* it.

If we add $\frac{3}{4}$ and $\frac{1}{2}$ by expressing both as twelfths, we have $\frac{3}{4}+\frac{1}{2}=\frac{9}{12}+\frac{6}{12}=\frac{15}{12}=1\frac{3}{12}=1\frac{1}{4}$. But we could have expressed both fractions as quarters thus: $\frac{3}{4}+\frac{1}{2}=\frac{3}{4}+\frac{2}{4}=\frac{5}{4}=1\frac{1}{4}$. It is simplest to express the fractions with a denominator which is the *L.C.M.* of the given denominators.

If there are whole numbers they can be added separately thus : $2\frac{1}{3}+3\frac{1}{4}=2\frac{4}{12}+3\frac{3}{12}=5\frac{7}{12}$. Improper fractions should be expressed as mixed numbers before adding them.

EXERCISE VIII (b)

1. What fraction of 1 hr. is 20 min.? 30 min.? 50 min.? Use your result to find the value of $\frac{1}{3}+\frac{1}{2}$. How many sixths is $\frac{1}{3}$? $\frac{1}{2}$?

2. What fraction of 1 hr. is 5 min.? 40 min.? 45 min.? Hence find $\frac{1}{12}+\frac{2}{3}$. How many twelfths in $\frac{2}{3}$? $\frac{3}{4}$?

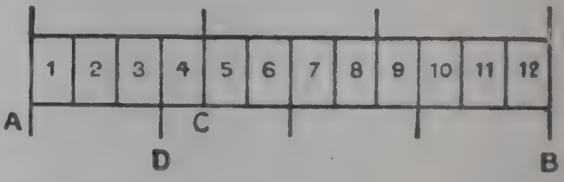
3. What fraction of £1 is 4s., 5s., 6s., 15s.? Hence find $\frac{1}{5}+\frac{1}{4}+\frac{3}{10}$. How many twentieths in $\frac{1}{5}$? $\frac{1}{4}$? $\frac{3}{10}$? $\frac{3}{4}$?

Simplify

4. $\frac{2}{3}+\frac{1}{4}+\frac{5}{6}$.
5. $1\frac{2}{3}+\frac{3}{4}$.
6. $\frac{1}{10}+\frac{2}{100}$.
7. $\frac{3}{100}+\frac{1}{1000}$.
8. $\frac{3}{10}+\frac{2}{100}$.
9. $\frac{1}{6}+\frac{2}{3}+\frac{5}{12}$.
10. $2\frac{3}{4}+1\frac{2}{9}+\frac{5}{36}$.
11. $2\frac{2}{3}+1\frac{5}{6}+\frac{7}{12}$.
12. $3\frac{2}{5}+2\frac{1}{10}+4\frac{2}{15}$.
13. $1\frac{7}{8}+2\frac{1}{12}+3\frac{3}{16}$.
14. $2\frac{5}{6}+1\frac{1}{9}+3\frac{5}{12}$.
15. $4\frac{3}{4}+1\frac{2}{9}+3\frac{5}{24}$.
16. $\frac{a}{3}+\frac{a}{4}+\frac{a}{6}$.
17. $\frac{x}{12}+\frac{x}{8}$.
18. $a+\frac{a}{4}$.
19. $2a+\frac{a}{6}$.
20. $a+\frac{b}{2}$.
21. $\frac{2a}{3}+3b+\frac{b}{2}$.
22. $\frac{1}{a}+\frac{1}{b}$:
23. $\frac{1}{2a}+\frac{1}{3b}$.
24. $3\frac{1}{4}+2\frac{2}{3}+1\frac{1}{8}+4\frac{5}{6}$.
25. $2\frac{1}{16}+3\frac{5}{48}+1\frac{5}{12}+2\frac{3}{8}$.
26. $\frac{26}{7}+1\frac{8}{35}+\frac{34}{15}$.
27. $\frac{29}{9}+\frac{4}{27}+\frac{8}{3}$.
28. $\frac{12}{7}+4\frac{7}{9}+\frac{2}{3}$.
29. $\frac{15}{9}+\frac{1}{2}+\frac{36}{18}$.
30. $\frac{7}{10}+\frac{8}{15}+\frac{23}{20}+\frac{15}{4}$.
31. $3{\cdot}2+4\frac{1}{5}+6{\cdot}7$.
32. $4{\cdot}8+3\frac{1}{4}+{\cdot}6$.
33. $\frac{31}{8}+4\frac{3}{16}+5\frac{1}{16}$.
34. $2\frac{4}{9}+3\frac{5}{12}+\frac{11}{8}$.
35. $3\frac{1}{11}+\frac{19}{11}+\frac{4}{22}$.

Subtraction

The distance AB is divided into 12 equal parts, also into thirds (above) and into fourths (below).



From the scale state what fraction of AB is AC and also AD. Hence deduce the value of $\frac{1}{3}-\frac{1}{4}$. Also find the values of $1-\frac{1}{3}$, $\frac{2}{3}-\frac{1}{4}$.

Example.—Subtract $1\frac{2}{5}$ *from* $3\frac{3}{4}$.

The L.C.M. of 5 and 4 is 20.

$$3\frac{3}{4}-1\frac{2}{5}=3\frac{15}{20}-1\frac{8}{20}=2\frac{15-8}{20}=2\frac{7}{20}.$$

Example.—Subtract $1\frac{2}{5}$ *from* $3\frac{1}{4}$.

$$3\frac{1}{4} - 1\frac{2}{5} = 3\frac{5}{20} - 1\frac{8}{20}.$$

Since $\frac{8}{20}$ is greater than $\frac{5}{20}$, we write $2\frac{25}{20}$ for $3\frac{5}{20}$.

$$= 2\frac{25}{20} - 1\frac{8}{20}.$$

$$= \underline{1\frac{17}{20}}.$$

EXERCISE VIII (c)

Evaluate

1. $\frac{1}{5} - \frac{1}{8}$. **2.** $\frac{1}{4} - \frac{1}{9}$. **3.** $\frac{2}{3} - \frac{1}{4}$. **4.** $\frac{7}{12} - \frac{3}{8}$.

5. $1 - \frac{4}{11}$. **6.** $3\frac{2}{3} - 1\frac{1}{5}$. **7.** $2\frac{1}{4} - 1\frac{5}{8}$. **8.** $3\frac{2}{3} - 1\frac{4}{5}$.

9. $\frac{1}{10} - \frac{1}{100}$. **10.** $\frac{1}{20} - \frac{1}{30}$. **11.** $\frac{1}{100} - \frac{1}{1000}$. **12.** $6\frac{1}{10} - 4\frac{1}{15}$.

13. $3\frac{1}{4} + 2\frac{1}{8} - 1\frac{1}{12}$. **14.** $2\frac{1}{2} + 1\frac{3}{4} - \frac{2}{3}$. **15.** $2\frac{1}{8} - 1\frac{1}{4} + 3\frac{1}{2}$.

16. $3\frac{1}{2} - (\frac{1}{2} + 1\frac{2}{3})$. **17.** $2\frac{2}{3} - (3\frac{1}{2} - 1\frac{1}{4})$. **18.** $3\frac{3}{4} - (2\frac{5}{6} + \frac{3}{12})$.

19. $\frac{a}{2} - \frac{a}{3}$. **20.** $\frac{3a}{2} - \frac{2a}{3}$. **21.** $\frac{1}{a} - \frac{1}{b}$. **22.** $\frac{2}{a} - \frac{3}{b} + \frac{1}{ab}$.

23. A man left $\frac{1}{3}$ of his money to his wife, $\frac{1}{5}$ to each of two sons, and to his daughter. What fraction of his money did this amount to ?

24. A boy ate $\frac{1}{2}$ of a cake one day and $\frac{1}{3}$ the next day. What raction of his cake was left ?

25. A man painted $\frac{1}{3}$ of a pole red, $\frac{1}{5}$ black, and the rest white. Vhat fraction was white ?

26. A plumber cut off $5\frac{5}{8}$ ft. from a pipe $12\frac{1}{2}$ ft. long. How much vas left ?

27. A draper sells $4\frac{2}{3}$ yd. of cloth, then $5\frac{1}{6}$ yd., and has $10\frac{1}{2}$ yd. eft. What length had he originally ?

28. During a campaign $\frac{1}{5}$ of a regiment are killed, $\frac{1}{3}$ die of disease, nd $\frac{1}{25}$ are taken prisoners. What fraction of the regiment is left ?

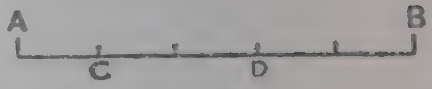
29. A man digs $\frac{1}{3}$ of a garden the first day and $\frac{1}{4}$ the next. If the est amounts to 80 sq. yd., what was the size of the garden ?

30. A traveller goes $\frac{1}{2}$ a journey by train, $\frac{1}{5}$ by motor, and walks he rest, which amounted to 9 miles. What was the total length of he journey ?

31. One tap can fill a bath in 15 min. What fraction of the bath s full in 1 min. ? Another tap can fill it in 20 min. What fraction f the bath is full in 1 min. if both run together ?

32. One man can sweep a road in 16 min., another could do it in o min. What fraction of the road is swept in 1 min. if both work ogether ?

Multiplication

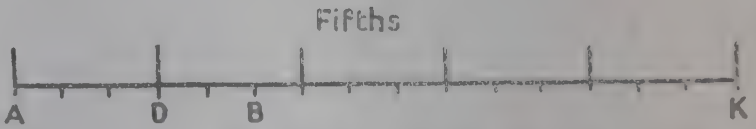


If AB is divided into 5 equal parts, then AC $=\frac{1}{5}$ of AB.

3 times AC $=$ AD $=\frac{3}{5}$AB;

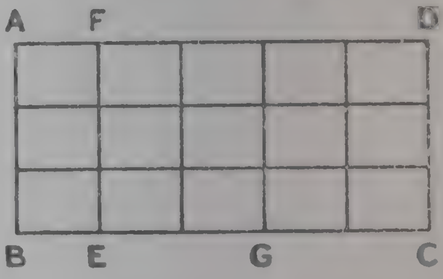
i.e. $\frac{1}{5}\times 3=\frac{3}{5}$.

We have seen that $3\times 2=2\times 3$, and in fact that $ab=ba$ when a and b are integers. We therefore say that $\frac{1}{5}\times 3=3\times\frac{1}{5}$, and that when we multiply 3 by $\frac{1}{5}$ we obtain $\frac{3}{5}$.



If the line AB is extended to K so that AK $=3$AB, and we take $\frac{1}{5}$ of 3AB (*i.e.* $\frac{1}{5}$ of AK), we get AD, which is $\frac{3}{5}$ of AB; *i.e.* $\frac{1}{5}$ of $3=\frac{3}{5}$, so that $\frac{1}{5}$ of 3 is the same as $\frac{1}{5}\times 3$.

When we multiply by a proper fraction the effect is to diminish the number; thus $4\times\frac{1}{2}=2$, just as when we say one-half of 4d. is 2d.



The rectangle ABCD is divided into 15 equal parts. What fraction of the whole is ABEF ?

What fraction of the whole is $\frac{1}{3}$ of ABEF ? Hence evaluate $\frac{1}{3}$ of $\frac{1}{5}$.

From the figure find $\frac{1}{3}$ of $\frac{3}{5}$, $\frac{1}{5}$ of $\frac{2}{3}$, $\frac{2}{3}$ of $\frac{1}{5}$.

Example.—Multiply $2\frac{2}{3}$ *by* 5.

5 times $\frac{2}{3}=\frac{10}{3}=3\frac{1}{3}$.

5 times $2=10$, $\therefore\ 2\frac{2}{3}\times 5=13\frac{1}{3}$.

If we have to multiply by a fraction it is easier to change the mixed number into an improper fraction.

Example.—Multiply $2\frac{2}{3}$ *by* $\frac{1}{5}$.

$2\frac{2}{3}=\frac{8}{3}$.

$\therefore\ 2\frac{2}{3}\times\frac{1}{5}=\frac{8}{3}\times\frac{1}{5}=\frac{8}{15}$.

Simplification.—It has been shown that $\frac{1}{3}\times\frac{3}{5}=\frac{3}{15}$ or $\frac{1}{5}$.

Here the numerator and denominator have a common factor, and it is usual to divide both by this factor as a first step thus :

$$\frac{1}{\underset{1}{\cancel{3}}}\times\frac{\overset{1}{\cancel{3}}}{5}=\frac{1}{5}.\qquad\text{Similarly }\frac{2}{3}\times\frac{9}{14}=\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{3}{\cancel{9}}}{\underset{7}{\cancel{14}}}=\frac{3}{7}.$$

EXERCISE VIII (d)

1. Mark off a line AB (on squared paper) equal to $\frac{7}{10}$ in. Produce it to C so that $AC = 3AB$. Hence show that $\frac{7}{10} \times 3 = 2\frac{1}{10}$.

2. Take a line AB, 4 in. long, and mark off $AC = \frac{1}{10}$ AB. Hence show that $\frac{1}{10}$ of $4 = \frac{4}{10} = \frac{2}{5}$.

3. Draw a rectangle ABCD with AB 7 in. long and AD 3 in. long, and divide it into 21 equal parts, as above.

Hence show that $\frac{1}{3}$ of $\frac{1}{7} = \frac{1}{21}$. $\frac{2}{3}$ of $\frac{3}{7} = \frac{2}{7}$. $\frac{1}{7}$ of $\frac{2}{3} = \frac{2}{21}$.

Simplify

4. $\frac{2}{11} \times 2$. 5. $\frac{5}{7} \times 3$. 6. $\frac{3}{5}$ of 6. 7. $\frac{2}{3}$ of 9.
8. $\frac{3}{4}$ of $\frac{2}{3}$. 9. $\frac{2}{3} \times \frac{5}{6}$. 10. $\frac{1}{4}$ of $\frac{7}{8}$. 11. $\frac{3}{7} \times \frac{14}{15}$.
12. $\frac{2}{9} \times \frac{5}{12}$. 13. $3\frac{1}{4} \times 2$. 14. $5 \times 2\frac{2}{3}$. 15. $\frac{1}{10}$ of 100.
16. $\frac{1}{10} \times \frac{1}{10}$. 17. $\frac{1}{10} \times \frac{1}{100}$. 18. $\frac{1}{1000} \times 10$. 19. $3\frac{2}{3} \times 1\frac{1}{4}$.
20. $2\frac{5}{6} \times 1\frac{3}{4}$. 21. $5\frac{1}{7} \times 3\frac{1}{9}$. 22. $\frac{5a}{3b} \times \frac{6b}{10}$. 23. $\frac{12a^2}{3b} \times \frac{9b^2}{4a}$.
24. $\frac{2a^2b}{3c} \times \frac{c^2}{4ab^2}$. 25. $\frac{5pm}{3qn} \times \frac{9q^2}{10p^2}$. 26. $5\frac{1}{9} \times 2\frac{1}{3}$. 27. $3\frac{1}{5}$ of 5.
28. $2\frac{1}{11}$ of $\frac{22}{23}$. 29. $3\frac{1}{4} \times 4\frac{1}{3}$. 30. $1\frac{3}{4} \times 2\frac{2}{7}$.

Application to Areas and Volumes

Example.—Find the area of a rectangle whose sides are $\frac{5}{6}$ ft. and $2\frac{1}{3}$ ft.

$\frac{5}{6}$ ft. = 10 in., $2\frac{1}{3}$ ft. = 28 in. ∴ the area is 280 sq. in. $= \frac{280}{144}$ sq. ft. $= \frac{35}{18}$ sq. ft.

It will now be seen that the rule "area = length × breadth" still holds good for $\frac{5}{6} \times 2\frac{1}{3} = \frac{5}{6} \times \frac{7}{3} = \frac{35}{18}$.

The longer method shown above is obviously quite unnecessary, for it is equivalent to

Length $= \frac{5}{6} \times 12$ in. Breadth $= \frac{7}{3} \times 12$ in.

Area $= \frac{5}{6} \times 12 \times \frac{7}{3} \times 12$ sq. in. $= \frac{5}{6} \times 12 \times \frac{7}{3} \times 12 \times \frac{1}{144}$ sq. ft. $= \frac{5}{6} \times \frac{7}{3}$ sq. ft.

A similar treatment may be applied to volumes.

EXERCISE VIII (e)

1. Find the area of a rectangle $2\frac{3}{4}$ ft. by $3\frac{2}{3}$ ft.
2. A garden lawn is 25 ft. 6 in. by 18 ft. 8 in. Find its area

3. A page of a book is $6\frac{3}{4}$ in. by $4\frac{1}{4}$ in. Find the area of paper used for a book of 240 pages.

4. What is the value of $\frac{2}{3}$ of $\frac{3}{8}$ of £4 ?

5. A lawn 32 ft. 8 in. by 24 ft. 9 in. has a path 3 ft. 4 in. all round it. Find the area of the path.

6. Find the volume of a brick $9\frac{2}{3}$ in. by $3\frac{1}{2}$ in. by $2\frac{2}{3}$ in.

7. The size of books is given in inches by: Demy 16mo, $5\frac{5}{8} \times 4\frac{3}{8}$. Foolscap, $6\frac{3}{4} \times 4\frac{1}{4}$. Demy 8vo, $8\frac{3}{8} \times 5\frac{5}{8}$. Foolscap quarto, $8\frac{1}{2} \times 6\frac{3}{4}$. Demy folio, $17\frac{1}{2} \times 11\frac{1}{4}$.

Find the area of each kind.

8. State, without working, which of the following are obviously incorrect, and give reasons: (i) $33\frac{2}{7} \times \frac{8}{9} = 34\frac{1}{63}$; (ii) $12\frac{9}{10} \times 3\frac{1}{2} = 32\frac{1}{10}$; (iii) $14\frac{2}{3} \times \frac{1}{4} = 58\frac{2}{3}$; (iv) $6\frac{4}{5} \times \frac{3}{2} = 10\frac{1}{5}$; (v) $12\frac{5}{6} \times \frac{18}{19} = 12\frac{3}{19}$.

Simplify

9. $\frac{2}{3}(\frac{1}{4} + \frac{1}{2})$. **10.** $2\frac{3}{4} \times (1\frac{5}{6} - \frac{2}{3})$. **11.** $3\frac{1}{8}$ of $(4\frac{1}{2} + \frac{2}{3} - \frac{1}{6})$.

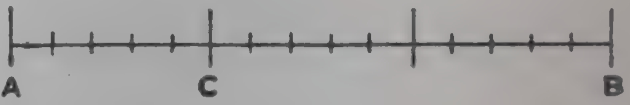
12. $\frac{4}{5}$ of $(2\frac{1}{2} - 1\frac{3}{8})$.

Division

When we divide 12 by 3 we find how many times 12 contains 3, *i.e.* $\frac{12}{3} = 4$.

Otherwise we can say "What number multiplied by 3 makes 12 ?"

When we divide 1 by $\frac{1}{4}$ we find how many times 1 contains $\frac{1}{4}$, *i.e.* $\frac{1}{\frac{1}{4}} = 4$: for instance, there are 4 farthings in one penny. Or we can say that the number which when multiplied by $\frac{1}{4}$ will make 1 is 4, for $4 \times \frac{1}{4} = 1$.



Again, if AB is divided into 15 equal parts, AC $= \frac{1}{3}$AB, and $\frac{1}{3}$AB is contained in AB three times, *i.e.* $\frac{1}{\frac{1}{3}} = 3$.

Example.—What is the value of $\frac{5}{6} \div \frac{3}{4}$?

To divide $\frac{5}{6}$ by $\frac{3}{4}$ is to find how many times $\frac{3}{4}$ is contained in $\frac{5}{6}$.

If we find how many times $\frac{3}{4}$s. is contained in $\frac{5}{6}$s., *i.e.* how many times 9d. is contained in 10d., the answer is $\frac{10}{9}$ times.

$$\therefore \frac{5}{6} \div \frac{3}{4} = \frac{10}{9}.$$

Now $\frac{5}{6} \div \frac{3}{4}$ is $\frac{\frac{5}{6}}{\frac{3}{4}}$.

Since a fraction is unaltered in value by multiplying both numerator and denominator by the same number,

$$\therefore\ \frac{\frac{5}{6}}{\frac{3}{4}}=\frac{\frac{5}{6}\times 12}{\frac{3}{4}\times 12}=\frac{10}{9}.$$

It will be noted that the number selected as a multiplier (12) is the L.C.M. of 6 and 4.

Or again. We say that the number of times 5 contains $\frac{1}{4}$ is 5×4 times.

So also we may say that the number of times $\frac{5}{6}$ contains $\frac{1}{4}$ is $\frac{5}{6}\times 4$ times.

But $\frac{3}{4}$ is three times as large as $\frac{1}{4}$.

∴ $\frac{5}{6}$ will contain $\frac{3}{4}$ only $\frac{1}{3}$ as many times as it contains $\frac{1}{4}$. ∴ the number of times $\frac{5}{6}$ will contain $\frac{3}{4}$ is $\frac{5}{6}\times 4\times\frac{1}{3}=\frac{20}{18}=\frac{10}{9}$;

$$i.e.\ \frac{5}{6}\div\frac{3}{4}=\frac{5}{\underset{3}{\cancel{6}}}\times\frac{\overset{2}{\cancel{4}}}{3}=\frac{10}{9}.$$

Reciprocals.—When the product of two numbers is 1, each is called the **reciprocal** of the other: thus 3 and $\frac{1}{3}$ are reciprocals, so are $\frac{2}{3}$ and $\frac{3}{2}$.

We can obtain the reciprocal of a number by dividing 1 by the number: thus the reciprocal of $\frac{2}{3}$ is $1\div\frac{2}{3}=1\times\frac{3}{2}=\frac{3}{2}$.

It will now be seen that dividing by a fraction is the same as multiplying by the reciprocal.

Example.—Divide $\frac{3}{4}$ by $2\frac{1}{7}$.

$$\frac{3}{4}\div\frac{15}{7}=\frac{\overset{1}{\cancel{3}}}{4}\times\frac{7}{\underset{5}{\cancel{15}}}=\frac{7}{20}.$$

Example.—Multiply £3, 2s. 3d. by $5\frac{2}{3}$.

£3	2	3			
		5			
15	11	3	6	4	6
2	1	6			
£17	12	9			

To multiply by $\frac{2}{3}$, multiply by 2 and put the result at the side, then divide by 3 and add to the product by 5.

Or by Practice.

$\frac{1}{3}$	£3	2	3
			5
	15	11	3
	1	0	9
	1	0	9
	£17	12	9

Example.—What fraction of £2, 3s. 4d. is 19s. 3d.?

£2, 3s. 4d. $=43\frac{1}{3}$s; 19s. 3d. $=19\frac{1}{4}$s; $19\frac{1}{4}$ is $\frac{19\frac{1}{4}}{43\frac{1}{3}}$ of $43\frac{1}{3}$. ∴ the required fraction is $\frac{19\frac{1}{4}}{43\frac{1}{3}}$ or $\frac{77}{4}\times\frac{3}{130}=\frac{231}{520}$.

Example.—What fraction of $3\frac{1}{4}$ *is* $1\frac{2}{3}$?
Express both fractions as twelfths.

$$3\frac{1}{4}=\frac{13}{4}=\frac{39}{12};\quad 1\frac{2}{3}=\frac{5}{3}=\frac{20}{12}.$$

Now 20 twelfths is $\frac{20}{39}$ of 39 twelfths. ∴ required fraction is $\frac{20}{39}$.

EXERCISE VIII (f)

Simplify

1. $1\div\frac{1}{10}$. 2. $10\div\frac{1}{100}$. 3. $\frac{2}{3}\div\frac{1}{2}$. 4. $\frac{3}{4}\div\frac{1}{5}$. 5. $\frac{2}{3}\div 3$.
6. $\frac{1}{10}\div 10$. 7. $\frac{1}{10}\div 100$. 8. $2\frac{2}{3}\times 1\frac{1}{4}$. 9. $3\frac{1}{2}\times 2\frac{2}{3}$. 10. $2\frac{3}{4}\div 1\frac{1}{2}$.
11. $3\frac{1}{2}\div 2\frac{2}{3}$. 12. $4\frac{1}{5}\div 2\frac{2}{3}$. 13. $3\frac{2}{9}\div\frac{1}{7}$. 14. $3\frac{1}{9}\div 4\frac{2}{3}$. 15. $\dfrac{5\frac{2}{3}}{4}$.
16. $\dfrac{2a^2}{b}\div\dfrac{4a}{b^2}$. 17. $\dfrac{5x^2y}{3z}\div\dfrac{10xy^2}{9z^2}$. 18. $\dfrac{4mn}{p^3}\div\dfrac{m^2}{n^2p^2}$.

Find the values of

19. $\frac{2}{3}$ of £1, 5s. 6d. 20. $1\frac{2}{3}$ of £2, 4s. 9d. 21. £3, 2s. 8d. $\times 3\frac{3}{4}$.
22. £4, 16s. 3d. $\times 4\frac{2}{3}$. 23. £2, 3s. 2d. $\div 3\frac{1}{2}$. 24. £20, 12s. 1d. $\div 4\frac{3}{5}$.

State, without working, which of the following are obviously incorrect; give reasons:

25. (i) $2\frac{1}{2}\div\frac{3}{4}=1\frac{7}{8}$. (ii) $3\frac{1}{2}\div\frac{11}{12}=3\frac{9}{11}$. (iii) $4\frac{3}{8}\times\frac{5}{6}=5\frac{1}{4}$.
(iv) $3\frac{4}{7}\div\frac{5}{7}=5$.

26. By what must $\frac{34}{95}$ be multiplied to make $\frac{119}{57}$?

27. Simplify $\dfrac{2\frac{2}{3}-1\frac{1}{4}}{\frac{5}{6}}$.

28. When a certain number is divided by $7\frac{1}{5}$ the result is $4\frac{1}{3}$. What is the number?

29. Simplify $(12\frac{1}{4}-10\frac{1}{2})(12\frac{1}{4}+10\frac{1}{2})$.

30. Which is the greater, $\dfrac{3}{4\frac{1}{2}}$ or $\dfrac{2}{3\frac{1}{4}}$, and by how much?

31. A pad of paper is 4 in. thick, and each sheet of the pad is $\frac{1}{20}$ in. thick. How many sheets are in the pad?

32. A man walks from A to B, $\frac{1}{4}$ of a mile. He then walks $\frac{4}{5}$ of the way back. How many yards has he walked altogether?

33. If the area of a rectangle is $9\frac{1}{3}$ sq. ft., and one side is $3\frac{2}{3}$ ft., what is the length of the other side?

34. The volume of a box is $6\frac{1}{3}$ cu. ft. Its length is $2\frac{1}{4}$ ft., breadth $1\frac{7}{12}$ ft. What is its height?

35. What fraction of (i) 22 yd. is $5\frac{1}{2}$ yd., (ii) $15\frac{3}{4}$ is $4\frac{1}{2}$?

36. What fraction divided by $3\frac{3}{4}$ gives $\frac{8}{45}$?

37. By what must $2\frac{1}{4}$ be divided to give $\frac{1}{6}$?

38. What fraction multiplied by $\frac{3}{8}$ gives $\frac{4}{9}$?

39. Divide $\frac{5}{3}$ of the sum of $2\frac{1}{4}$ and $1\frac{1}{8}$ by $2\frac{1}{4}$ times the difference between $3\frac{1}{2}$ and $2\frac{2}{3}$.

40. By what number must $2\frac{11}{12}$ be multiplied to make $2\frac{1}{2}$?

41. Express the first fraction as a fraction of the second in the following pairs :

(i) $1\frac{1}{4}$, $2\frac{2}{3}$. (ii) $3\frac{1}{4}$, $5\frac{1}{3}$. (iii) $1\frac{2}{3}$, $3\frac{1}{3}$.
(iv) $5\frac{1}{6}$, $6\frac{1}{2}$. (v) $\frac{3}{15}$, $\frac{4}{5}$. (vi) $4\frac{2}{7}$, $7\frac{1}{7}$.

42. What fraction of £1, 18s. 6d. is £1, 12s. 8d.?

43. What fraction of 16s. 4d. is £1, 17s. 4d.?

44. What fraction of 3 yd. 2 ft. 3 in. is 2 yd. 3 in.?

MISCELLANEOUS EXAMPLES M (III)

1. Simplify (i) $\dfrac{0{\cdot}02 \times 0{\cdot}08}{0{\cdot}16}$, (ii) $\frac{3}{4} \div 0{\cdot}25$.

2. ABC is a straight line; AB $=3{\cdot}81$ cm., BC $=2{\cdot}43$ cm. How far is B from the mid-point of AC ?

3. A triangle was drawn from given dimensions by several boys and an unknown side measured; the lengths obtained in cm. were 8·23, 8·29, 8·17, 8·21, 8·2. What is the average of their measurements ?

4. By what must $2\frac{1}{2}$ be multiplied to give $3\frac{1}{2}$?

5. From a stick, one-third of its length is cut away; from the remainder, one-quarter of that is cut away. What fraction of the original length remains ?

6. Express as a decimal, ignoring all figures beyond the fourth decimal place in the working and beyond the third place in the answer, $\frac{1}{3}+\frac{3}{7}+\frac{4}{5}-\frac{8}{11}$.

7. Simplify $\frac{1}{2}-\frac{1}{3}-\frac{1}{5}+\frac{1}{29}$.

8. What is the least number which, when divided by 3 or 4 or 5 or 6, leaves a remainder 1 ?

9. How many people can receive 12s. 6d. if £9 is available for distribution ? What sum is left over ?

10. A man pays 16 fr. 50 c. per day for board and lodging at a Swiss hotel. How long can he stay if his bill is not to exceed 300 fr.?

11. A map is drawn on a scale of 4 in. to the mile; how many km. does a line of length 2·4 in. represent? Take 1 mile = 1·61 km.

12. Taking 1 metric ton = 0·9842 ton, find in lb., neglecting decimals of a lb., the amount by which 1 ton exceeds 1 metric ton.

13. The product of two numbers is $3\frac{1}{2}$, one of them is $2\frac{1}{3}$. What is their sum?

14. How many pieces of string each $5\frac{3}{4}$ in. long can be cut from a length of $12\frac{1}{2}$ ft., and how much is over?

15. The weights in two scale-pans of a balance are 17·37 grams and 19·13 grams. What weight must be transferred from one scale-pan to the other to make them balance?

16. If the French exchange were 87 fr. 40 c. to the £, what would a man receive for £6, 15s.?

17. How many $1\frac{1}{2}$d. stamps can be bought for 13 shillings?

18. What is the value of $\frac{1}{4}(5-3x)$ (i) if $x=\frac{5}{7}$, (ii) if $x=1{\cdot}8$?

19. A motor bicycle runs 45 miles to the gallon of petrol. How much does it use on a journey of 378 miles?

20. From a sheet of cardboard 11·4 cm. long, 8·5 cm. wide, a rectangle 9·5 cm. long, 6·2 cm. wide is cut out. What is the area of the portion left?

21. What is the smallest number greater than 3000 which is a multiple of 173?

22. A man buys eggs at 1s. $9\frac{1}{2}$d. per dozen and sells them at $2\frac{1}{4}$d. each. What does he gain per gross?

23. In a certain sum the divisor is ·305, the quotient is 7·4, and the remainder is ·143. What is the dividend?

24. Arrange in ascending order of magnitude

$$\frac{4}{7},\ \frac{7}{13},\ 0{\cdot}55,\ \frac{19}{33},\ \frac{27}{50},\ \frac{69}{125}.$$

25. The average of ten numbers is 23·17, and the average of nine of them is 24·3. What is the other number?

26. Given that 1 ton = 1016 Kg., express 1 cwt. 3 qr. in Kg.

27. Express £220, 10s. in guineas.

28. How many cu. cm. blocks can be cut from a block 0·05 m. high, 0·72 m. long, 0·3 m. wide?

29. Evaluate $(1{\cdot}7)^2-(1{\cdot}4)^3$.

30. Find the value of $\dfrac{a+b}{a-b}$ if $a=2\frac{1}{2}$, $b=1\frac{2}{3}$.

31. Given that 1 Kg. = 2·205 lb., express 3 Hg. 5 Dg. in oz.

32. What days in August are always the same day of the week as October 20 ?

33. Express in £, s. d. the value of $(\frac{2}{3}+\frac{1}{4}+\frac{3}{5})$ of £4.

34. A penny weighs $\frac{1}{3}$ oz., and a halfpenny weighs $\frac{1}{5}$ oz. What is the difference in weight of 5 shillings' worth of pennies and 5 shillings' worth of halfpennies ?

35. A boy spends $\frac{1}{10}$ of his money in the first week and $\frac{1}{6}$ of the remainder in the second week of the term: he has then £1, 7s. left. What did he start with ?

36. What is the least integer which is a perfect square and is divisible by 24 ?

37. A cubic foot of stone weighs 150 lb. What is the size of a block which weighs a ton and a half ?

38. The scale of a map is 4 in. to the mile. What is the length on the map of a portion of road 1000 yd. long, neglecting thousandths of an inch ?

39. A vessel full of water weighs 2·27 Kg., and when half full weighs 1·89 Kg. What will it weigh when empty ?

40. Express 0·73 cwt. in lb., oz., neglecting fractions of an ounce.

41. How many parallel lines at intervals of 0·15 in. can be engraved on a bar 4·3 in. long ?

42. A rectangle $3\frac{3}{4}$ in. long, $2\frac{5}{6}$ in. wide, is divided by a network of lines parallel to each of its sides so that each of the meshes is a square. What is the least number of squares ?

43. A carpet 9 ft. 6 in. long, 7 ft. 4 in. wide, lies on the floor of a room 12 ft. square. What area is uncarpeted ?

44. Find the value of (i) $\frac{3}{5}\times(2\frac{1}{2}\div 1\frac{1}{2})$; (ii) $\frac{3}{5}\div(2\frac{1}{2}\times 1\frac{1}{2})$; (iii) $\frac{3}{5}\div 2\frac{1}{2}\times 1\frac{1}{2}$.

45. Find in prime factors the L.C.M. of $2^3\times 3^2\times 7\times 11$; $2^2\times 3\times 7^3$; 3×7; 22^2. Write down in factors the quotients obtained by dividing the L.C.M. by each of these numbers in turn.

46. Find the value of $1\frac{3}{7}\times 51+51\times 2\frac{6}{7}-\frac{2}{7}$ of 51.

47. A bar of iron weighs 2 qr. 13 lb. What is the greatest number of such bars whose total weight is less than $1\frac{1}{2}$ tons ?

48. A book of 234 sheets is 1·63 in. thick. Is the thickness of each sheet greater or less than $\frac{7}{1000}$ in. ?

49. Prove that $\frac{3+5}{7+11}$ lies between $\frac{3}{7}$ and $\frac{5}{11}$. Which is the greatest number ?

50. A train is running up an incline of $\frac{1}{100}$ at 30 miles an hour. What vertical height does it rise in $2\frac{1}{2}$ minutes ?

REVISION PAPERS

(R 11–20)

R 11

1. Find the cost of 143 acres at £15, 14s. 9d. per acre.

2. A 12-gallon cask of cider costs £2. Find the price per half-pint glass.

3. If a soldier takes 120 paces per minute, each of 33 in., how far does he go in half an hour ?

4. A rectangular field is 23 chains long and 14 chains wide. How many square yards does it contain ?

5. Write in decimal form (1) five-hundred thousandths and three thousandths ; (2) forty-two hundredths ; (3) ten thousandths.

6. Simplify (i) $\frac{1}{2}+1\frac{1}{3}+\frac{2}{9}$; (ii) $3\frac{4}{9}-1\frac{4}{7}$.

R 12

1. The marks obtained by five boys, A, B, C, D, E, in an examination were as follows ; find the totals and arrange them in order of merit.

	Latin.	Greek.	History.	English.	French.	Mathematics.
A	59	61	43	48	62	35
B	84	79	61	47	19	13
C	23	17	34	86	38	92
D	63	51	42	38	41	7
E	39	33	47	62	58	41

2. A man, who has £100, makes the following payments : £7, 8s. 11d. ; £11, 3s. 4d. ; £29, 17s. ; £19, 12s. 8½d. ; £23, 16s. 5d. What is his balance ?

3. One clock gains 2 seconds an hour and a second clock loses 8 seconds a day. Both are put right at mid-day on Sunday. What is the difference between the times they show at 9 a.m. on Wednesday ?

4. Find the cost of 240,000 overcoats at £3, 12s. 9d. each.

5. How many meshes are there in 1 sq. metre of wire gauze if each mesh is 2 cm. long and 1 cm. wide ?

6. A tank 6 ft. long, 3 ft. wide, 4 ft. deep is half full of water. Three dozen leaden 1-foot cubes are submerged in it. What height does the water-level rise ?

R 13

1. Divide 105 yd. by 27.

2. Express $3\frac{5}{6}$ sq. yd. and 2·48 tons as compound quantities.

3. A picture-frame is 38 in. long, 26 in. broad. The width of the vood is 2 in. Find the area of the front of the frame.

4. One man takes a stride of 33 in., another takes a stride of 36 in. How many strides does the first take more than the second in walking a mile ?

5. How many hundredths are there in (i) ·0162, (ii) 3·24375 ; and how many thousandths in (i) ·314, (ii) 1·2623 ?

6. Find the L.C.M. of 240, 270, and 150.

R 14

1. What number exceeds 12,438 by as much as 5692 falls short of it ?

2. Multiply 3s. $2\frac{1}{2}$d. by 239.

3. Twelve men are employed to repair a road ; nine of them eceive 38s. 6d. a week, two of them receive 41s. a week, and one eceives 44s. a week. The work takes 11 weeks, and the material ised costs twice as much as the wages. How much does it cost to epair the road ?

4. What fractions of £1 are the following : (i) 12s., (ii) 6s. 8d., iii) 17s. 6d., (iv) 1s. 3d., (v) 8s. 4d. ? Express the results also as lecimals to 3 places.

5. A lawn 100 yd. long, 50 yd. wide is surrounded by a path 2 yd. vide. What is the area of the path ?

6. A rectangular stack of coal is 20 yd. long, 8 ft. wide, 6 ft. high. Vhat is its value if 1 ton of coal is worth £1, 12s. 6d. and if 45 cu. ft. f coal weigh a ton ?

R 15

1. How long does it take a clock to tick a million times, if it ticks wice a second ?

2. By using brackets write down the value of the remainder when welve times the difference between a and b is subtracted from five imes the sum of x and y, given that a is less than b.

3. If 14 lb. of tea worth 1s. $9\frac{1}{2}$d. a lb. is mixed with 16 lb. of tea vorth 2s. 5d. a lb., what is the cost of a lb. of the mixture ?

4. Divide £320, 9s, 4d. into 19 equal parts.

5. Express as fractions in their lowest terms (i) ·35, (ii) ·015.

6. Express as decimals $\frac{6 \cdot 2}{10^2}, \frac{14}{10^4}, \frac{17}{25}$.

R 16

1. A man pays for a railway ticket with a £5 note and receives £3, 17s. 4d. change. How much change would he have received if he had bought three of these tickets?

2. Find the cost of 100 pairs of stockings at 11s. 9d. per pair.

3. Divide 5000 by $11 \times 7 \times 3$ by *short* division and check your result by multiplication.

4. Divide £16, 4s. 4d. by $1\frac{2}{5}$.

5. An oak fence costs 18s. per yard length. What will it cost to enclose a rectangular garden 40 ft. wide and 90 yd. long?

6. The outside measurements of a box without a lid are as follows: length 30 cm., breadth 22 cm., depth 9 cm. The wood is 1 cm. thick. Find the number of cubic centimetres of wood used in making the box.

R 17

1. If a profit of 4s. 8d. is made on each £, find the total profit on £237, 18s. $1\frac{1}{2}$d.

2. Find the value of (i) $40 \cdot 03 \times \cdot 024$, (ii) $\frac{28 \cdot 42}{\cdot 014}$.

3. I buy 2 gross of eggs at 10d. a dozen. Eighteen are broken, and the rest are sold at 9 a shilling. How much do I gain?

4. If thunder is heard 16 seconds after the flash is seen, find how far away is the storm in miles and yards, assuming the flash is seen instantaneously and that sound travels 1140 ft. per second.

5. An inch of rain falls over an area of $2\frac{1}{2}$ sq. miles. What will it weigh to the nearest thousand tons, taking 1 cu. ft. of water to weigh 1000 oz.?

6. How many times could I fill a bucket holding x cu. in. of water from a tank x yd. by y ft. in which the water was z in. deep?

R 18

1. The population of a country in 1911 was 42,893,417. From 1911 to 1921 there were 6,347,204 deaths and 6,815,612 births. The population in 1921 was 48,537,322. How many more people entered the country than emigrated during this period?

2. The circumference of each of the large wheels of a tricycle is 132 in., and the circumference of the small wheel is 55 in. How many more times does the small wheel revolve than the large wheel in a mile?

3. (i) Divide £18, 15s. 11d. by 13; (ii) what decimal of £1 is 17s. 5d.? (4 places).

4. A man travels 10 miles at 10 miles an hour, then 10 miles at 20 miles an hour, then 10 miles at 30 miles an hour, then 10 miles at 60 miles an hour. How long did the whole journey take, and what was his average speed?

5. How many paving-stones, each 2 ft. by 1 ft., are required to pave a walk 100 yd. long, 5 ft. wide?

6. Sixty-six loads of gravel are laid evenly on a path 5 ft. wide and $\frac{1}{4}$ mile long. Each load contains 25 cu. ft. Find the thickness of the layer.

R 19

1. How many bicycles, each costing £10, 10s., can be bought for £273?

2. A railway rises 1 ft. in every 360 ft. of the track for 6 miles and then descends 1 ft. in every 440 for 4 miles. How many feet is it then above the original level?

3. A glazier puts 12 panes in a window each measuring 12 in. by 14 in. What is the cost of the glass at 8d. per square foot?

4. Express 828 in prime factors. What is the least multiple of 828 which is a perfect square?

5. How many cubic feet of water will it take to flood a yard 64 ft. by 20 ft. to a depth of 6 in.? To what depth would the same amount of water flood a yard half as long and half as broad?

6. The average age of a class of 13 boys was 14 yr. 2 mo. When one of them left the average age was 14 yr. 1 mo. What was the age of the boy who left?

R 20

1. A clock will run for 200 hours after it has been wound up. What is the least number of times it must be wound up during a year (365 days) if it is not to stop?

2. Ten tons of coal are divided equally between 7 lorries. Find in tons, cwt., qr., lb. the weight of each load.

3. Find the cost of 54 pairs of sheets at £1, 3s. 9d. per pair.

4. If the factors of one number are a and x and the factors of another number are b and x, show that x is a factor of the difference between the numbers. What is the other factor?

5. How many strips of linen 3 in. wide, 10 in. long can be cut from a rectangular sheet 13 ft. by 6 ft., and how much is wasted?

6. A canal of rectangular section is 15 yd. wide and the water is 6 ft. deep. If the water flows at 1 mile an hour, find the number of cubic feet of water that flow under a bridge in a quarter of an hour.

PROBLEM PAPERS

(P 11–20)

P 11

1. How much time would have been gained if troops had marched 130 paces of 33 in. each every minute, over a journey which took them 1 hr. 39 min. when marching every minute 125 paces of 30 in. each?

2. A hogshead of wine contains 57 gallons. How many bottles can be filled from it if 6 bottles hold four pints?

3. A reduction of 3d. in the price of a book makes a difference of £20, 10s. 3d. in the receipts. How many books were sold?

4. A piece of wood whose area is 2 sq. ft. weighs 1224 grams. What would be the weight of a frame made of this wood 2 in. wide showing a glass surface of 16 in. by 14?

5. If xy^2 has always the same value and x is ·32 when y is 12, what is x when y is 8?

P 12

1. To settle a hotel bill, A pays £9, 12s., B pays £7, 17s., C pays £3, 13s. If it is to be shared equally, how must they settle up amongst themselves?

2. What is the least number by which 150 must be multiplied to make the result (i) a perfect square, (ii) a perfect cube, (iii) both a perfect square and a perfect cube? Give your answers in factors; do not multiply out.

3. In a schoolhouse there are 10 hip-baths in use at a time, and each lasts 2 terms; each bath costs 24s. What is the cost per year (3 terms to the year) in wear and tear of baths?

4. Fruit-trees are planted in rows in a field 200 yd. long, 120 yd wide, so that adjacent trees form the corners of a square of side 5 ft.

and no tree is less than 10 ft. from the edge of the field. What is the largest number of trees that can be planted in the field ?

5. A wooden tee-box measures externally 12 in. long, 12 in. broad, 10 in. high ; it has no lid, and is made of wood 1 in. thick. How many cubic inches of sand does it hold when just full ? How many cubic inches of wood are used to make the box ?

P 13

1. What is the smallest integer which, when multiplied by 21, gives a product of which each digit is 1 ?

2. By what must (i) two times one-fifth be multiplied to give $2\frac{1}{5}$, (ii) $a \times \frac{1}{b}$ be multiplied to give $a + \frac{1}{b}$?

3. A quantity of tea is packed in 500 boxes whose internal dimensions are 4 ft., 3 ft., and 3 ft. How many boxes would have been required if the dimensions had been 5 ft., 4 ft., and 4 ft. ?

4. The length of a box is twice its width, and the area of the four sides is 24 sq. ft. If it is 2 ft. deep, find the area of the base.

5. I write a cheque for sixteen guineas to pay 3 bills, the first of which is £2 less than the second, and the second £2 less than the third. What were the bills ?

P 14

1. A milestone on the main road between Winchester and Southampton reads, "To Winchester 5 miles ; to Southampton 7 miles." Find the sum of all the numbers on all the milestones *between* Winchester and Southampton.

2. In 1917 there were 900,000 workers engaged in making munitions ; one-third of these received £3, 6s. 8d. each per week, and the remainder received £1, 19s. 6d. each per week. Find the total wage-bill per week.

3. If the unit of length is 9 in., find the number of units of area in a rectangle 10 yd. long, 7 yd. wide, and the number of units of volume in a block 12 ft. by 9 ft. by 6 ft.

4. The external dimensions of an open tank are as follows : length 9 ft., breadth 6 ft., depth 4 ft. Find the cost of painting the whole of the outside at 6d. per square yard.

5. Two rooms contain equal quantities of air. The floor area of one is 390 sq. ft. and its height is 14 ft. ; the height of the other is 10 ft. 6 in. What is its floor area ?

P 15

1. If a certain number is added to 76,143 the result is 5436 greater than if the number had been subtracted from 76,143. Find the number.

2. How many degrees of longitude are there between 25° W. and 47° E.?

3. What is the greatest power of 2 which will divide an exact number of times into the product of the first twenty whole numbers ?

4. A domestic servant stayed in a post from January 1, 1915, to August 20. She earned wages at 24s. a month of 4 weeks, and paid insurance of 3d. per week. Find her net earnings.

5. Cauliflowers are planted in rows in a rectangular garden 1 ft. apart and 1 ft. from the edges. There are 33 cauliflowers in a row and 47 rows. Find the greatest number of bushes which could be planted in this [illegible] ey are at least 2 ft. apart and at least 2 ft. from the [illegible] n time wou

3 in. each

P 16

1. What [illegible] the least number that must be added to 73,841 to make the result divisible by 291 ?

2. A hotelkeeper buys 120 pairs of blankets (first quality) at 36s. 6d. per pair and 80 pairs of blankets (second quality) at 27s. 9d. per pair. How much would he have saved if instead he had bought 80 pairs of the first quality and 120 pairs of the second quality ?

3. A and B measure the length of a fence with their walking-sticks. A finds that it is between 52 and 53 times as long as his stick ; B finds that it is between 57 and 58 times as long as his stick. A's stick is 38 in. long and B's stick is 35 in. long. Between what limits (in feet and inches) must the length of the fence be ?

4. Which requires more paper : a book of 220 pages, each page being 8 in. by 5 in., or a book of 164 pages, each page being 9 in. by 6 in.?

5. A slab of metal 3 ft. 6 in. by 2 ft. 6 in. by 1 ft. 5 in. is melted down and cast into cubes, some of edge 2 in. and some of edge 3 in. How many can be made if there are (i) an equal number of each, (ii) three times as many of the smaller as there are of the larger ?

P 17

1. A train 110 yd. long comes to a viaduct 550 yd. wide. If it takes $7\frac{1}{2}$ seconds to pass a telegraph post, how long does it take to pass completely over the viaduct ?

2. A person bought 2 tons of sugar at $2\frac{1}{4}$d. per lb.; 6 cwt. 84 lb. of it was damaged, and sold at $1\frac{1}{4}$d. per lb.; the rest was sold at 3d. per lb. What did he gain or lose?

3. A pipe whose cross-section is 2 sq. in. delivers 42 gallons of water per minute at a steady speed. If a gallon of water is 280 cu. in., find the speed of flow.

4. The cost of the railway fare and a holiday of 7 days is £11, 2s., but if I stay away 12 days the cost is £17, 2s. Assuming the same cost per day, find the railway fare.

5. A rectangle of wood 4 in. by 9 in. weighs 180 grams. What would be the weight of this wood required to make a shelf whose area is 2 sq. ft.?

P 18

1. The year x A.D. contains $\frac{x}{6}$ days. What is x?

2. The petrol consumption of a car depends on its speed. It runs 18 miles to the gallon when the speed is 20 m.p.h., and 15 miles to the gallon when the speed is 36 m.p.h. How much is used on a ourney of 3 hours at 20 m.p.h. and $2\frac{1}{2}$ hours at 36 m.p.h.?

3. A shopkeeper made the same total amount of profit by selling 3 articles at 3s. $7\frac{1}{2}$d. each as he did by selling 5 of them at 3s. each. How much did each article cost him?

4. A field is 50 yd. longer than it is broad; one of the shorter sides s bounded by a hedge and the other 3 sides by a fence. The length f the fence is 370 yd. What is the area of the field?

5. The area of the surface of a pond is 2 acres, and it is covered vith ice 4 in. thick. A cubic foot of ice weighs 56 lb. Calculate the otal weight of the ice in tons.

P 19

1. Show that the greatest number of digits in the product of two umbers having 19 and 37 digits respectively is 56, and the least umber of digits is 55.

2. Sound travels 1140 ft. per second through air. If it travelled t the same rate from the sun we should hear an explosion on the un fourteen years after it occurred. What distance does this ssume for the sun to the nearest million miles?

3. There are 3 paths leading from a village X to another village Y. n how many ways can I walk from X to Y and come back by a th different from the one by which I went?

4. A train starts from A to B at the same time as one from B to A. The first travels 56 miles an hour, and in $1\frac{1}{2}$ hours meets the other. If the second travels 42 miles an hour, when will it reach A? How far is A from B?

5. A sum of £12, 10s. is divided among 13 men and 4 boys, so that a man gets twice as much as a boy. How much does each man receive?

P 20

1. A man has 5000 red cards, 6000 blue cards, and 7000 green cards. He wishes to make up a number of packets, each of which is to contain 12 red, 14 blue, 16 green cards. How many complete packets can he make up, and how many cards of each colour will be left over?

2. Plymouth is 226 miles from London. Five men go down from London to Plymouth, and three of them return the next day. What is the total railway fare at the rate of $1\frac{1}{2}$d. per mile?

3. A map is drawn on a scale of 8 in. to the mile. What area on the map represents an estate of 180 acres?

4. A lawn is 110 yd. long by 42 yd. wide. How long will it take to mow it with a 14-in. machine if the man pushes it at 3 miles an hour?

5. The rain-water from a flat roof 16 yd. long, 20 ft. wide is collected in a closed tank 8 ft. long, 6 ft. wide. How much does the water-level rise in the tank after $\frac{1}{4}$ in. of rain has fallen?

CHAPTER IX

UNITARY METHOD

Direct Proportion

Example.—5 *tons of coal cost £12. How much coal can be bought for £9?*

£12 buys 5 tons of coal;

£1 buys $\frac{1}{12} \times 5$ tons of coal;

£9 buys $\frac{9}{12} \times 5$ tons of coal $= \frac{5 \times 3}{4} = \frac{15}{4} = \underline{3\frac{3}{4}\text{ tons.}}$

Note.—(i) Always write down a proper sentence in the top line.
(ii) Arrange the top line so that the quantity required comes last.
(iii) State the units in the answer.
(iv) The second step should be omitted as soon as the method is understood.

Example.—A train travels x miles in t hours. How far will it go n y hours at the same rate ?

In t hours it travels x miles; in 1 hour it travels $\frac{1}{t} \times x$ miles; in hours it travels $\frac{y}{t} \times x$ miles $= \frac{yx}{t}$ miles.

EXERCISE IX (a)

1. Six gallons of spirit weigh 48 lb. What is the weight of 1 gallon, gallons ?

2. Six gallons of spirit weigh 48 lb. How much spirit weighs lb., 72 lb.?

3. A motor runs 120 miles on 5 gallons of petrol. How many miles vill it run on 1 gallon, 3 gallons ?

4. A motor runs 120 miles on 5 gallons of petrol. How much does t use in 300 miles ?

5. A boy can write 30 lines in 6 minutes. How long does he take o write 100 lines ?

6. The charge for a telegram is at the rate of 12 words a shilling. Vhat is the charge for 20 words ?

7. A beam of wood 15 ft. long weighs 300 lb. What is the weight f an 11-foot length of it ?

8. Taking 8 Km. as equivalent to 5 miles, express (i) 36 Km. in miles, i) 3 miles in kilometres.

9. What is the value of 15 Swiss francs if 25 Swiss francs are orth £1 ?

10. The shadow of a man 6 ft. high is 4 ft. long. What is the height f a tower whose shadow at the same moment is 60 ft. long ?

11. Two dozen note-books cost 18s. What is the cost of ten note-ooks ? How many can be bought for one guinea ?

12. A bicycle wheel revolves 12 times in 30 yd. How many evolutions does it make in 100 yd.? How far will 100 revolutions arry it ?

13. A man reads a novel at the rate of 3 pages every 4 minutes. ow long does he take to read 450 pages ?

14. The duty on 600 cigarettes is 15s. For what number does the ity amount to £5 ?

15. Three tons of anthracite cost ten guineas ; find the cost of tons. How much can be bought for £14 ?

16. A furnace consumes 3 cwt. of fuel every 2 hours. How long will 2 tons last?

17. A cook's wages amount to £65 for 52 weeks. What is she paid for 6 weeks?

18. Ten shop eggs cost 1s. 3d. How many can be bought for 5s. 3d.?

19. A dozen bottles of claret cost two guineas. How many bottles can be obtained for £3, 10s.?

20. A gallon of water weighs 10 lb. What is the weight of 3 pints of water?

21. A train is running at 30 miles an hour. How many seconds does it take to go 220 yd.?

22. The first-class fare for a journey of 350 miles is £3, 10s. What would it be for 380 miles?

23. A man pays £17 each year to insure his life for £1000. What would he be paying if he had insured it for £1200?

24. A man who has £100 in the Savings Bank receives each year £2, 10s. interest. What interest would he receive on £160?

25. The interest on £100 being £5 per annum, find the interest on (1) £500 for (i) 1 year, (ii) 3 years; (2) £400 for 6 years; (3) £250, 10s. for 3 years.

26. If I receive £3 interest on £100 each year, on how much money do I receive £90 interest in the course of 5 years?

27. If the interest is £4 per £100 per annum, how many years must elapse before I have received £200 in interest on a sum of £1200?

28. If I gain 5s. by selling an article which cost 20s., what should I gain at the same rate in shillings if the article had cost 100s.? What should I gain in £ if the article had cost £100?

29. If I lose 2d. by selling an article for 6d., what should I lose in pence at the same rate if the article had cost 100d.? What should I lose in shillings if the article had cost 100s., and what in £ if the cost had been £100?

30. I buy an article for £15 and sell it for £16, 5s. What should I have gained at the same rate if the article had cost £100?

31. If a articles cost £x, how much will b of them cost?

32. If I buy x lb. for y shillings, how many lb. can I buy for (1) z shillings, (2) z pounds?

33. If p yd. cost a pence, what should q yd. cost (1) in pence, (2) in shillings?

34. I buy t tons for £x. How many shillings should m cwt. cost?

35. I walk x yd. in m min. How many feet can I walk in h hours?

36. If a ft. cost x pence, how many £ will b yd. cost ?

37. If I buy for £x and sell for £y, how much should I have gained if the cost had been £100 and I had sold at the same rate ?

38. If I sell for £x and gain £a, how much should I gain if the cost were £100 and I gain at the same rate ?

Inverse Proportion

Example.—A bookcase holds 600 *books if each book is* 3 *cm. wide. How many will it hold if each book is* 2 *cm. wide ?*

If the width of each book is 3 cm., there is room for 600 books.
" " " 1 " " " 600×3 books.
" " " 2 " " " $\frac{600 \times 3}{2}$ "
$= 300 \times 3 =$ **900 books.**

Note.—Always consider whether the quantity at the end of the line is to be increased or decreased. Here it is obvious that if each book is narrower there is room for a larger number.

EXERCISE IX (b)

[This exercise includes examples both of direct and of inverse proportion. There are also some questions to which the method of proportion is obviously not applicable ; *in the latter case point this out, and do not attempt to give any further answer.*]

1. A man can address 200 envelopes in 3 hours. How long will he take to address 300 envelopes ?

2. Six men can address a pile of envelopes in 6 hours. How many men are required to address them in 2 hours ?

3. Twelve men can mow a field in 5 days. How long will 20 men take ?

4. The first six chapters of a book contain 150 pages. How many pages are there in the first eight chapters ?

5. Eight similar cricket balls weigh 44 oz. What is the weight of 10 cricket balls of the same kind ?

6. Three men can clear the snow off an ice-rink in 10 hours. How long will 5 men take to do so ?

7. There is enough food in a camp to last 600 men for 24 days. How long will it provide for 800 men ?

8. A man weighs 10 stone when he is 20 years old. What does he weigh when he is 60 years old ?

9. A chronometer loses 3 seconds in 36 hours. How much does it lose in a week ?

10. A cyclist riding 10 miles an hour takes $2\frac{3}{4}$ hours to make a journey. How long would he have taken if he had travelled at 11 miles an hour ?

11. A train travelling at a steady rate passes 56 telegraph posts in 6 minutes. How many does it pass in one-quarter of an hour ?

12. A golfer takes 20 strokes for the first 4 holes. How many does he take for the first 9 holes ?

13. In a sale, a sofa is reduced from thirty guineas to £28. If all reductions are made on the same scale, what is the ordinary price of a set of chairs reduced in the sale to £40 ?

14. The maximum speed of a 10 horse-power car is 40 miles an hour. What is the maximum speed of a 12 horse-power car ?

15. One-quarter of a pound of fine snuff costs £6. What is the price of one-third of a pound ?

16. A large tank is filled by 6 equal pumps in 8 hours. How long would it take if one pump was out of action ?

17. In every 5 tons of a certain coal delivered there is half a ton of shale which is worthless. How much coal must be ordered so as to obtain 180 tons of useful fuel ?

18. On a map whose scale is 5 in. to the mile two churches appear 12 in. apart. How far apart would they appear on a map of scale 3 in. to the mile ?

19. On a map whose scale is 5 miles to the inch two windmills appear $1\frac{1}{2}$ in. apart. How far apart would they appear on a map of scale 3 miles to the inch ?

20. Thirty miles an hour is approximately the same speed as 4 Km. every 5 minutes. How many miles are there in 100 Km.?

21. How many sheets of paper each of area 80 sq. in. have the same weight as 1200 sheets of the same kind of paper each of area 96 sq. in.?

22. If a men take x hours over a job, how long should b men take ?

23. If I can finish a journey in x hours by walking a miles an hour, how long should I take if I walked b miles an hour ?

24. I buy x yd. of material when the price is y shillings per yard. How much can I buy with the same money if the price changes to z shillings per yard ?

25. If a train takes x hours over a journey when travelling p miles an hour, how long would it take when travelling q miles an hour (i) when p is greater than q, (ii) when p is less than q ?

CHAPTER X

RATIO AND PERCENTAGE

Ratio

When we wish to compare two or more quantities of the same kind we often say that they are in a certain ratio to one another. A man 6 ft. high is twice as tall as a child of 3 ft.; in other words, the ratio of the man's height to the child's is 2 to 1, or $\frac{2}{1}$, while the child's height is to the man's in the ratio of 1 to 2, or $\frac{1}{2}$.

The ratio of a to b is conveniently expressed by the fraction $\frac{a}{b}$. This fraction is generally written in its lowest terms or as a decimal.

The ratio of a to b is also written $a : b$ (read a is to b). This method is most convenient when more than two quantities are compared. The ratio of a to b to c is written $a : b : c$.

Any set of numbers is said to be **proportional** to another set of numbers if the ratio of each pair in the first set is equal to the ratio of the corresponding pair in the second set. Thus the numbers 6, 10, 12, 18, 24 are proportional to the numbers 15, 25, 30, 45, 60, and this is often written $6 : 10 : 12 : 18 : 24 = 15 : 25 : 30 : 45 : 60$.

It is impossible to express any ratio between quantities of different kinds, *e.g.* there is no relation between £6 and 3 ft.

A clear distinction must be drawn between a comparison shown by estimating a **difference** and another by estimating a **ratio.**

Compare, for instance, the relation between a change from £90 to £100 and a change from £900 to £1000. In the first case the **difference** is £10, in the second it is £100, but the ratio of the former to the latter is 9 to 10 in both cases.

If a quantity changes from 1 to 5, and then from 5 to 10, which change is the greater? In the first case the difference is 4, in the second 5, but the ratios are $5 : 1$ for the first and only $2 : 1$ for the second.

Multiplying Factor.—The fraction by which a number must be multiplied in order to change its value in a given ratio is called the multiplying factor.

In order to increase a wage of £200 in the ratio of $5 : 6$, the multiplying factor is $\frac{6}{5}$ and the new wage is £$200 \times \frac{6}{5}$.

Example.—Find the ratio of 4s. 6d. to 7s. 6d.

$$\text{The ratio} = \frac{4\frac{1}{2}}{7\frac{1}{2}} = \frac{9}{15} = \frac{3}{5}.$$

That is to say, 4s. 6d. is $\frac{3}{5}$ of 7s. 6d., or 7s. 6d. is $\frac{5}{3}$ of 4s. 6d., so that this example answers the question "What fraction of 7s. 6d. is 4s. 6d.?"

Example.—Express in its simplest form the ratio $2\frac{1}{2} : 3\frac{1}{3}$.

$$\frac{2\frac{1}{2}}{3\frac{1}{3}} = \frac{6 \times 2\frac{1}{2}}{6 \times 3\frac{1}{3}} = \frac{15}{20} = \frac{3}{4}.$$

This answers the question "What fraction of $3\frac{1}{3}$ is $2\frac{1}{2}$?"

Example.—By what must 24 be multiplied to give 30?

$24 \times \frac{30}{24} = 30.$ ∴ the *multiplying factor* $= \frac{30}{24} = \frac{5}{4}$.

Example.—Express in whole numbers the ratio of $\frac{1}{2}$ *to* $\frac{1}{3}$ *to* $\frac{1}{5}$.

Multiplying by 30, the ratio is 15 : 10 : 6. This multiplication does not affect the ratio for $\frac{\frac{1}{2}}{\frac{1}{3}} = \frac{30 \times \frac{1}{2}}{30 \times \frac{1}{3}} = \frac{15}{10}$, and so on.

Example.—What decimal of £2, 9s. 8d. is £1, 2s. 0½d.?

Rough check.—£1, 2s. 0½d. is about ½ of £2, 9s. 8d. ∴ approx. answer is 0·5.

£2, 9s. 8d. = £2·4833 and £1, 2s. 0½d. = £1·102. ∴ the ratio is $\frac{1·102}{2·4833}$ = 0·444 (to 3 places); *i.e.* £1, 2s. 0½d. = 0·444 of £2, 9s. 8d.

EXERCISE X (a)

Express the following ratios in their simplest forms:

1. A weight of 6 lb. to a weight of 9 lb. What fraction of 9 lb. is 6 lb.?
2. A length of 12 cm. to a length of 15 cm. What decimal of 15 cm. is 12 cm.?
3. An area of 1 sq. yd. to an area of 6 sq. ft.
4. The value of a florin to that of a half-crown.
5. A speed of 12 miles an hour to a speed of 16 miles an hour.
6. A salary of £30 a month to a salary of £300 a year.
7. A charge of 8d. per foot to a charge of 1s. 6d. per yard.

Express the following ratios as fractions in their lowest terms

8. 6 : 9. **9.** 9 : 6. **10.** 10 : 2. **11.** 25 : 100. **12.** 5 : 10.
13. 10 : 5. **14.** 7500 : 5000. **15.** 1 : $\frac{3}{4}$. **16.** $\frac{1}{2}$: 1. **17.** $\frac{1}{2}$: $\frac{1}{3}$.

18. $3\frac{1}{3} : 1\frac{1}{4}$. **19.** $\frac{1}{6} : \frac{2}{3}$. **20.** $1\frac{1}{2} : 2\frac{1}{4}$. **21.** $\frac{1}{4} : \frac{3}{4}$. **22.** $\frac{1}{8} : \frac{1}{2}$.
23. ·75 : 1. **24.** ·4 : 1·2. **25.** 1·2 : 10. **26.** ·75 : 1·5. **27.** 1·5 : 2·5.
28. 3 in. to 1 ft. **29.** 1 ft. to 2 yd. **30.** 16 sq. cm. to 4 sq. cm.
31. 5s. to £1, 10s. **32.** 4d. to 2s. **33.** 2s. 4d. to 3s. 6d.
34. £1, 2s. to £5, 4s. **35.** 2 ft. 6 in. to 1 yd. 2 ft.
36. 2 m. 4 dm. to 1 m. 8 dm. **37.** 1 ac. 2 ro. to 4 ac. 2 ro.
38. 2 qt. 1 pt. to 3 qt. 1 pt.

Fill in the blanks in the following equalities :

39. $\frac{4}{8}=\frac{\quad}{14}$. **40.** $\frac{9}{6}=\frac{15}{\quad}$. **41.** $\frac{1}{8}=\frac{\frac{1}{2}}{\quad}$. **42.** $\frac{30}{20}=\frac{\quad}{1}$. **43.** $\frac{1}{5}=\frac{\quad}{7}$.
44. $\frac{3}{7}=\frac{3^2}{\quad}$. **45.** $\frac{1}{x}=\frac{x^2}{\quad}$. **46.** $\frac{x}{1}=\frac{\quad}{xy}$.

47. What multiplying factor changes 12 into 15 ? What fraction of 15 is 12 ?

48. By what must 20 be multiplied to increase it (i) by 10, (ii) by 5 ?

49. What multiplying factor changes (i) 75 into 100, (ii) 100 into 125 ?

50. In what ratio must 50 be increased to make it 75 ?

51. In what ratio must 100 be decreased to make it 75 ?

52. A man starts the year with £12,500 and ends it with £15,000. In what ratio has his property increased ?

53. A's income is to B's in the ratio 5 : 3. If B's income is £210 a year, what is A's ?

54. The distance from A to B is to the distance from A to C in the ratio 5 : 3. A is 210 miles from B. How far is A from C ?

55. One train travels 45 miles in an hour and another travels 30 miles in 40 minutes. Find the ratio of their rates.

56. Increase £150 in the ratio 5 : 6.

57. One side of a rectangle is reduced from 24 in. to 21 in. ; reduce the other side of 14 in. in the same ratio.

58. One man's salary of £5 a week is increased to £6 a week increase another salary of £500 a year in the same ratio.

59. Two sums of money are in the ratio 3 : 4 ; the smaller is £6, what is the larger ? The larger is £6, what is the smaller ?

60. A boy's mark of 154 is increased to 182. What should a mark of 132 become if altered in the same ratio ?

61. A bonus of 45 is added to a mark of 225. What bonus should be added to 150 to preserve the proportion ?

62. A table priced at £20 is reduced in a sale by £4, 16s. What should be the sale price of a suite, originally marked £50, if reduced at the same rate ?

63. What multiplying factor will increase 100 by 6 ?

64. What is the ratio of the area of a square of side 6 in. to that of a square of side 3 in. ? What decimal of the larger area is the smaller ?

65. Find the ratio of the volumes of two cubes whose edges are 8 in. and 6 in. in length.

66. In a fight 351 men are killed and 2223 are wounded. What is the ratio of killed to total casualties ?

67. What number bears the same ratio to x as a does to b ?

68. A rectangle whose sides are a in. and b in. is reduced in area in the ratio $p : q$. Find the new area. [$p > q$.]

69. A sum £x is increased in the ratio $a : b$ [$a < b$], and the result is again increased in the same ratio. Find the final result.

70. The speed of a train is reduced from 40 m.p.h. to 30 ft. per second. In what ratio is the speed reduced ?

71. The horizontal intensity of the earth's magnetic field in latitude 50° N. is 0·195 at long. 10° E. and 0·201 at long. 20° E. What is it at long. 16° E., assuming it changes uniformly per degree of longitude ?

72. The linear dimensions of the wheels of a carriage are in the ratio 9 : 7. When the larger revolves 315 times, how many times does the smaller revolve ?

73. A clock loses 8·5 seconds an hour when the fire is alight and gains 5·1 seconds an hour when the fire is out. On the whole it keeps correct time. For what fraction of each day is the fire burning ?

74. Taking 32° F. = 0° C. and 212° F. = 100° C., express 50° F. in Centigrade and 65° C. in Fahrenheit.

75. The lengths of two fields are in the ratio 3 : 2, and their breadths in the ratio 6 : 5. What is the ratio of their areas ?

76. What are the simplest numbers to which 96, 48, 84 are proportional ?

77. Express as simply as possible : (i) $\frac{1}{2} : \frac{1}{3} : \frac{1}{6}$; (ii) $\frac{2}{3} : 1\frac{1}{2} : 1\frac{1}{3} : \frac{3}{4}$.

78. In a certain system for advertising the charges are 30 words for 14 shillings, 38 words for 16 shillings, 44 words for £1. Are the charges proportional to the number of words ?

79. What fraction of $3\frac{1}{4}$ is $2\frac{2}{3}$? What is the ratio of $2\frac{2}{3}$ to $3\frac{1}{4}$?

80. What multiplier of $4\frac{1}{4}$ will make $5\frac{2}{3}$? What is the ratio of $4\frac{1}{4}$ to $5\frac{2}{3}$?

Give the answers to the following correct to 3 places of decimals :

81. What decimal of £2, 3s. 8d. is £1, 0s. 9d.? What decimal is £1, 0s. 9d. of £2, 3s. 8d.?

82. What decimal must £3, 2s. 8d. be multiplied by to give £1, 19s. 4d.?

83. Express 11s. 3d. as a decimal of 16s. 8d.

84. Give the first of the following quantities as a decimal of the second : (*a*) (i) £1, 5s. 9d, (ii) £2, 13s. 10d. ; (*b*) (i) £1, 14s. $2\frac{1}{4}$d., (ii) £3, 0s. $8\frac{1}{2}$d. ; (*c*) (i) 8s. $7\frac{1}{2}$d., (ii) £1, 6s. 3d. ; (*d*) (i) 4 cwt. 2 qr., (ii) 1 ton 2 cwt. ; (*e*) (i) 3 yd. 2 ft. 6 in., (ii) 5 yd. 1 ft. 4 in.

Example.—A map is drawn on a scale of 6 *in. to the mile. What ratio represents this scale ?*

$$1 \text{ mile} = 1760 \text{ yd.} = 1760 \times 36 \text{ in.}$$

$$\therefore \text{ ratio} = \frac{6}{1760 \times 36} = \frac{1}{1760 \times 6} = \frac{1}{10{,}560}.$$

Note.—Abroad, maps are usually made on a scale which is expressed by a ratio of the form, 1 : multiple of 10, such as $\frac{1}{250{,}000}$, $\frac{1}{100{,}000}$, $\frac{1}{50{,}000}$. The fraction $\frac{1}{n}$ is called the **Representative Fraction,** or R.F. of the map.

Example.—Express the ratio of 15 *to* 20 *in the form* $n : 1$.

The ratio $\frac{15}{20} = \frac{1 \cdot 5}{2} = \frac{\cdot 75}{1}$, *i.e.* **·75 : 1.**

Example.—Divide £49 *between two men in the ratio of* 5 *to* 2.

If we divide £49 into $(5+2)$ equal portions, one man receives 5 of these portions and the other receives 2 of these portions.

$\therefore$ the shares are $\frac{5}{7}$ of £49 = **£35,** and $\frac{2}{7}$ of £49 = **£14.**

Example.—Divide ten shillings into three shares in the ratio 4 : 5 : 6.

If we divide ten shillings into $(4+5+6)$ equal portions, the first share contains 4 of these portions and the second share contains 5 of these portions and the third share contains 6 of these portions.

$\therefore$ the shares are $\frac{4}{15}$ of 10s., $\frac{5}{15}$ of 10s., $\frac{6}{15}$ of 10s., or $2\frac{2}{3}$s., $3\frac{1}{3}$s., 4s., or 2s. 8d., 3s. 4d., 4s. *Check.*—2s. 8d. + 3s. 4d. + 4s. = **10s.**

EXERCISE X (b)

Express in the form $n : 1$ the following ratios :

1. 3 : 2. **2.** 87 : 100. **3.** 100 : 40. **4.** $5\frac{1}{2} : 1\frac{1}{4}$.

5. 1 year : 1 month. **6.** 1 cwt. : 1 ton 5 cwt.

By expressing both ratios in the form $n : 1$, find which is the greater of the following pairs :

7. 3 : 2 or 4 : 3. **8.** 6 : 5 or 8 : 7. **9.** 11 : 9 or 10 : 8·5.

10. 64 : 53 or 95 : 79. **11.** $2 : 3$ or $2^2 : 3^2$. **12.** $5 : 3$ or $5^2 : 3^2$.

13. A number is increased by $\frac{3}{7}$ of itself. What is the ratio of its new value to its old value ?

14. In what ratio is a number increased if it is made half as large again ?

15. A number is diminished by $\frac{1}{4}$ of itself. In what ratio has it been changed ?

16. If 5s. is the cost of 2 lb., by what factor must 5s. be multiplied to give the cost of 3 lb.?

17. The price of a loaf is increased by one-third of itself. What is the ratio of the new price to the old ?

18. What multiplying factor increases a number by two-thirds of itself ?

19. What multiplying factor reduces a number by two-fifths of itself ?

20. Two loads are 12 tons and 8 tons. Both are reduced by 5 cwt. Is the ratio of the first load to the second load greater or less than originally ? What is it now ?

21. Two bank deposits are £120, £80. Each is increased by £20. Is the ratio of the two deposits increased or diminished ?

22. A number is increased by adding to it one-third of itself. In what ratio must the result be reduced to obtain the original number ? What fraction of the increased number must be subtracted to give the original number ?

23. A man's income is reduced by one-fifth. What fraction of his reduced income must be added in order to make it up to the original amount ?

24. What multiplying factor is required in the following ? (i) To diminish a number by $\frac{1}{20}$ of itself ; (ii) to increase 50 by 10 ; (iii) to change a price in shillings per yard to a price in pence per foot ; (iv) to change a speed of miles per hour to a speed of feet per second ; (v) to convert a charge of farthings per lb. into £ per ton.

25. What is the R.F. of a map if (i) a distance of 12 miles is represented by 2 in. ; (ii) 5 cm. on the map represents 5 Km. on the ground ; (iii) 6·4 in. on the map represents $5\frac{1}{2}$ miles ?

26. What is the ratio of the area of a field to the area shown on a map whose R.F. is $\frac{1}{10,000}$?

27. If the R.F. of a map is $\frac{1}{50,000}$, find (i) the distance apart in yards of two places 2·42 in. apart on the map ; (ii) the distance apart in Km. of two places 4·3 cm. apart on the map.

28. The scale of a map is $\frac{1}{n}$. (i) How far apart in miles are two places which are x in. apart on the map? (ii) How far apart in Km. are two places which are x cm. apart on the map?

29. Divide £1 between two boys in the ratio 3 : 2.

30. Divide a rod 2 ft. long in the ratio 4 : 1.

31. The point P divides AB in the ratio 2 : 3. Find the ratio of (i) AP to AB, (ii) AB to PB.

32. A sum of £5 is divided between two men in the ratio 3 : 7. What does each receive?

33. The sides of a rectangle are in the ratio 3 : 2; its area is 2646 sq. ft. What is its perimeter?

34. Divide £92 in the ratio $\frac{2}{5} : \frac{3}{4}$.

35. Divide a rod 3 ft. long in the ratio 2 : 3 : 4.

36. Three men divide the profits (£900) of a business in the ratio 3 : 5 : 7. How much is the largest share?

37. An examiner wishes to mark four questions in the ratio 3 : 4 : 5 : 8. Full marks for the questions are to be 100. What marks should he give to each?

38. A stick is broken in two pieces in the ratio 2 : 3. The smaller is again broken in the ratio 4 : 5. What is the proportion of the three parts to each other?

39. On a map of scale 1 : 10,000 a straight road crosses the contour lines 100 metres, 120 metres at P, Q; the length of PQ on the map is 2·3 cm. Find the average slope of the road from P to Q. [Answer in form 1 in n.]

40. Out of a capital of £10,000, A provides £3000, B £4300, C £2700. The year's profits are £2600. What should each receive?

41. A road up a hill ascends for 600 yd. at a gradient of 1 in 18, for 300 yd. at 1 in 12, for 150 yd. at 1 in 9, for 200 yd. at 1 in 8 What is the average gradient for the whole ascent?

42. The following table gives the horse-power at which an engine must work to draw a train of a certain weight at a certain steady speed along the level.

Weight of train in tons.	200	250	300
30 ft. per second .	110	135	160
40 ft. per second .	145	180	215
50 ft. per second .	180	225	270

At what horse-power would you expect an engine to work if it is drawing a train (i) of weight 240 tons at 44 ft. per second; (ii) of weight 260 tons at 36 ft. per second?

43. If £a is divided in the ratio $x : y$, what is the value of each share?

44. In what ratio has x been diminished if y is subtracted from it?

45. Divide £124 in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{5}$.

46. What is the ratio of $3\frac{1}{2}$ to $2\frac{1}{4}$? If $2\frac{1}{4}$ lb. cost 1s. $8\frac{1}{4}$d., what will $3\frac{1}{2}$ lb. cost?

47. What is the ratio of $\frac{5}{12}$ to $\frac{3}{7}$? If $\frac{3}{7}$ of a pole is 6 ft., what is the length of $\frac{5}{12}$ of it?

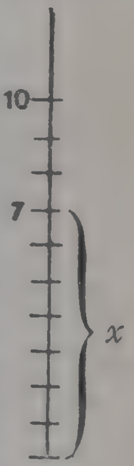
48. What is the ratio of $1\frac{1}{4}$ to $2\frac{2}{3}$? If $2\frac{2}{3}$ acres cost £32, what will $1\frac{1}{4}$ acres cost?

When a quantity has been altered in a given ratio and we are given the final value, the original value should always be found by considering its ratio to the final value.

Example.—If $\frac{7}{10}$ of a number is x, what is the number?

If the original number is divided into 10 equal parts, 7 of these will equal x. The ratio of the original number to the part x is 10 : 7.

I.e. the original number is $\frac{10}{7}$ of $x = \frac{10x}{7}$.



Example.— After paying a tax of 2s. in the £, a man has £720 left. What had he originally?

Out of 20s. originally he has 18s. left. ∴ the original amount to what is left is 20 : 18. ∴ the original amount is $\frac{20}{18}$ of £720 = £800.

EXERCISE X (c)

1. If $\frac{5}{7}$ of a number is x, what is the number?

2. If $\frac{11}{12}$ of a number is 121, what is the number?

3. A number is reduced by $\frac{1}{3}$ of itself and is then 162. What was it originally?

4. After spending $\frac{1}{6}$ of his holiday, a boy has 35 days left. How long was the holiday?

5. After paying 2s. 6d. in the £, a man has £490 left. What had he originally?

6. One-seventh of a field is fenced off, leaving 4200 sq. yd. What was the original area?

7. A boy gives up an answer of 0·625 and is told that it is $1\frac{1}{4}$ times too much. What was the correct answer?

8. If after 2 oz. in every lb. has been wasted there remains 2 cwt. 3 qr. 7 lb., how much was there originally?

9. An entertainment has to pay 9s. $7\frac{1}{2}$d. out of every £ in expenses. If the balance was £72, 12s. 6d., what were the gross receipts?

10. After a deposit has been increased by a shilling in the pound, it amounts to £214, 12s. 9d. What was it originally?

11. A liquid when heated expands ·00236 of its original volume. If it is finally 624·7 c.c., what was it before heating?

12. A sum of money has been reduced by $\frac{1}{3}$. What fraction of the reduced amount must be added to bring it back to its original value?

13. A boy's savings are increased by 1s. 3d. for every £ he has saved: he then has £46, 15s. What had he originally?

14. One-eighth of each side of a square is cut away and the area is then 441 sq. in. What was the original area?

15. After paying duty of 2s. $5\frac{1}{2}$d. in the £, the cost of an article is increased to £13, 9s. 6d. What was the original price of it?

16. Two cwt. of slack is delivered with every ton of coal. How many tons must be delivered to provide 90 tons which are not slack?

17. A ton of ore is reduced to a residue of 4 cwt. by washing. How much ore must be washed to provide 250 tons of residue?

18. When dried, every cwt. of grain loses 7 lb. in weight. How much must be dried to produce 25 tons after the operation?

Questions involving proportion may be solved by means of ratios instead of by means of unitary method, as explained on p. 114.

Example.—*$7\frac{1}{2}$ tons of coal cost £16, 10s. Find the cost of $6\frac{1}{4}$ tons of coal at the same rate.*

$$\frac{\text{Cost of } 6\frac{1}{4} \text{ tons}}{\text{Cost of } 7\frac{1}{2} \text{ tons}} = \frac{6\frac{1}{4}}{7\frac{1}{2}}. \quad \therefore \text{cost of } 6\frac{1}{4} \text{ tons} = \frac{6\frac{1}{4}}{7\frac{1}{2}} \text{ of } £16\frac{1}{2}$$

$$= £\frac{25}{4} \times \frac{2}{15} \times \frac{33}{2} = £\frac{55}{4} = \underline{£13,\ 15\text{s.}}$$

Example.—*Twelve pumps working 9 hours a day can raise 250,000 gallons of water in 10 days. How long will 16 pumps take to raise 800,000 gallons working 8 hours a day?*

12 pumps can raise 250,000 gallons in 10 × 9 hours.

∴ 12 ,, ,, ,, 800,000 gallons in $\frac{80}{25} \times 10 \times 9$ hours.

∴ 16 ,, ,, ,, 800,000 gallons in $\frac{12}{16} \times \frac{80}{25} \times 10 \times 9$ hours.

$$\frac{12}{16} \times \frac{80}{25} \times 10 \times 9 = 216 \text{ hours} = \frac{216}{8} \text{ days} = \underline{27 \text{ days.}}$$

EXERCISE X (d)

1. If 12 yd. of cloth cost £2, what will 15 yd. cost?

2. If the third-class fare for a journey of 24 miles is 3s., what is the fare for a journey of 40 miles?

3. If 3 equal taps can fill a cistern in 30 minutes, how long will 2 of the taps take?

4. A rail journey takes 48 minutes if the train travels at 28 miles an hour. How long does it take if the average speed is 35 miles an hour?

5. A man receives £2, 4s. for a week of 48 hours' work. How much should he receive if he is absent 15 hours in the week?

6. Ten men can dig a certain trench in 6 days. How long would 4 men take?

7. A garrison of 900 men have enough food for 50 days. How long will it last if they receive a reinforcement of 350 men?

8. The rates on a house are proportional to the rent. If the rates on a house rented at £150 amount to £90, what are the rates on a house rented at £55?

9. A housekeeper buys 50 lb. of tea at 2s. 3d. a lb. How much could she obtain for the same money of a quality costing 2s. 6d. a lb.?

10. A strip of lace 3 ft. 4 in. long costs 2 guineas. What is the cost of a strip of the same lace 6 ft. 3 in. long?

11. Three men take 2 days to paint a fence. How long will 4 men take to paint a fence of the same height but twice the length?

12. If 6 men can mow 6 acres of grass in 6 days, how many acres would 10 men mow in 9 days?

13. Seven men earn £42, 10s. in 16 days, working 10 hours a day. How much would 6 men earn in 14 days, working 8 hours a day?

14. If an 8-penny loaf weighed 48 oz. when wheat was at 39s. per quarter, what should a 6½d. loaf weigh when wheat is at 50s. per quarter?

15. The expenses of 5 men for 8 days amount to £30. What should be the expenses of 6 men for 9 days at the same rate?

16. Three thousand tiles are needed for a floor 15 ft. long, 8 ft. wide. How many are needed for a floor 20 ft. long, 12 ft. wide?

17. Fifty tons of anthracite are needed for 5 furnaces every 20 days. How much is consumed by 8 furnaces each week (7 days)?

18. A plate of steel 6 ft. long, 3½ ft. wide weighs 4 tons. What is the weight of a plate of steel 9 ft. long, 4 ft. 8 in. wide, and three times as thick?

19. Water is standing to a depth of 15 in. in a cistern 12 ft. long, 8 ft. wide; it is run off into a tank 20 ft. long 18 ft. wide. What will be the depth in the tank?

20. The driver of a water-cart finds that he can water 400 yd. of road with 32 gallons if he drives at $2\frac{1}{4}$ miles an hour. How much water will he need for 750 yd. if he drives at 3 miles an hour, if the water runs out at the same rate in gallons per second?

Percentage

If a fraction is transformed so that its denominator is 100, it is said to be expressed as a percentage. Thus $\frac{1}{4}=\frac{25}{100}$, and we say that $\frac{1}{4}$ is the same as 25 per cent. (Lat. *per centum*). This is often written 25%. If we wish to compare the values of two fractions, it is often convenient to express each as a percentage; for instance, A gets 48 marks out of 64, B gets 32 out of 40. Which is the better performance? The ratio of A's mark to the maximum is $\frac{48}{64}=\frac{3}{4}=\frac{75}{100}$. The ratio of B's mark to the maximum is $\frac{32}{40}=\frac{4}{5}=\frac{80}{100}$. B gets 80% and A gets 75% of the maximum.

Example (i).—*Express* $\frac{3}{5}$ *as a percentage.*

$$\frac{3}{5}=\frac{\frac{3}{5}\times 100}{100}=\frac{60}{100}=60\%.$$

We can therefore find the percentage by multiplying the fraction by 100. $\frac{3}{5}=\frac{3}{5}\times 100\%=60\%$.

Example (ii).—*What is* 40% *of* £55?

$$40\% \text{ of } £55=£\tfrac{40}{100}\times 55=£\tfrac{2}{5}\times 55=\underline{£\mathbf{22}}.$$

Example (iii).—*What percentage is* 8*d*. *of* 2*s*. 6*d*.?

$$2s.\ 6d.=30d. \quad \therefore\ 8d.=\tfrac{8}{30} \text{ of } 2s.\ 6d.$$

$$\text{Now } \tfrac{8}{30}=\tfrac{8}{30}\times 100\%=\tfrac{80}{3}\%=26\tfrac{2}{3}\%.$$

Example (iv).—63 *is* 14% *of a certain number. What is the number?*

The required number bears to 63 the ratio of 100 to 14.

$$\therefore \text{ number}=63\times\tfrac{100}{14}=\underline{\mathbf{450}}.$$

Example (v).—*A tradesman sells an article for* 5% *less than it cost. If the cost was* £30, *find the selling price.*

The ratio of the selling price to the cost price is 95 : 100. ∴ the multiplying factor is $\frac{95}{100}$.

$$\therefore \text{ selling price}=£\tfrac{95}{100}\times 30=£28{\cdot}5=\underline{£\mathbf{28,\ 10s.}}$$

EXERCISE X (e)

1. Express as fractions (i) 50%, (ii) 75%, (iii) 60%, (iv) 35%, (v) 150%, (vi) $33\frac{1}{3}$%, (vii) $2\frac{1}{2}$%, (viii) x per cent.

2. Express as percentages (i) $\frac{4}{5}$, (ii) $\frac{1}{20}$, (iii) $\frac{1}{10}$, (iv) $\frac{2}{3}$, (v) $\frac{3}{4}$ (vi) $\frac{5}{2}$, (vii) $\frac{a}{b}$.

3. Find the values of (i) 25% of £60; (ii) 40% of 5 ft.; (iii) 5% of 1 mile; (iv) $12\frac{1}{2}$% of 3 gallons; (v) 70% of 450 boys.

4. What percentage is (i) 3d. of 1s.; (ii) 3s. 6d. of £1; (iii) 2 qr. of 1 ton; (iv) 4 chains of 1 mile?

5. If 25 per cent. of a number is 16, what is the number?

6. If 12 per cent. of a sum of money is £30, what is the sum?

7. If a man spends 85 per cent. of his income, what percentage does he save?

8. A boy scores 27 marks out of 60 on a paper; what percentage is this? In order to pass it is necessary to obtain 40 per cent.; what is the lowest mark for passing?

9. The marked price of a chair is £5, 10s., and a dealer deducts 15 per cent. for cash. What is the cash price?

10. The duty on an article is 12 per cent. of its value; the duty is 18s. What is its value?

11. A man invests £2500 in house property and sells it when it has risen 18 per cent. in value. What does he sell it for?

12. What is the percentage error if an error of $1\frac{1}{2}$ in. is made in measuring a length of 5 ft.?

13. State in shillings the value of 5 per cent. of £x.

14. Two men each start with a salary of £320 a year; one then receives an increase of 8 per cent. and the other an increase of 13 per cent. What is now the difference between their yearly salaries?

15. To a lump of pure gold is added 20 per cent. of alloy. What fraction of the mixture is pure gold?

16. In sawing timber there is a waste of $2\frac{1}{2}$ per cent.; a block containing 280 cu. ft. is cut up. How many cubic feet are available for use?

17. Eight gallons of old worn-out wine are fortified by adding 150 per cent. of new wine. How many gallons does the mixture contain?

18. Full marks for three papers are 200, 300, 250, and a boy scores 40 per cent., 60 per cent., 58 per cent. on them respectively. What is his percentage score on all three papers together?

19. There were two candidates at an election: one of them received 59 per cent. of the votes cast and so secured a majority of 54 votes. How many voted?

20. A commercial traveller made 76 per cent. of his year's income in the first half of the year; in the second half of the year he made £63. What was his year's income?

21. x per cent. of a number is 10. What is the number?

22. A man sells y per cent. of his estate. What percentage does he keep?

23. A man mixes x tons of anthracite with $3x$ tons of coke. What percentage of the mixture is anthracite?

24. A house is valued at £P, and the tenant pays r per cent. of its value as rent each year. How much rent does he pay in T years?

25. A gas company raises its charges by 20 per cent., and a year later increases the new charges by 20 per cent. If the total increase had been made all at once, to what percentage would it have been equivalent?

26. A man sells an article and loses 5 per cent. What is the ratio of the money he gave for it to the price for which he sold it? If he sold it for £19, what did he give for it?

27. A tradesman sold an article for 12 per cent. more than it cost him. If he sold it for £14, what did it cost him?

28. A bicycle after losing 10 per cent. of its original value is worth £8, 2s. What was its original value?

29. A man loses 12 per cent. by selling an article for £11. What was it really worth?

30. A bill has $2\frac{1}{2}$ per cent. deducted from it and then amounts to £2, 3s. $10\frac{1}{2}$d. What was it originally?

CHAPTER XI

AREAS

British Units

44 (= 12^2) square inches (sq. in.) = 1 sq. foot (sq. ft.)
9 (= 3^2) square feet = 1 sq. yard (sq. yd.)
84 (= 22^2) square yards = 1 sq. chain
10 square chains = 1 acre (ac.)
40 acres = 1 sq. mile (sq. m.)

4840 sq. yd. = 1 acre.

$30\frac{1}{4}$ (= $(5\frac{1}{2})^2$) sq. yds. = 1 sq. pole.
40 sq. poles = 1 rood.
4 roods = 1 acre.

A square foot is the area of a square whose side is 1 ft. or 12 in.; it therefore contains $12\times12=144$ sq. in. Similarly a square yard contains 3×3 sq. ft.

Since 1 mile $=1760$ yd., 1 sq. mile $=1760\times1760$ sq. yd. $=\frac{1760\times1760}{4840}$ acres $=640$ acres.

Land agents use roods and square poles; they call the latter, however, simply poles.

Acre: derivation Anglo-Saxon *acer* = a field. Note that an acre is the area of a plot of ground of length 1 furlong (10 chains) and breadth 1 chain.

Metric System

100 sq. millimetres (sq. mm.) = 1 sq. centimetre (sq. cm.)
100 sq. centimetres = 1 sq. decimetre (sq. dm.)
100 sq. decimetres = 1 sq. metre (sq. m.)
100 sq. metres = 1 sq. decametre (sq. Dm.)
100 sq. decametres = 1 sq. hectometre (sq. Hm.)
100 sq. hectometres = 1 sq. kilometre (sq. Km.)

1 sq. decametre = 1 are (a.).

100 centiares (ca.) **= 1 are.**
100 ares = 1 hectare (Ha.).

1 hectare is about $2\frac{1}{2}$ acres.

For other constants, see p. xiv.

Example.—A plank is 16 ft. 6 in. long and 1 ft. 8 in. wide. Find its area.

Length $=16\frac{1}{2}$ ft.; breadth $=1\frac{2}{3}$ ft. ∴ area $=16\frac{1}{2}\times1\frac{2}{3}$ sq. ft.

$$=\frac{33}{2}\times\frac{5}{3}\text{ sq. ft.}=\frac{55}{2}\text{ sq. ft.}=\underline{\mathbf{27\tfrac{1}{2}\text{ sq. ft.}}}$$

Example.—A man requires 165 sq. ft. of carpet: it is made up from a roll of carpet 2 ft. 9 in. wide. What length of the roll must he buy?

Area $=165$ sq. ft.; width $=2\frac{3}{4}$ ft. ∴ length $=165\div2\frac{3}{4}$ ft.

$$=165\div\frac{11}{4}\text{ ft.}=165\times\frac{4}{11}\text{ ft.}=\underline{\mathbf{60\text{ ft.}}}$$

Example.—How many tiles 5 in. by 3 in. are needed to pave the floor of a hall 20 ft. long, 15 ft. wide?

Area of hall $=20\times15$ sq. ft. Area of each tile $=\frac{5}{12}\times\frac{1}{4}$ sq. ft.

$$\therefore\text{ number of tiles }=\frac{20\times15}{\frac{5}{12}\times\frac{1}{4}}=20\times15\times\frac{12}{5}\times\frac{4}{1}=\underline{\mathbf{2880\text{ tiles.}}}$$

EXERCISE XI (a)

1. A piece of squared paper is ruled in inches and tenths of an inch. How many small squares are there in a square of side 1 in.? What is the length of the side and the area of one of the small squares?

2. If you are given a square of side 1 in., how would you construct a square whose area is (i) $\frac{1}{4}$ sq. in., (ii) $\frac{1}{9}$ sq. in.?

3. A rectangle is 2·4 cm. long and 1·5 cm. wide. Into how many squares, each of side 1 mm., can it be divided?

4. Draw a figure to show the number of sq. cm. in 1 sq. dm., and express 1 sq. cm. as a decimal of a sq. dm.

5. Express in sq. m. (i) 385 sq. cm., (ii) 6 sq. dm., (iii) 100 sq. cm.

6. Express in sq. cm. (i) 1·7 sq. dm., (ii) 9 sq. mm., (iii) 5 sq. dm. 4 sq. mm.

7. Express in sq. m. (i) 15 sq. Km., (ii) 15,000 sq. cm., (iii) 3 sq. Dm.

Find the area and perimeter of the rectangles in Ex. 8–15.

8. A sheet of paper 6·5 in. wide, 8·2 in. long.

9. A pane of glass 1 ft. 6 in. wide, 2 ft. 8 in. high.

10. A courtyard 30 yd. long, 40 ft. wide.

11. A blackboard 3 ft. 4 in. high, 5 ft. 3 in. wide.

12. A strip of cloth 2 ft. 6 in. wide, 8 yd. long.

13. A bandage 6 ft. long, 2 in. wide.

14. A field 2 Hm. long, 3 Dm. wide.

15. A bench 4 metres long, 35 cm. wide.

16. A sheet of postage stamps is 24 cm. wide and 30 cm. long: each stamp is 2 cm. wide and 3 cm. long. How many stamps are there in the sheet?

17. The floor of a room, 16 ft. long, 10 ft. wide, is to be covered with tiles 2 ft. square. How many are needed?

18. Tiles 8 in. by 6 in. are used to cover a floor. What is the area of the floor if 1200 tiles are required? Is it possible for the floor to be square if no tiles have to be broken? If so, what is its size, and how could the tiles be arranged?

19. A plank is 9 in. wide and 8 ft. long. How many are required for the floor of a room 16 ft. long, 9 ft. wide?

20. A packet of paper contains 100 sheets, each 12 in. by 8 in.; a second packet contains 175 sheets, each 9 in. by 6 in. Which contains the greater area of paper, and by how much?

21. What area of thin cardboard is needed to make a pack of 52 cards and 2 extra cards, if each card is 2 in. by 3 in.?

22. A hall 84 ft. long, 28 ft. wide is paved with stones 9 in. by 7 in. How many stones are required ?

23. How many turfs 3 ft. by 1 ft. does it take to turf a piece of ground 24 yd. long, 15 yd. wide ?

24. How many panes of glass, each 10 in. by 7 in., can be cut from a sheet of glass 50 in. square, and how much is left over ?

25. A rectangular box measures externally 4 ft. by 3 ft. by 2 ft. How many square feet of canvas are needed to cover it completely outside ?

26. A garden wall costs £1000 per mile length. Find, in square yards, the area of a square plot of ground that can be completely walled in for (i) £100, (ii) £200.

27. An acre equals 4840 sq. yd. ; a mile equals 1760 yd. How many acres are there in 1 sq. mile ?

28. How many dish-cloths, 2 ft. by 3 ft., can be made from 30 sq. yd. of cloth ?

29. A tennis court is 78 ft. by 36 ft. If it is to be netted in by a surround 40 yd. by 20 yd., what extra area of ground is required ?

30. A rug is 4 ft. wide and its area is 4 sq. yd. What is its length ?

31. The perimeter of a square field is 1 mile. What is its area in acres ?

32. The floor of a room 10 ft. wide is paved with tiles 8 in. by 4 in. If 810 tiles are needed, what is the length of the room ?

33. 40 sq. yd. of lint are cut up into bandages 3 in. wide. What is the total length ?

34. A sheet of cardboard weighs $\frac{3}{4}$ lb. and is 8 in. wide. What is its length if each square foot weighs 4 oz. ?

35. A carpet 25 ft. 6 in. long, 17 ft. 6 in. wide is made up from strips cut from a roll $3\frac{1}{2}$ ft. wide. What length of roll is required ?

36. The area of a rectangle is 6 sq. in. and its width is half an inch. What is its length ?

37. A roller towel is $2\frac{1}{2}$ ft. wide and requires 25 sq. ft. of linen. At what height must the roller be fixed if the bottom of the towel is 1 ft. above the floor ? [Neglect the size of the roller.]

38. A rectangular field is $7\frac{1}{2}$ acres : it is one-quarter of a mile long. What is its width ?

39. A map is on the scale of 4 in. to the mile. What area in acres is represented by a rectangle $2\frac{1}{2}$ in. long, $1\frac{1}{2}$ in. wide ?

40. A map is $2\frac{1}{2}$ ft. long, $1\frac{3}{4}$ ft. wide: it is drawn on a scale of 2 miles to the inch. What area does it represent?

41. On a map an estate of 2000 acres is represented by a rectangle 5 in. long, $2\frac{1}{2}$ in. wide. What is the scale of the map?

42. What is the cost of a carpet 5 yd. long, 10 ft. wide at 22s. 6d. per square yard?

43. A carpet 15 ft. long, 12 ft. wide costs £18. What is the cost per square yard?

44. How many circles, each of diameter 2 in., can be drawn on one side of a sheet of paper 16 in. long, 10 in. wide, if no two circles cut?

45. How many sheets of paper, 6 in. by 4 in., will be needed to cover inside the sides and bottom of an open box 3 ft. 2 in. long, 2 ft. 4 in. wide, 2 ft. 7 in. deep?

46. If the unit of length is 4 in., find the number of units of area in a rectangle 3 ft. by 2 ft.

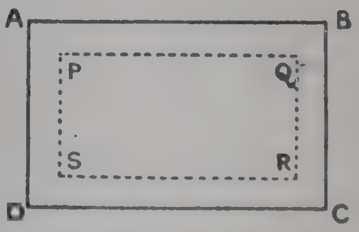
47. A rectangle is x ft. long and y yd. wide. What is its area in (1) square feet, (2) square yards?

48. A rectangle has an area of x sq. yd. If one side is x ft., what is the length of the other side in feet?

49. The perimeter of a rectangle is x ft. and its breadth is y ft. What is its area?

50. A carpet x ft. long, y ft. wide is made up from strips cut from a roll z ft. wide. What length of roll is required?

Example.—ABCD is a rectangular courtyard 20 yd. long, 12 yd. wide. A flagged path 5 ft. wide runs round the edge, and the inner part PQRS is turfed. Find the area of the path.



AB = 20 yd. = 60 ft. and AD = 12 yd. = 36 ft.

∴ since the path is 5 ft. wide, PQ = 60 − 5 − 5 = 50 ft.

PS = 36 − 5 − 5 = 26 ft.

∴ area of PQRS = 50 × 26 = 1300 sq. ft.

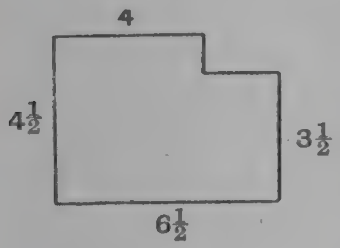
But area of ABCD = 60 × 36 = 2160 sq. ft.

∴ area of path = 2160 − 1300 = 860 sq. ft.

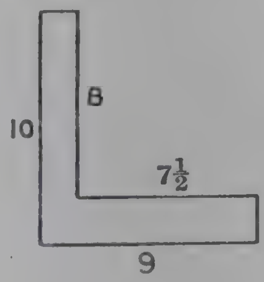
Note.—It is better to use the subtraction method in an example of this kind, instead of dividing the path into several rectangles and finding the area of each separately.

EXERCISE XI (b)

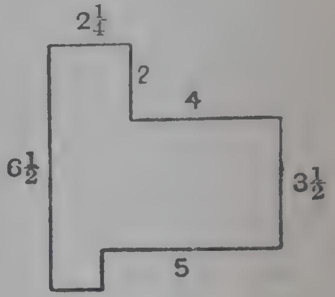
Find the area of the figures in Ex. 1–8, the dimensions being given in inches and the angles being all right angles.



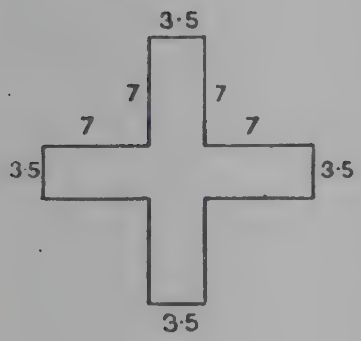
1.



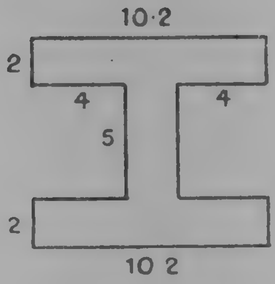
2.



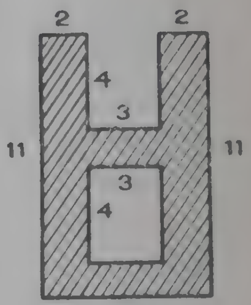
3.



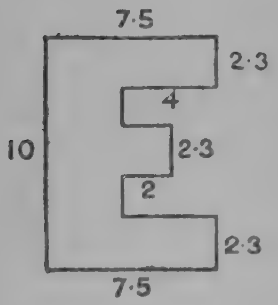
4.



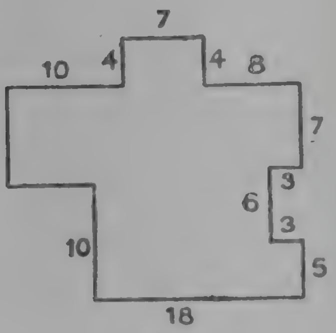
5.



6.



7.



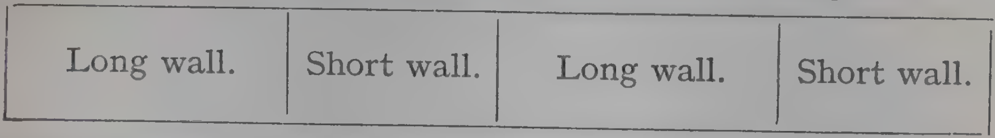
8.

9. A rectangular lawn 80 ft. long, 52 ft. wide is surrounded by a gravel path 4 ft. wide. Find the area of the path. What is the length of the outer edge of the path ?

10. Two gravel walks, each 5 ft. wide, cross each other at right angles : their lengths are 33 ft., 25 ft. Find their total area.

11. Four holes, each 2·5 ft. by 3·8 ft., are pierced in an iron plate 12·4 ft. by 8·5 ft. What is the area of the surface of the plate?

12.

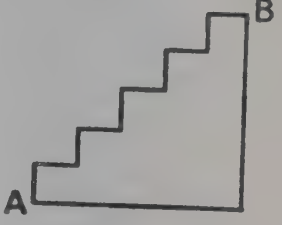


The figure represents the four walls of a room folded out flat (*i.e.* the net of the four walls). The room is 15 ft. long, 12 ft. wide, 8 ft. high: (i) find the total area of the four walls; (ii) find the area of the floor; (iii) draw the figure freehand and show on it a frieze 1 ft. deep which runs round the top of the room and find its area.

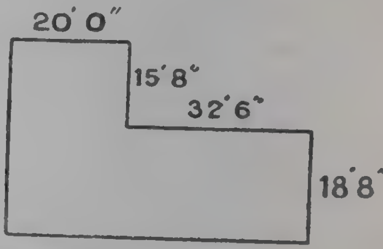
13. A wall of a room contains two windows each 5 ft. 4 in. by 3 ft. 9 in., and a door 3 ft. 6 in. by 7 ft. 4 in.; the rest is distempered. The wall is 18 ft. 8 in. long and 8 ft. 10 in. high. What area is distempered?

14. The figure represents a vertical section of a flight of 5 steps, each of which is 7 in. high and 10 in. deep. Find the area of the cross-section.

A stair carpet starts at A and ends at B and is 3 ft. wide. What is the area of the carpet?



15. The figure represents the ground-plan of a house; it is surrounded by a path 4 ft. wide. Find the area of the path.



16. A carpet 12 ft. 6 in. by 8 ft. 9 in. is laid in a room 15 ft. 8 in. by 10 ft. 9 in. and the border is stained. What is the cost of staining it at a halfpenny per square foot?

17. A garden is enclosed by a fence 6 ft. high. The perimeter of the garden is 100 yd. What is the area of the fence required?

18. How many strips of paper 5 in. square can be cut from a sheet 3 ft. by 4 ft., and how much is left over?

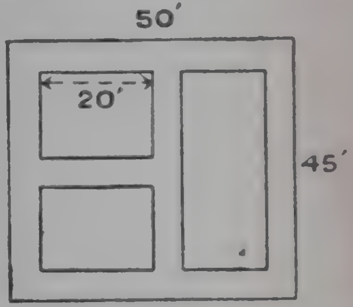
19. A room is 16 ft. long, 12 ft. wide. Find the cost of staining a border 1½ ft. wide all round the edge of the room if a pint of the staining mixture costs 5s. and does 15 sq. ft.

20. A fence runs all round a rectangular plot 180 yd. long, 110 yd. wide: it is surrounded by a pavement 3 yd. wide and a roadway outside the pavement 10 yd. wide. Calculate the area of the pavement and the area of the roadway.

21. An ordinary Bryant & May's wooden matchbox is 6 cm. long, 4 cm. wide, 2 cm. deep. What is the area of matchwood used in making it (*i.e.* for the two parts together)?

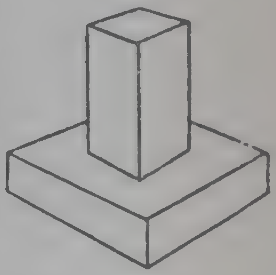
22. How many rows of potatoes may be set lengthwise in a garden 12 yd. long, 8 yd. wide, allowing 1 yd. between the rows and half a yard margin on the outside? If the potatoes are placed 1 ft. apart in a row, and a margin of 1 ft. at each end is allowed, how many potatoes will there be in the whole plot?

23. The figure represents three grass plots surrounded by gravel paths. All the paths are 5 ft. wide, and the external dimensions of the enclosure are 50 ft. by 45 ft. Find the area of (i) the grass, (ii) the paths.



24. An open box 8 in. long, made of thin cardboard, has its two ends 6 in. square, and is fitted with a cardboard lid which covers the top, the front, and the two square sides. What area of cardboard is used to make the box?

25. A wooden paper-weight (in which lumps of lead ore are embedded) consists of two rectangular blocks of wood glued together. The lower slab is 5 in. by 5 in. by 3 in., and the upper slab is 2 in. by 2 in. by 5 in. When it rests with its base on a level table, find the area of surface exposed to the air.



26. A rectangular lawn is x ft. long and y ft. wide. It is surrounded by a path z ft. wide. What is the area of the path? [Leave the result in brackets.]

27. A rectangular walled garden has a path x ft. wide round it next to the wall. If the lengths of the walls are a yd. and b yd., find the area of the cultivated part.

28. A thin cardboard box without a lid is x in. long, y in. broad, and z ft. deep. What area of cardboard is used in making it?

29. A field $3x$ yd. long, $2x$ yd. wide is enclosed by a fence x in. high. What is the area of the fence?

30. In the L-shaped figure in question 2 the lengths of the outer sides are each x in., and the lengths of the inner sides are each y in. What is the area?

Areas with Decimals

If the dimensions of a rectangle are measured correct to 3 significant figures, it is desirable to indicate that the fourth figure is unknown by putting a × for it; this will show that the area cannot be found beyond a limited degree of accuracy.

Example.—Find the area of a rectangle whose sides are 3·24 cm. and 4·56 cm.

Rough check 12 sq. cm.

```
  324×
     456×
 ---------
1296×
 1620×
   1944×
      ××××
 ---------
1477
```

Since the remainder from the fifth column is uncertain, we cannot give the result more accurately than 14·8 sq. cm.

A similar procedure should be adopted with division.

Example.—Find the length of a rectangle whose area is 46·4 sq. cm. and breadth 8·37 cm.

Rough check 5 cm.

Breadth **5·5 cm.**

```
            55
      -----------
837× | 464××
       4185×
       ---------
        45×××
        4185×
        --------
          3×××
```

EXERCISE XI (c)

Find the areas of the following rectangles:

1. (i) Length 4·42 cm., breadth 8·37 cm.; (ii) length 6·28 cm., breadth 5·43 cm.; (iii) length 12·82 cm., breadth 13·64 cm.; (iv) length 14·9 cm., breadth 16·6 cm.

Find the other dimensions of the following rectangles:

2. (i) Area 6·24 sq. cm., length 1·43 cm.; (ii) area 12·8 sq. cm., length 3·62 cm.; (iii) area 14·62 sq. in., length 5·28 in.; (iv) area 146·4 sq. in., breadth 13·4 in.

3. A closed box is 16·2 in. long, 12·4 in. wide, and 8·3 in. high. Find the total area of the outside surface.

4. A picture 18·6 cm. by 14·4 cm. is pasted on a sheet of paper 24·4 cm. by 19·8 cm. Find the area of paper uncovered.

5. A rectangle 24·6 m. by 11·4 m. is marked out on grass. If a fence is built round it at a distance of 4·2 m. away, what area lies between the fence and the rectangle?

6. A rectangular sheet of cardboard 26·4 cm. by 19·7 cm. is cut down so that its length is reduced by 4·8 cm. and its breadth by 3·9 cm. What area is cut away ?

7. A tank is 6·24 m. long, 4·38 m. wide, and 3·76 m. deep. When water in it has a depth of 1·87 m., what is the area of the inside which is not wetted ?

8. The walls of a room 8·6 m. by 12·4 m., which is 4·8 m. high, are papered with paper 1·2 m. wide. What length of paper is required ?

9. The circumference of a roller is 3·62 m. and its width is 72 cm. How many turns must it make to cover an area of 540 sq. m.?

10. A roller 86 cm. wide in 10 turns covers an area of 42·4 sq. m. Find its circumference.

CHAPTER XII

VOLUMES

Example.—How many cubic feet of water are required to fill a bath 40 *ft. long,* 30 *ft. wide to a depth of* 8 *ft.? Taking* 1 *cu. ft.* $=6\frac{1}{4}$ *gallons, find the number of gallons required.*

$$\text{Volume of water} = 40 \times 30 \times 8 \text{ cu. ft.} = \underline{\textbf{9600 cu. ft.}}$$

$$\text{Number of gallons} = 9600 \times 6\tfrac{1}{4} = 9600 \times \tfrac{25}{4} = 2400 \times 25 = \underline{\text{60,000 gals.}}$$

Example.—A swimming-bath is 50 *ft. long,* 24 *ft. wide. How much does the water-level rise when* 10,000 *gallons of water are pumped into it?*

$$10{,}000 \text{ gallons} = 10{,}000 \div 6\tfrac{1}{4} \text{ cu. ft.} = \frac{10{,}000 \times 4}{25} \text{ cu. ft.}$$

The area of the surface of the water $= 50 \times 24$ sq. ft.

$$\therefore \text{ the water-level rises } \frac{10{,}000 \times 4}{25} \div (50 \times 24) \text{ ft.}$$

$$= \frac{10{,}000 \times 4}{25 \times 50 \times 24} \text{ ft.} = \frac{4}{3} \text{ ft.} = \underline{\textbf{16 in.}}$$

EXERCISE XII (a)

1. Express in cu. ft. (i) 4 cu. yd. 12 cu. ft., (ii) 2 cu. yd. 864 cu. in.

2. Express in cu. m. (i) 18 cu. dm., (ii) 6000 cu. cm.

3. Express in cu. dm. (i) 5 cu. m., (ii) 700 cu. cm., (iii) 4 cu. m. 500 cu. cm.

4. What is the weight of (i) 1 cu. dm. of water, (ii) 1 litre of water, (iii) 1 cu. m. of water, given that 1 cu. cm. of water weighs 1 gram. ?

5. Find the volumes of the following rectangular blocks: (i) 6 ft. long, 4 ft. wide, 6 in. high; (ii) 20 cm. long, 12 cm. wide, 3 mm. high; (iii) 2 m. long, 4 dm. wide, 5 cm. high.

6. Find the volume of air in a room 16 ft. 8 in. long, 13 ft. 6 in. wide, 8 ft. 4 in. high.

7. A packet of paper is 25 cm. long, 16 cm. wide, 3 cm. high: it contains 100 sheets. Find the volume of each sheet.

8. A packing-case is 6 ft. long, 4 ft. wide. How high must it be if it is required to hold (i) 18 boxes each of which is a 2-foot cube, (ii) 480 boxes each of which is a cube of edge 6 in.?

9. 19 cu. ft. of water is poured into a tank 6 ft. 4 in. long, 4 ft. 6 in. wide. What is the depth of the water?

10. A dormitory is 20 ft. long, 15 ft. wide, $10\frac{1}{2}$ ft. high. How many boys can sleep in it if each requires 600 cu. ft. of air?

11. A packet of postcards measures 10 cm. by 8 cm. by 4 mm. How many packets can be placed in a box 4 dm. by 4 dm. by 2 dm.?

12. A bar of metal is 12 ft. long, 3 in. wide, $\frac{1}{2}$ in. thick. What does it weigh if a cubic inch of the metal weighs $\frac{3}{8}$ lb.?

13. A sheet of metal is $\frac{1}{10}$ in. thick. What is the weight per square foot of surface if a cubic foot of the metal weighs 504 lb.?

14. A wooden beam weighs 144 lb.: the area of its surface is 12 sq. ft. What is the thickness of the beam if the timber weighs 18 lb. per cubic foot?

15. Water is standing to a depth of 18 in. in a cistern 3 ft. long, 2 ft. wide. How does the water-level alter if (i) a 1-foot cube of lead is placed in the water, (ii) 2 cu. ft. of water are drawn from the cistern?

16. The water in Ex. 15 is poured into a tank whose base is 2 ft. square. What will be its depth?

17. Three hundred gallons of water are poured into an empty tank whose base is $4\frac{1}{2}$ ft. square. What will be the depth of the water? (1 cu. ft. $=6\frac{1}{4}$ gallons.)

18. How many gallons of water are needed to fill a bath 60 ft. long, 40 ft. wide, 12 ft. deep?

19. Find the weight of a beam 15 ft. long, 16 in. wide, 6 in. deep if the timber weighs 18 lb. per cubic foot.

20. Cubes of wood of edge 5 in. are cut from a 1-foot cube. How much wood is unused?

21. A trench 20 ft. long, 3 ft. wide, 4 ft. deep is made, and the soil which has been removed is spread out evenly to a depth of 4 in. What area of ground will it cover?

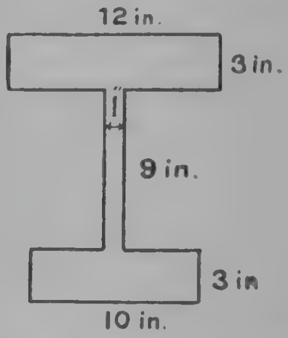
22. A stream is flowing at half a mile an hour in a channel 10 ft. wide, 3 ft. deep. How many gallons of water pass under a bridge in 5 minutes?

23. Eighteen cubic feet of gravel go to the load. How many loads must be ordered to gravel a path 36 yd. long, 8 ft. wide to a depth of 3 in.?

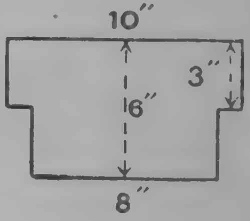
24. A coal-cellar measures 10 ft. by 8 ft. by 7 ft. Reckoning 90 lb. of coal to the cubic foot, find the cost of filling the cellar at 36s. per ton.

25. A rectangular tank 4 m. long, 2 m. broad is full of water. If 400 litres of water are drawn off, how much does the water-level fall?

(i)



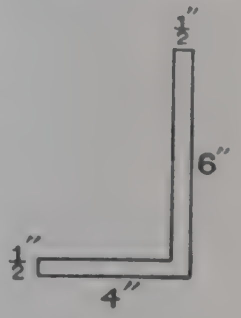
(ii)



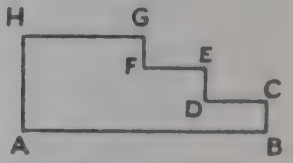
26. Find the volume per foot-run of the girders whose cross-sections are represented above. If the girders are of steel and 15 ft. long, find their weight, given that 1 cu. ft. of steel weighs 4 cwt.

27. Find the weight of an angle bar of cast iron, 4 ft. long, whose section is represented in the diagram, given that 1 cu. in. of cast iron weighs $\frac{1}{4}$ lb.

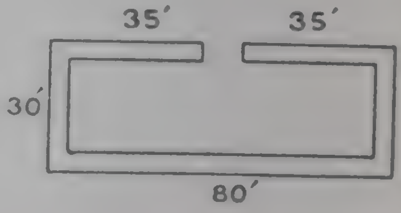
28. A steel rail is 10 yd. long and weighs half a ton. Find the area of its cross-section, given that 25 cu. in. of steel weighs 7 lb.



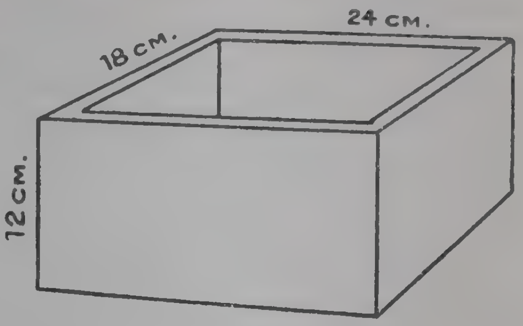
29. The diagram represents the section of 3 steps: BC = ED = FG = 6 in., CD = EF = 12 in., GH = 2 ft. The steps are 6 ft. wide. Find the volume.



30. The diagram represents the ground-plan of a stone parapet $2\frac{1}{2}$ ft. high and 2 ft. broad all the way round. Find its volume.



Example.—A box without a lid is 24 cm. long, 18 cm. wide, 12 cm. high (outside dimensions). If the wood is 1 cm. thick, find the volume of wood in the box.



If the box were solid the volume of wood would be $24 \times 18 \times 12 = 5184$ c.c.

The inside dimensions are: length $24-2=22$ cm., breadth $18-2=16$ cm., height $12-1=11$ cm.

∴ volume of inside $=22 \times 16 \times 11=3872$ c.c.
∴ volume of wood $=5184-3872=$ **1312 c.c.**

EXERCISE XII (b)

1. The external dimensions of a box without a lid are 12 in. long, 10 in. wide, 8 in. high: it is made of wood $1\frac{1}{2}$ in. thick. What are the internal dimensions of the box? How many cubic inches of water will the box hold? How many cubic inches of wood are used in making the box?

2. The external dimensions of a closed box are 20 cm. by 12 cm. by 10 cm.: the box is made of wood $\frac{1}{2}$ cm. thick. What are the internal dimensions of the box? How many cubic centimetres of air does the box contain? How many cubic centimetres of wood are used in making the box?

3. A box with a lid is made of wood 1 in. thick. The outside dimensions are: length 2 ft. 6 in., breadth 1 ft. 9 in., and height 1 ft. 4 in. Find the volume of the wood.

4. A box with a lid is made of wood 1 in. thick. The outside dimensions are: 2 ft. 3 in.; 2 ft.; height 1 ft. 8 in. Find the volume of wood.

5. An open tray for holding foolscap paper measures externally 24 cm. (length), 18 cm. (breadth), 5 cm. (height): the wood is 1 cm. thick. Find the volume of wood required to make the tray.

6. A corridor 4 ft. wide is built round the outside of a rectangular room; the length of the corridor in one direction is 35 ft., and in the other 28 ft., both measurements being taken from wall to wall. If the height is 14 ft., find the air-space it contains.

7. A box 12 cm. by 8 cm. by 6 cm. is placed inside another whose external dimensions are 24 cm. by 16 cm. by 10 cm., which is made of wood $\frac{1}{2}$ cm. thick. Find the volume of the vacant space when the lid of the outer box is closed.

8. A gravel walk 6 ft. wide runs round a grass plot 60 ft. long, 40 ft. wide. The gravel costs 12s. per cubic yard. Find the cost of a coat of gravel 2 in. deep.

9. A cistern 3 ft. deep on a base 5 ft. square receives the rain-water from roofs containing a horizontal area of 100 sq. yd. What depth of rainfall will fill the tank ?

10. Water flows into a rectangular tank through a square section pipe 4 in. square at the rate of 9 in. per second. The tank is 20 ft. long, 12 ft. broad. Find the depth of water in the tank at the end of 1 hour.

11. Water runs into a tank 90 ft. long, 40 ft. wide at the rate of 120 gallons per minute. Taking 1 gallon as equivalent to $\frac{1}{6}$ cu. ft., find the rate at which the water-level rises in inches per hour.

12. A hole whose section is a square of side $\frac{3}{4}$ in. is bored straight through a cube of side $3\frac{3}{4}$ in., and a hole whose section is a square of side 3 in. is bored straight through a cube of side $4\frac{1}{2}$ in. Which of the two solids so formed has the greater volume ?

Volumes with Decimals

EXERCISE XII (c)

1. Find the volume of a cuboid with the following dimensions. (Answer to 3 significant figures only) :

6·43 by 4·72 by 3·84 cm.
14·8 by 12·5 by 10·3 in.
5·62 by 4·83 by 3·76 m.

2. Find the other dimensions of the following cuboids :

2·43 by 3·62 cm., vol. 15·6 c.c.
12·3 by 14·2 cm., vol. 232·4 c.c.
14·6 by 16·7 cm., vol. 216 c.c.

3. What volume of water is required to fill a dock 162·4 m. long, 62·5 m. broad, 12·2 m. deep ?

4. A tank is 2·4 m. long, 1·8 m. wide. How much will the water-level rise if 621 litres of water are poured into it ?

5. A beam is 5·6 m. long and 34 cm. square in section. What will it weigh if 10 cu. dm. of the timber weighs 8·6 Kg.?

6. A stream of water is flowing at 1·4 m. per second along a channel 3·6 m. wide and 2·4 m. deep. How many litres pass a given point in 12 seconds ?

7. The soil from a trench 32·6 m. long, 1·4 m. wide, and 1·8 m. deep is spread out evenly to a depth of 8·4 cm. What area of ground would it cover if the volume remained unaltered ?

8. If ·74 cu. m. of gravel make a load, find the number of loads required to cover a path 42·4 m. long, 2·0 m. wide to a depth of 8 cm.

9. An open tray measures externally 26·2 cm. by 18·4 cm. and is 4·7 cm. high; if the wood is 1·2 cm. thick, find the volume of the wood.

10. A closed box is made of wood 7 mm. thick. The outside dimensions are 28·4 cm. long, 24·6 cm. wide, and 14·6 cm. high. Find the volume of wood.

11. An open box is 16·2 cm. long, 15·4 cm. wide, and 14·5 cm. high, outside dimensions. If the metal is $\frac{1}{2}$ cm. thick, and each c.c. weighs 8·4 grams, find its weight.

MISCELLANEOUS EXAMPLES

M (IV)

1. A clock which goes steadily is noticed at 10 a.m. on Sunday to be 140 seconds slow. On the succeeding Saturday afternoon, at 24 minutes past 3, it is found to be 109 seconds fast. When was it exactly right?

2. One knot is a velocity of 51·43 cm. per second. Express a velocity of 10 knots in miles per hour to the 1st place of decimals. (1 m. = 39·37 in.)

3. A man has a square piece of cardboard whose edge is 12 in. He cuts a strip off all round of width 1 in., and then finds he has only reduced the area by half the amount he wished, so he cuts off another strip whose width is 1 in. By how much per cent. must the area still be reduced to get the card to the desired size?

4. Which is the greater, $\frac{1234\frac{1}{2}}{6789\frac{1}{2}}$ or $\frac{1234}{6789}$?

5. Simplify $\dfrac{\dfrac{9}{1\frac{1}{2}}+\dfrac{6}{1{\cdot}2}}{\dfrac{1\frac{1}{2}}{9}+\dfrac{1{\cdot}2}{6}}$.

6. A man walks at x miles per hour. If his stride is y inches, find the number of strides he makes in a minute.

7. Instead of dividing by 1·6, a correct result may be obtained by multiplying by a decimal fraction. What is the fraction?

8. A rectangular piece of cardboard 6 in. by 3 in. weighed 27·4 gr. What would be the area of a figure cut out from this cardboard which weighed 17·8 gr.?

9. A barometer reads 76·72 cm. Express this in inches, given 1 mm. = 0·03937 in.

10. A machine took 5 min. 20 sec. to bore through a plate $\frac{5}{8}$ in. thick. How long should it take to bore through $1\frac{3}{8}$ in.?

11. Wages when reduced by 5 per cent. amount to £3, 6s. 6d. a week. What were they before reduction, and by how much per cent. must they be increased to return to their original amount?

12. Coming down a flight of steps two at a time there is 1 over, three at a time there are 2 over, four at a time 3 over, five at a time 4 over. How many steps were there at least?

13. One-eighth of a plank is damaged, and a workman cuts off $\frac{5}{6}$ of the remaining part. The piece he cuts off is 4 ft. $4\frac{1}{2}$ in. long. What was the length of the original plank?

14. Two boys have the same amount in the bank. One draws out $\frac{1}{8}$ of his money and the other $\frac{2}{3}$. The difference between these amounts is £1, 6s. What had they each originally in the bank?

15. A room is 18 ft. long, 16 ft. broad, and $11\frac{1}{2}$ ft. high. If 30 sq. ft. be allowed for doors and windows, etc., find the cost of the paper required for papering the room, the paper being in pieces 12 yd. long and 21 in. broad, costing 2s. 4d. each.

16. Simplify $\dfrac{6{\cdot}07 - 2{\cdot}4 \times 1{\cdot}8}{1{\cdot}625 \div 0{\cdot}65 + 4}$.

17. What is the ratio of the diameter of the moon to the diameter of the earth in the form 1 : n (to 2 decimal places), given diameter of moon 2160 miles and of the earth 7920 miles.

18. If 1 ton cost £2·64, find the cost of 3 tons 15 cwt. 1 qr. 14 lb.

19. After buying a farm a man's estate is increased by 10 per cent. and then amounts to 495 acres. What was the area of the farm?

20. Divide £210, 4s. 2d. in the ratio of 2 : 3 : 5.

21. How many numbers are there between 500 and 800 which are multiples of 7?

22. A watch is wound up in the morning at 7 o'clock and again at night. In the morning 6 turns are required, and in the evening 9 turns. At what time in the evening is the watch wound up?

23. A and B run a race of 100 yd., and if A gives B 5 yd. start the race is a dead-heat. What would be the result if A had started 5 yd. behind the line and B on the line, assuming uniform speed for both?

24. Simplify $\dfrac{\frac{1}{2} + \frac{1}{3} \times \frac{2}{5}}{\frac{1}{2} \times \frac{1}{3} + \frac{2}{5}}$.

25. A debtor pays 11s. 8d. in the £. How much does he pay a creditor to whom he owed £246, 14s.?

26. If $4\frac{2}{3}$ yd. cost 17s. 8d., what is the cost of $5\frac{1}{4}$ yd.?

27. If I travel 32 miles an hour a journey takes 4 hr. 20 min. How long should it take if I travelled 39 miles an hour ?

28. Divide a line of length 26·4 cm. in the ratio of 1·2 : 2·8 : 2·6.

29. If £5 interest is paid on £100 every year, what would be the interest on £315, 10s. for $4\frac{1}{2}$ years ?

30. Two trains are travelling towards each other, one at 40 miles an hour and the other at 30 miles an hour. At what rate are they approaching one another ? If they start 224 miles apart, after what time will they meet ?

31. Find the next two numbers above 5955 and 12,307 respectively which have the same H.C.F. as these two.

32. The sides of a triangle are in the ratio of 2 : 2·5 : 3·5. If the perimeter is 12 in., find the sides.

33. A map is drawn to a scale of 1 in. to 2 miles. What is the area in acres of a wood which on the map covers an area of ·86 sq. in. ?

34. A man is asked to pay duty on 2 Kg. 420 grams at 7s. 4d. per lb. Find the amount payable, taking 1 Kg. = 2·2 lb.

35. A is greater than B by $\frac{1}{3}$ B. By how much of A is B less than A ?

36. What number divided by $\frac{3}{7}$ gives $5\frac{2}{9}$?

37. A tank is 24 in. × 17 in. × 9 in. How many gallons does it hold, taking 1 gallon = 277 cu. in., and what does it cost to fill at 1s. 10d. a gallon ?

38. If the price of an article were increased by $1\frac{1}{2}$d., the additional receipts on the same sales would amount to £25, 12s. 6d. How many articles were sold ?

39. If 2·5 cm. of rain falls on an area of 1500 sq. m., what would be its weight in Kg.?

40. The inhabitants of a town speak either Welsh or English or both: 72 per cent. speak Welsh and 84 per cent. speak English. What percentage speak both ?

41. Three travellers, A, B, C, set out each with provisions for 40 days. A goes to a point where he can fill up B and C and return to his base. B goes on with C to a point where he can fill up C and return to his base. How many days does (i) A, (ii) B travel from his base ?

42. After paying away 1s. 8d. in the £, a man has £246, 19s. left. What had he at first ?

43. Divide £57, 4s. among 18 men and 14 women, giving each man £1 more than a woman.

44. A is $\frac{2}{3}$ of B and $\frac{3}{5}$ of C. Find the ratio of A : B : C.

45. Given 1 verst = 0·6629 mile, and 1 Km. = 0·6214 mile, find the difference between 1520 versts and 1520 Km.

46. Write the decimal fraction 0·4343 in the form $\frac{1}{n}$.

47. A train travelling 45 miles an hour is $15\frac{1}{2}$ miles behind another which is travelling in the same direction at 30 miles an hour. What time will elapse before it passes the slower one, and how far apart will they be 20 minutes afterwards?

48. One man could dig a garden in 12 hours, another one could do it in 10 hours. If both are put to work on it, how much of the garden would be dug in 1 hour, and how long would they take together to finish the job?

49. At an election the number of electors was 11,250, but only 74 per cent. of them voted. The successful candidate got in with a majority of 2003. How many votes did each get?

50. In a billiard match of 12,000 up A gave B a start of 3000. When A had scored 8400 B's score was 9200. Which has the better of the game, and how many more points ought the other to have scored to be on level terms?

REVISION PAPERS

(R 21–30)

R 21

1. Simplify (i) $1\frac{1}{2} - \frac{3}{4}$ of $\frac{8}{9}$; (ii) $(1\frac{1}{2} - \frac{3}{4}) \times \frac{8}{9}$; (iii) $\frac{1}{\frac{1}{2} + \frac{1}{3}}$.

2. If $\frac{2}{3}$ of a stick is 4 ft. long, what is the length of three-quarters of the rest of it?

3. On a map whose scale is 2 miles to the inch two places are 4·38 in. apart. What is the true distance in miles and yards?

4. Given 1 lb. = 0·4536 Kg., express 8 lb. $14\frac{1}{2}$ oz. in Kg. (to nearest decagram).

5. Find the value of 5 per cent. of £124, 13s. 4d.

6. One side of a rectangle is 3 ft. 4 in. and the area is 14 sq. ft. 24 sq. in. Find the length of the other side.

R 22

1. How many copies of a book, each costing 4s. 6d., can be bought for £10, and how much money will be left over?

2. Simplify (i) $\dfrac{\frac{1}{2}+\frac{7}{15}}{\frac{5}{6}-\frac{3}{5}}\div\frac{30}{7}$; (ii) if $x\div7\frac{1}{5}=4\frac{1}{3}$, what is x ?

3. Show that $-3{\cdot}12$ equals $-4+0{\cdot}88$, and express the following in the form "minus an integer plus a decimal": (i) $-5{\cdot}691$; (ii) $(-3+{\cdot}71)+(-2+{\cdot}56)$; (iii) $-7{\cdot}216\div4$.

4. How many slips of paper 3·8 cm. long, 1·4 cm. wide can be cut from a sheet 19 cm. long, 21 cm. wide ?

5. A bar of iron is 24 ft. long, 3 in. wide, ½ in. thick. What does it weigh if the weight of the iron is 480 lb. per cubic foot ?

6. One car runs 24 miles to the gallon of petrol and a second car runs 30 miles to the gallon. How far can the first car run on the same amount of petrol that the second car uses for a journey of 75 miles ?

R 23

1. Write each of the following (1) correct to 2 places of decimals, (2) correct to 3 significant figures :

(i) ·06849; (ii) 1·0538; (iii) 10·019.

2. Find the H.C.F. and L.C.M. of 14, 35, 84 by expressing them in their prime factors.

3. Multiply £6, 1s. 4d. by $5\frac{5}{7}$.

4. A boy spent $\frac{2}{3}$ of his savings on a book and $\frac{1}{2}$ of what was left on a knife. He then had 2s. 6d. How much had he originally ?

5. If a man takes 2 hr. 42 min. over a journey when he travels at 10 miles an hour, how long should he take when he travels at 15 miles an hour ?

6. B's share of a prize is 20 per cent. greater than A's, and C's share is 20 per cent. greater than B's. If A's share is £25, what is C's share ?

R 24

1. A bus company carried in one week 3262 passengers at 9d. each, 4874 at 6d. each, and 5180 at 3d. each. What were the takings for the week ? The expenses (including depreciation) were £212 ; what profit was made ?

2. There are 15 lamps in a house, each of which burns on the average $\frac{1}{3}$ of a pint of oil per day. How long will a 30-gallon cask of oil last ?

3. One mile = 1·609 Km. Find in kilometres the equivalent of 3 miles 7 furlongs.

4. Simplify (i) $(1\frac{2}{3}+3\frac{2}{5})\div(1\frac{2}{3}+1\frac{1}{2})$; (ii) $\frac{2}{2\frac{1}{2}}+\frac{2}{1\frac{2}{3}}$.

5. A damask table-cloth 36 in. square is advertised at 3s. 6d. The same qualities in other sizes are: 45 in. square, 5s. 6d.; 60 in. square, 9s. 11d.; 66 in. by 84 in., 14s. 11d. What should these prices be if the charge per square inch of damask is uniform?

6. A square yard of iron sheeting weighs 720 lb. Find the thickness of the sheet if 1 cu. ft. of iron weighs 480 lb.

R 25

1. Find the cost of 4·50 Kg. at 25 fr. 15 c. per kilogram.

2. How many books, each worth 5 fr. 75 c., can you buy for 150 fr., and how much money will be left over?

3. Express £$3\frac{7}{32}$ as a compound quantity.

4. Simplify (i) $2\frac{1}{5}-\frac{2}{5}$ of $2\frac{1}{3}+1\frac{2}{3}$; (ii) $(2\frac{1}{5}-\frac{2}{5})$ of $2\frac{1}{3}+1\frac{2}{3}$; (iii) $2\frac{1}{5}-\frac{2}{5}$ of $(2\frac{1}{3}+1\frac{2}{3})$.

5. Three-quarters of an alloy is lead and $\frac{1}{6}$ is antimony. If it contains 16 lb. of lead, what is the weight of antimony in it?

6. A rectangular room is 13 yd. long and 8 yd. wide, excluding the platform. How many people will it seat, allowing 12 per cent. of the floor-space for gangways and each person 20 in. by 32 in.?

R 26

1. A hotelkeeper buys 7 doz. pillow-cases at 13s. 9d. per dozen; 4 doz. bedspreads at 7s. 6d. each; and 6 doz. pairs of sheets at 23s. 6d. per pair. What is the total cost?

2. If x ft. of cloth cost y shillings, how many pence would you pay for z yd.?

3. The price of a Jhanzie tea share is quoted as £$3\frac{11}{16}$. Express this in £, s. d.

How many shares could be bought for £100, and what money would be left over?

4. (i) Multiply 3·07 by 80·04; (ii) divide 11·1 by 0·075.

5. Casement cloth 50 in. wide is advertised at 2s. 9d. per yard. What is the cost of the material for 6 pairs of double-width (*i.e.* 100 in.) curtains, each 8 ft. long?

6. Water standing at a depth of 2 ft. 6 in. in a tank 12 ft. long, 8 ft. wide is run off into an empty tank 10 ft. long, 6 ft. wide. What will be the depth in this tank?

R 27

1. A difference of 15° in longitude makes a difference of 1 hour in time. What time is it at New York (74° W.) when it is noon at Greenwich ?

2. Find the cost of 26 cwt. 3 qr. 18 lb. at £3, 7s. 4d. per cwt.

3. If $5\frac{1}{2}$ yd. cost 2s. $6\frac{1}{4}$d., what is the cost of $6\frac{1}{2}$ yd.?

4. Show clearly, without quoting any rule, that $\frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$.

5. Find the smallest multiple of 48 which is also a multiple of 300, 525, and 588.

6. Find the external depth of a closed tank which is to hold 200 litres if its outside length is 85 cm., breadth 120 cm., and thickness of metal 2 cm.

R 28

1. What is the difference in price between two Yorkshire hams, one of 15 lb. at 1s. 10d. per lb., and the other of 16 lb. at 1s. 9d. per lb.?

2. Express as vulgar fractions in their lowest terms, (i) 0·75, (ii) 0·125, (iii) 0·05, (iv) 0·36, (v) 0·0025.

3. Taking a kilogram as equal to 2·2 lb., find the number of kilograms in 11 cwt.

4. Simplify (i) $3\frac{3}{4}-\frac{1}{4}$ of $5\frac{1}{3}-1\frac{1}{3}$; (ii) $1\div\left(\frac{6}{0\cdot5}-\frac{9}{1\cdot2}\right)$.

5. A railway track is 24 miles long and 22 ft. wide. What is its area in acres ?

6. The cross-section of a pipe is 5 sq. in. and the water is coming out at the rate of 90 gallons a minute. Find in feet per second the rate of flow of the water. [Take 1 gallon =275 cu. in.]

R 29

1. Find the value of $3516\times98\cdot8$ without multiplying by any of the figures in the question.

2. Correct any of the following statements which are wrong: (1) $(\cdot01)^2=\cdot001$, (2) $(2\cdot6\times1\cdot4)10=26\times14$, (3) $(2\cdot6+1\cdot4)10=26+14$, (4) $4(\cdot2)^2=(\cdot4)^2$, (5) $10(\cdot2)^2=\cdot4$, (6) $100(\cdot3)^2=(3)^2$.

3. The duty on an article came to 2s. 5d. per cwt. If the duty was $\frac{7}{20}$ of the value of the article, find its value per cwt.

4. Find the cost of 6413 tons of coal at £1, 18s. 4d. per ton.

5. Find the greatest number which will divide without remainder into 561, 1275, 1768.

6. If $\frac{x}{y^2}$ is always constant and if $x=9$ when y is 16, find x when y is 14.

R 30

1. A man draws from the bank £20 in silver in two bags, each containing £10. The first bag consists entirely of florins; the second bag contains shillings and half-crowns in equal numbers, and one odd sixpence. Which bag contains most coins, and by how many?

2. Arrange the following in ascending order of magnitude: $\frac{8}{23}$, $\frac{1}{3}$, $\frac{2}{7}$, $\frac{7}{19}$, 0·35.

3. (i) Divide $8\frac{3}{4}$ by $2\frac{1}{3}$; (ii) multiply 7·04 by $2\frac{3}{8}$; (iii) simplify $\frac{2}{4\frac{2}{3}}+\frac{3}{5\frac{1}{4}}-\frac{2}{2{\cdot}4}$.

4. How many strips of wood 4·7 cm. long can be cut from a strip of length 3 metres, and how much is left over?

5. A path 5 ft. wide surrounds a field 100 yd. long, 80 yd. wide. A fence is constructed each side of the path. What is the total length of fence required? Find also the area of the path.

6. How many cubical blocks of wood, edge 3 in., can be cut from a block 40 in. long, 35 in. wide, 25 in. deep. How much wood is wasted?

PROBLEM PAPERS

(P 21–30)

P 21

1. If $\frac{2}{5}$ of a sum of money is £43, 15s., find the value of $\frac{3}{7}$ of it.

2. A bar of metal when heated increased its length by ·0064 times its original length. If it was finally 14·62 ft., what was it originally?

3. What is the least number by which 17,325 must be multiplied to make it a perfect square?

4. One-fifth is added to a fraction and the sum is multiplied by 40. If the result is $34\frac{2}{3}$, find the fraction.

5. A cask of sugar worth £5, 6s. 2d. is made up into 2-lb. bags. How much sugar will be left over if sugar is worth $4\frac{1}{4}$d. a lb.?

P 22

1. The asterisks represent missing figures, (i) is an addition sum and (ii) is a subtraction sum. State what the missing figures are.

$$\text{(i)}\quad \begin{array}{r} 8\;3\;9\;8 \\ *\;*\;*\;6 \\ \hline *\;5\;1\;4\;* \\ \hline \end{array} \qquad\qquad \text{(ii)}\quad \begin{array}{r} 7\;2\;3\;5 \\ *\;*\;6\;* \\ \hline 3\;*\;8 \\ \hline \end{array}$$

2. What is the smallest power to which $\frac{1}{2}$ must be raised in order that the result may be less than 0·001 ?

3. A train travelling at 51 miles an hour takes $2\frac{1}{4}$ hours over a certain journey. Another train travels at $\frac{3}{5}$ of this speed and has to go $\frac{2}{3}$ of the distance. How long does it take ?

4. A cistern 6 ft. long, 4 ft. wide contains 300 gallons of water. What is the area of the wetted surface if 1 cu. ft. is taken as equivalent to $6\frac{1}{4}$ gallons ?

5. A man leaves in his will $\frac{1}{3}$ of his whole property to his wife and $\frac{1}{5}$ to each of two sons. He leaves £250, £100, £25 to three charities, and the residue to his only daughter. She receives £929. What was the value of the whole property ?

P 23

1. Find a man's income if he is no better off when his income is increased by £20 a year and at the same time his income tax is raised from 9d. to 1s. in the £.

2. The difference between $\frac{2}{3}$ and $\frac{3}{5}$ of a load of coal is 3 cwt. 2 qr. 4 lb. What was the weight of the load ?

3. Calculate correct to 4 decimal places the value of $1 + 1 + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} +$ etc.

4. What number, after being increased by ·034 and then multiplied by 2·3, gives 3·68 ?

5. A heap of earth weighing 249 tons is removed by 2 carts, one holding a load of 1 ton 10 cwt., the other 1 ton. The larger makes one more journey than the smaller. Find how often each cart is filled.

P 24

1. In a crest album there is room for 11 crests in each row and 14 rows on a page. A boy puts in to a new album 3200 crests. On what page will the last crest come, and what will be its position on that page ?

2. What is the least number which, when divided by 9 or 10 or 11 or 12, leaves a remainder of 5 ?

3. A hurdle is made of four horizontal bars and three upright bars, one at each end and one at the middle. The hurdle is 3 ft. high and 5 ft. wide. What length of wood is required to make sufficient hurdles to enclose a sheepfold 40 yd. long, 30 yd. wide ?

4. The inside measurements of a box with no lid made of wood 1 in. thick are 1 ft. 3 in. high, 2 ft. 6 in. broad, 4 ft. long. What is the area of the outside surface ?

5. It is stated that an inch of rain on an area of 1 acre weighs 100 tons. How many ounces does this statement assume for the weight of 1 cu. ft. of water ?

P 25

1. I divide $\frac{3}{5}$ of a sum of money into 7 equal parts, each amounting to £3, 12s. What was the original sum ?

2. Exeter is 35 miles from Barnstaple: on a map of Devon this is represented by 2·5 in. The area of the map is 13·2 sq. in. What is the area of Devon, to the nearest square mile ?

3. Evaluate $\frac{1}{2{\cdot}5}+\frac{1}{(2{\cdot}5)^2}+\frac{1}{(2{\cdot}5)^3}+\ldots$ etc. Correct to 4 decimal places.

4. If Venus and the earth revolve round the sun in 224 and 365 days respectively, at what intervals will they be in line ?

5. A number formed entirely of threes is exactly divisible by 19. Show clearly that the last 3 figures in the quotient must necessarily be 807.

P 26

1. A man when in work earns £3, 12s. 6d. a week and is able to save half a guinea a week. If his expenses are always the same, how many weeks can he be out of work in the year without getting into debt ?

2. How many parallel lines at intervals of ·005 of an inch can be engraved on a 6-inch rule ?

3. A piece of cardboard 24 cm. long, 15 cm. wide weighs 73·8 grams. What is the area of a piece of the same kind of cardboard that weighs 27·3 grams ? A box with a lid is made out of this cardboard. It is 30 cm. long, 24 cm. wide, 12 cm. high : find its weight.

4. A man buys a dog for £9, 12s. 6d. and sells it for £7, 15s. What would he have sold it for if he had gained twice as much as, in point of fact, he lost ?

5. The internal measurements of a tank are as follows : length 6 ft., breadth 4 ft., depth 3 ft., and it contains water to a depth of 1 ft. 10 in. How many bricks, measuring 8 in. by 4 in. by 3 in., must be put into the tank to make the water overflow ?

P 27

1. In finding the cost of a carpet sold in a roll 20 in. wide at 1s. 4d. per yard, a boy works as if the question read 1s. 4d. per foot. If his answer is £7, 10s. too large, what was the area of carpet?

2. Find the largest number which will give a remainder 1 when divided into 19 or 13.

3. A man goes from A to B at 4 miles an hour and returns at 5 miles an hour. What is his average rate for the whole journey?

4. A refiner has 10 lb. of gold 20 carats fine, and melts it with 16 lb. of gold 18 carats fine. How much pure gold must he put into it to make the whole 22 carats fine? [Twenty-four carats fine is pure gold, 20 carats is 20 parts gold and 4 parts alloy.]

5. One ruler is divided into 48 equal parts and another of the same length is divided into 36. If the end of the second is placed on the 14th division of the first, what division on the first is nearest the 14th division of the second?

P 28

1. What is the height of a pile of 250,000 sheets of paper if a packet of 600 sheets is $1\frac{1}{2}$ in. high?

2. When one-tenth part of a stick is cut off, the remainder is $1\frac{1}{2}$ ft. long. What would the length of the remainder have been if one-quarter of the stick had been cut off?

3. What is the difference between the areas of the inside and outside of a closed box, the outside measurements being 8 in. by 7 in. by 5 in. and the wood half an inch thick?

4. Beech-trees planted for timber are allowed 40 sq. yd. per tree. The average amount of timber per tree is 11 cu. ft., and the price of the timber is £1, 10s. per load, a load being 25 cu. ft. Find the value of the timber on an acre.

5. A tank has a rectangular base 9 ft. by 6 ft., and the sides are vertical. It is filled with water to a depth of 2 ft. How far will the water rise if a cube of lead of side 3 ft. is placed in the tank?

P 29

1. Find the greatest number that is contained an exact whole number of times in each of the numbers 10·5 and 5·95. How many times does it go into each?

2. Water is pumped at the rate of 1000 gallons per minute out of a reservoir with a surface of 5400 sq. ft. into one with a surface of 3600 sq. ft. How many minutes will it take to diminish the original difference between their levels by 1 in.? [$6\frac{1}{4}$ gallons = 1 cu. ft.]

3. The H.C.F. of two numbers each of 3 digits is 17, and their L.C.M. is 714. Find the numbers.

4. A man can make 40 articles of one kind or 60 of another kind in a week. How long will he take to make one of each? If he makes the same number of each kind in a week, how many will it be?

5. The weight in tons of rails for a mile of single line of railway equals the weight of a rail in pounds per yard multiplied by a constant. Find the constant.

P 30

1. Divide the sum of $\frac{2}{3}$ and $\frac{5}{6}$ by the sum of $\frac{1}{4}$ and $\frac{3}{8}$. From the result take $\frac{1}{5}$ of 7.

2. Two rods each 1 dm. long are divided, one into 10 equal parts and the other into 11 equal parts. They are placed side by side so that the sixth points of division in each coincide. What is the distance between their extremities?

3. Prove the following rule: multiply the price in farthings per lb. by $2\frac{1}{3}$, and you then get the price in £ per ton.

4. A canal lock is 100 ft. long and 17 ft. wide. To let a boat pass through up-stream, the water in the lock must rise 9 ft. The sluice to admit the water is such that the water rises in two minutes. How many gallons per second must the open sluice let through? [Take 1 cu. ft. $=6\frac{1}{4}$ gallons.]

5. A hole whose section is a square of side half an inch is bored straight through a cube of side $2\frac{1}{2}$ in., and a hole whose section is a square of side 2 in. is bored straight through a cube of side 3 in. Which of the two solids so formed is the bigger?

ARITHMETIC

PART III

CHAPTER XIII

GRAPHS AND DIRECT VARIATION

Graphs

Statistics.—Statistical information is much more clearly grasped when illustrated on a graph than it is when given in tabulated form.

The graph should be drawn on as large a scale as the paper will allow, subject to the condition that scales are selected which are easily read.

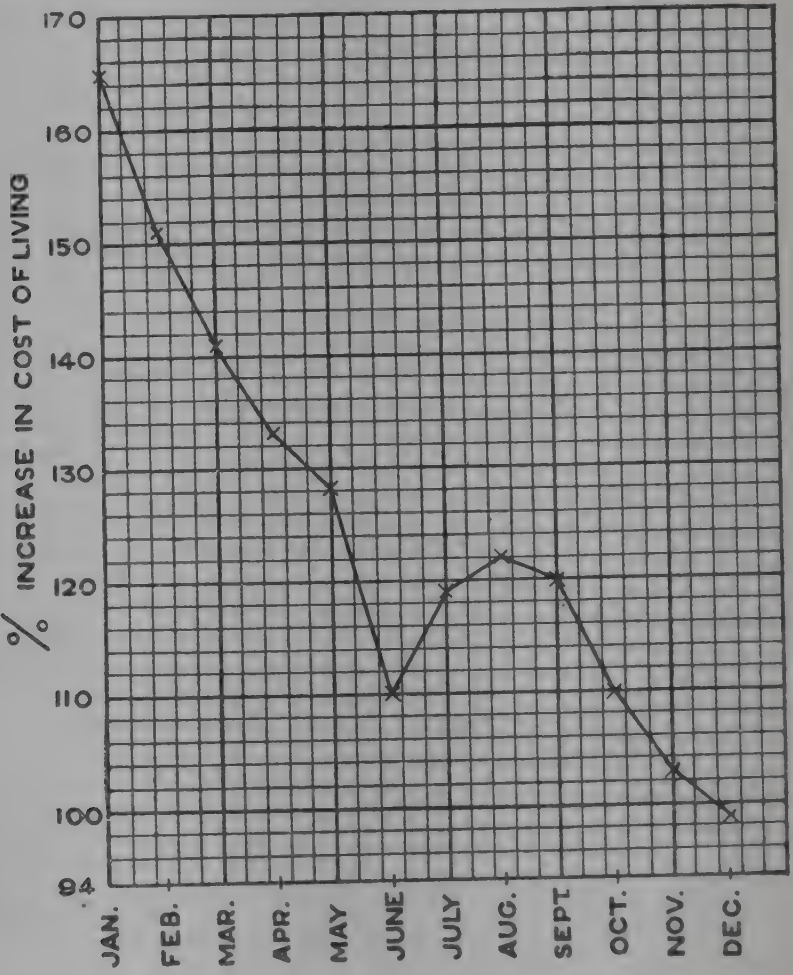
Large divisions on the paper should usually represent units or multiples of five.

The two illustrative examples which follow represent distinct types of graphs. The first corresponds to a set of isolated facts: the consecutive points are therefore joined by straight lines, *not* by a smooth curve, as there is no definite law giving the percentage at any given time. But the second graph is an example of continuous change; the volume and pressure are related by a physical law, ny slight increase in the pressure produces a corresponding slight ecrease in the volume. Hence in this case consecutive points are joined by a smooth curve. Familiar examples of graphs representing ata derived from observation or experiment are (i) the temperature hart of a feverish patient; (ii) a barograph, which records changes in he atmospheric pressure. We shall see later (Chapter XXIV) how raphs can sometimes be utilised for deducing an expression for the hysical law underlying data obtained by observation.

Example.—The average percentage increase in the cost of living during 1921, as compared with July 1914, is given by the following table :

Month 1921	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
Percentage increase	165	151	141	133	128	110	119	122	120	110	103	99

Represent the data by a graph.

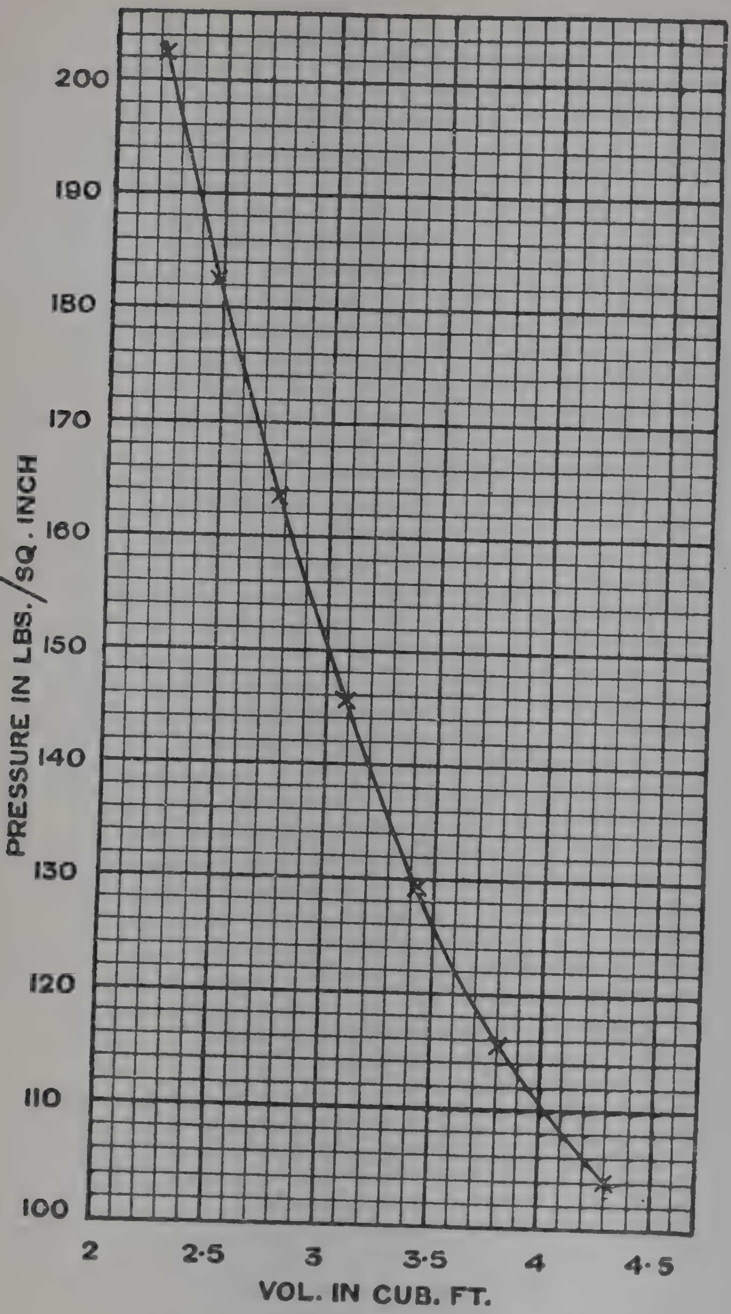


Note.—The scales employed are indicated on the figure.

Example.—The pressure and volume for 1 lb. of saturated steam are given by the following table :

Pressure in lb. per sq. in.	104·1	115·1	129·2	145·8	163·3	182·4	202·3
Vol. in cu. ft. .	4·29	3·82	3·42	3·07	2·76	2·48	2·24

Represent the data by a graph.



It is suggested that this graph should be used for some oral work, thus—

(i) What is the pressure if the volume is 2·4, 3·2, 3·5, 4·1 cu. ft. ?

(ii) What is the volume if the pressure is 110, 120, 140, 172, 193 lb. per sq. in. ?

EXERCISE XIII (a)

Show graphically the following statistics :

1. The mean temperature in degrees Fahrenheit for the various months :

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
38·6	39·4	41·8	47·2	53·3	59·4	62·7	61·6	57·2	50·0	43·5	39·9

At what time of year does the mean temperature increase and decrease most rapidly ?

2. Lighting-up time for bicycles :

	May 17.	May 31.	June 14.	June 28.	July 12.	July 26.
Time .	8.44	9.2	9.14	9.19	9.13	8.57

(i) Can a cyclist be summoned for not having lit his lamp by 9.10 p.m. on June 7 ? (ii) What was lighting-up time on July 9 ? (iii) On what dates is lighting-up time at 9 p.m. and 9.10 p.m. ?

3. Table showing the average price of wheat, shillings per quarter :

Year .	1914	1915	1916	1917	1918	1919	1920	1921	1922
Wheat .	34/11	52/10	58/5	75/9	72/10	72/11	80/10	71/6	47/10

For the same years the marriage rate per 1000 was

15·3	18·3	14·6	14·2	14·4	16·1	19·3	16·2	15·0

Plot these on the same graph : scale for price of wheat on the left, scale for marriage rate on the right.

Do the data justify the conclusion that as the price of wheat rises the marriage rate falls ?

4. Expectation of life for males of various ages :

Age . .	20	30	40	50	60	70	80	Yrs.
Expectation of life.	39·4	32·1	25·3	18·9	13·1	8·3	4·8	Yrs.

How long may a man aged 74 be expected to live ? At what age is the expectation of life 30 years ?

5. The population of Australia in millions :

Year .	1871	1881	1891	1901	1911	1921
Population in millions.	1·66	2·25	3·17	3·77	4·45	5·44

What was the probable population in 1896 ?

6. The number of crimes committed in London, and the population :

Year . . .	1919	1920	1921	1922	1923
Number of crimes in thousands.	13·4	17·0	16·9	15·4	14·8
Population in millions	6·73	7·25	7·46	7·47	7·55

Draw these on the same graph. Can you account for the increase in crime in 1920?

7. The records for running up to 1 mile :

Distance in yards	100	120	150	200	440	600	880	1 mile
Time in secs. .	$9\frac{3}{5}$	$11\frac{1}{4}$	$14\frac{1}{2}$	19	47	$70\frac{4}{5}$	$112\frac{1}{5}$	$250\frac{2}{5}$

What is the probable record for 130, 220, and 1000 yd. ?

8. The premiums paid for a Life Assurance of £100 and age of insurer :

Age .	21	25	30	35	40	45
Premium	£1 19s. 4d.	£2 3s. 1d.	£2 8s. 9d.	£2 15s. 9d.	£3 4s. 8d.	£3 16s. 0d.

What is the probable premium at an age of 23, 32, 50 ?

9. The number of persons receiving relief from the poor rate *in one day* in the winter of the following years :

Year .	1913	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923
No. .	785	754	756	681	634	585	554	574	660	1486	1530

What explanation can you give of the variation in these numbers ?

10. Value of coal exports of Great Britain :

Year . .	1916	1917	1918	1919	1920	1921	1922	1923
Millions of £'s	50·7	52·3	52·4	92·3	99·6	42·9	72·5	99·8

From this graph can you estimate the value of the exports in 1924 ?

11. A car P moves from A to B in a straight line for 30 min. with a uniform speed of 20 mls./hr. : it remains at B for 10 min., and then travels 20 miles further to C, at 30 mls./hr. After staying at C for 10 min., it returns to A at a uniform speed of 40 mls./hr.

Represent time along OX and distance from A along OY. Draw a graph showing the distance of the car from A at any time, and from it determine at what time a cyclist Q leaving A at the same time as P will meet P on its return journey 15 miles from A. At what uniform speed must Q move ?

12. Solve graphically : A train which travels at 50 mls./hr. leaves London at noon ; meets at 2.30 p.m. another train which has come from York, and arrives at York at 3.52 p.m. If the other train left York at 12.27 p.m., at what rate did it travel ? What is the distance of York from London ?

13. An express leaves London at 11 a.m., passes through Reading (36 miles) at 11.37 a.m., and arrives at Bristol at 1 p.m. A train running at an average rate of 30 mls./hr. between stations leaves Bristol at 11.10 a.m., stops at Bath (12 miles) for 5 min., at Swindon (40 miles from Bristol) for 5 min., and at Reading for 10 min. Find graphically the distance from Bristol to London, the average rate of the express, the time when the trains meet, and the time when the second train reaches London.

14. On a Centigrade thermometer the freezing point and boiling point of water are marked 0° and 100° respectively; on a Fahrenheit scale they are 32° and 212°. Draw a graph showing corresponding temperatures, taking Centigrade scales along OX. What temperature Fahrenheit corresponds to 76° C., and what is 100° F. on the Centigrade scale?

15. Two trains moving at 40 and 47 miles an hour pass simultaneously in opposite directions through a station at midday. Draw a graph to show their distance apart from 11 till 2. Find their distance apart at 12.50, and the time when they are 150 miles apart.

16. One man starts from A to walk to B, 30 miles away, at 4 mls./hr. Another starts at the same time from B to cycle to A, at 13 mls./hr. Draw a graph to show their positions at any time. If they start at midday, show from the graph (1) at what time they meet; (2) how far apart they are at 1.15 p.m.; (3) at what time they are 5 miles apart.

17. Calculate the squares of 1·2, 1·5, 1·8, 2·2, 2·5, 2·7, 3·5, 3·8, 4·4, 4·6. Draw a graph showing the squares of numbers from 0 to 5. From it write down the approximate values of $(2{\cdot}9)^2$, $(34)^2$, $\sqrt{17}$, $\sqrt{1{\cdot}62}$.

Direct Variation

If a body moves with a uniform speed of 3 ft. per second, it will travel 6 ft. in 2 sec., 9 ft. in 3 sec., and so on. The distance it travels is directly proportional to the time. If it travels x ft. in t sec., then $\frac{x}{t}=\frac{3}{1}=\frac{6}{2}=\frac{9}{3}$, etc. We say that in this case x *varies directly* as t, and write $x \propto t$. If the body travels x_1 ft. in t_1 sec., we know that $\frac{x}{t}=\frac{x_1}{t_1}$; this can also be written in the form $\frac{x_1}{x}=\frac{t_1}{t}$, because if $\frac{x}{t}=\frac{x_1}{t_1}$ then $xt_1=x_1t$, and therefore $\frac{x_1}{x}=\frac{t_1}{t}$.

Every example of direct proportion is an example of direct variation. The statement that x varies directly as t is equivalent to saying that if t alters in any ratio, then x alters in the same ratio; if t is doubled, x is also doubled, etc. If x varies directly as t, and if we plot x against t, we obtain a straight line graph through the origin. The example given above leads to the following table of values:

$t = 1$	2	3	4	5	6
$x = 3$	6	9	12	15	18

The reader should draw the graph corresponding to this relation between x and t.

Example.—A speed of x *miles an hour is the same as a speed of* y *ft. per sec. Express* y *in terms of* x. *Show that* y *varies directly as* x, *and draw a graph to convert speeds given in m.p.h. into speeds expressed in feet per second.*

x miles an hour is equivalent to $x \times 1760 \times 3$ ft. in 60×60 sec.

$$\text{or} \quad \frac{x \times 1760 \times 3}{60 \times 60} \text{ ft. in 1 sec.}$$

$$\text{or} \quad \frac{22x}{15} \text{ ft. per sec.}$$

$$\therefore y = \frac{22x}{15} \quad \text{or} \quad \frac{y}{x} = \frac{22}{15}.$$

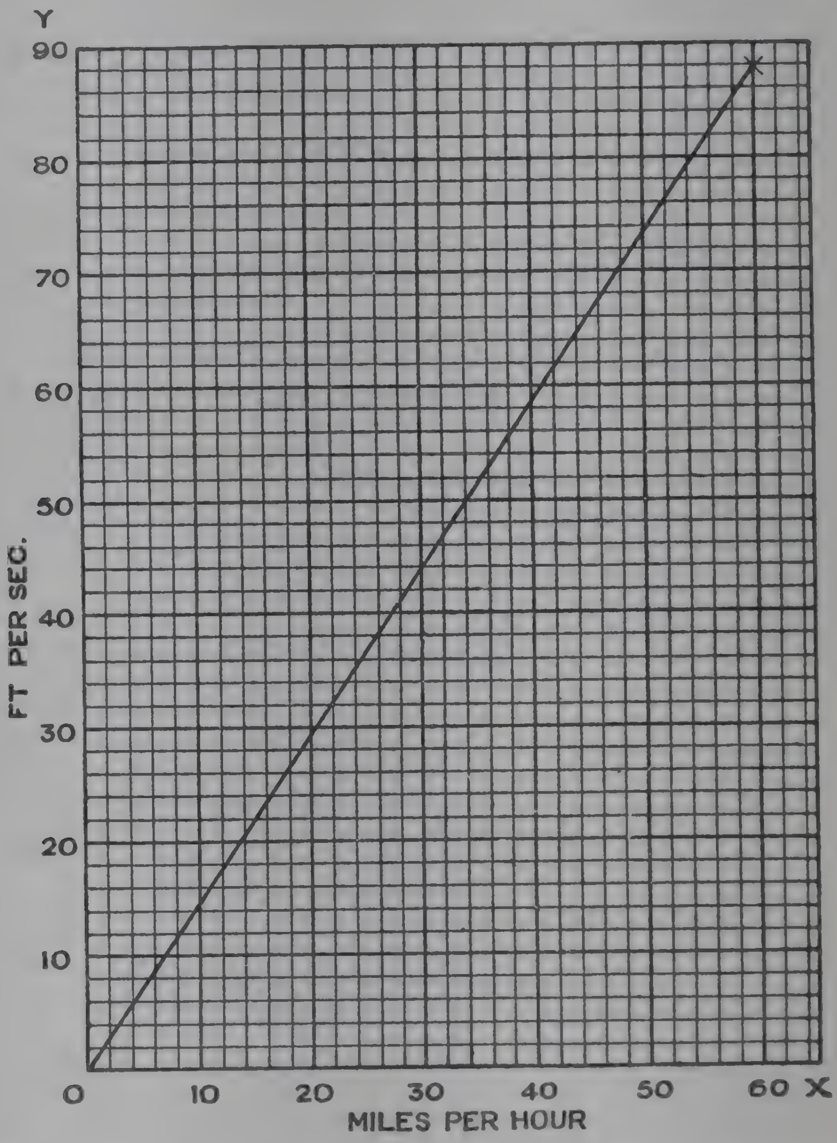
Therefore y varies directly as x.

If $x = 60$, $y = \frac{22 \times 60}{15} = 88$.

∴ A speed of 60 m.p.h. equals a speed of 88 ft. per sec.
For a table of values we have

$x = 0$	15	30	45	60
$y = 0$	22	44	66	88

But since y varies directly as x, it is unnecessary to plot all these values: we need merely join the origin ($x=0$, $y=0$) to the point $x=60$, $y=88$, which represents a speed of 60 m.p.h., or 88 ft. per second.



A few questions on this graph may be taken orally, such as—

(i) Express in ft. per sec., 12, 26, 48, 53 m.p.h.

(ii) Express in m.p.h., 20, 31, 50, 67 ft. per sec.

EXERCISE XIII (b)

1. Draw a graph for converting Kg into lb., given that 1 Kg. = 2·2 lb. Show values up to 10 Kg. Read off 4·5 Kg. in lb., and 7 lb in Kg.

2. Draw a graph for converting inches into cm., given that 1 in. $=2 \cdot 54$ cm. Show values up to 1 yd. Read off 27 in. in cm., and 40 cm. in inches.

3. Draw a graph which will give the result of dividing any number from 1 to 100 by 5·9. (Note $\frac{1}{5 \cdot 9}=0 \cdot 17$ approx.) Read off the value of $\frac{65}{5 \cdot 9}$.

4. Draw a straight line graph which will reduce marks running from 123 to 317 to a range of 0 to 100. Calculate the reduced mark corresponding to 240 and compare with the result given by the graph.

5. Draw a graph showing a duty of 22½ per cent. on articles up to a value of £100. Read off the duty on an article valued at £85. What is the value if the duty is £10 ?

6. Draw a graph showing the circumferences of circles whose diameters range from 0 up to 12 in. What is the circumference of a circle whose diameter is 3·2 in., and the diameter of a circle whose circumference is 17·3 in. ?

7. A man runs 100 yd. in 10 sec. Can you say how long he takes to run a mile ? Does the time vary directly as the distance ?

8. Does a person's height vary directly as his age ? Give numerical values to illustrate your answer.

9. Test whether x varies directly as y from the following data :

x	2·1	3·5	4·9	7·7
y	0·6	1·0	1·4	2·2

10. If y varies directly as x, what is the effect (i) on y of trebling x ; (ii) on x of halving y ; (iii) on y of increasing x by 40 per cent. ?

11. The area of a rectangle is 60 sq. in. ; its length is l in., and its breadth is b in. Make a table showing corresponding values of l and b, and another table for corresponding values of l and $\frac{1}{b}$. Does l vary directly as $\frac{1}{b}$?

12. Does the cost of an inland telegram vary directly as the number of words sent ? Is the cost doubled when the number of words is doubled ?

13. Circular pieces of cardboard of different sizes are cut from the same sheet. Do their weights vary directly as their diameters ?

14. A swimming-bath is filled by allowing water to run into it through a pipe of given circular section. Does the time required to fill the bath vary directly as the speed of the water in the pipe ?

15. Is it true to say (1) the circumferences of circles vary directly as their radii ; (2) the areas of circles vary directly as their radii ? Justify your answer by taking numerical examples.

CHAPTER XIV

HARDER FRACTIONS

When an expression consists of various parts, some of which are separated by + or − signs, and others by × and ÷ signs, the parts which are separated by + and − are called *terms* ; thus in the expression $2\frac{1}{2}\div\frac{1}{3}+\frac{3}{4}-\frac{1}{4}\times\frac{2}{7}$ there are 3 terms : (1) $2\frac{1}{2}\div\frac{1}{3}$; (2) $\frac{3}{4}$; (3) $\frac{1}{4}\times\frac{2}{7}$.

The terms must be simplified first :

(1) becomes $\frac{5}{2}\times 3=\frac{15}{2}=7\frac{1}{2}$; (3) becomes $\frac{1}{\underset{2}{\cancel{4}}}\times\frac{\overset{1}{\cancel{2}}}{7}=\frac{1}{14}$.

The expression is now

$$7\frac{1}{2}+\frac{3}{4}-\frac{1}{14}=7\frac{14}{28}+\frac{21}{28}-\frac{2}{28}=7\frac{14+21-2}{28}=7\frac{33}{28}=8\frac{5}{28}.$$

Expressions in brackets must be simplified first.

Example.—Simplify $(\frac{2}{3}+\frac{1}{5})\frac{1}{3}+\frac{2}{7}$ of $\frac{14}{15}$.

$$\frac{2}{3}+\frac{1}{5}=\frac{10}{15}+\frac{3}{15}=\frac{13}{15}$$
$$\frac{13}{15}\times\frac{1}{3}=\frac{13}{45}$$
$$\frac{2}{7}\text{ of }\frac{14}{15}=\frac{4}{15}.$$

The expression is therefore

$$\frac{13}{45}+\frac{4}{15}=\frac{13}{45}+\frac{12}{45}=\frac{25}{45}=\frac{5}{9}.$$

The solution should be written thus :

$$\left(\frac{2}{3}+\frac{1}{5}\right)\frac{1}{3}+\frac{2}{7}\text{ of }\frac{14}{15}=\left(\frac{10}{15}+\frac{3}{15}\right)\frac{1}{3}+\frac{2}{\cancel{7}}\text{ of }\frac{\overset{2}{\cancel{14}}}{15}=\frac{13}{15}\times\frac{1}{3}+\frac{4}{15}$$
$$=\frac{13}{45}+\frac{4}{15}=\frac{13}{45}+\frac{12}{45}=\frac{25}{45}=\frac{5}{9}.$$

Note that $\frac{1}{2}+\frac{1}{3}$ of $\frac{3}{4}$ means $\frac{1}{2}+(\frac{1}{3}$ of $\frac{3}{4})=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$,

whereas $(\frac{1}{2}+\frac{1}{3})$ of $\frac{3}{4}$ means $(\frac{3}{6}+\frac{2}{6})$ of $\frac{3}{4}=\frac{5}{6}\times\frac{3}{4}=\frac{5}{8}$.

Note also that $\frac{1}{2}+\frac{3}{4}=\frac{2}{4}+\frac{3}{4}=\frac{5}{4}$,

and does *not* equal $\frac{1+3}{2+4}=\frac{4}{6}=\frac{2}{3}$.

If two numbers are connected by the word "of" the product is regarded as a single expression, thus:

$4\div\frac{1}{4}$ of $\frac{2}{3}$ means $4\div(\frac{1}{4}\times\frac{2}{3})=4\div\frac{1}{6}=24$,

whereas $4\div\frac{1}{4}\times\frac{2}{3}$ means $4\times\frac{4}{1}\times\frac{2}{3}=\frac{32}{3}=10\frac{2}{3}$.

Note.—This example is inserted because unfortunately there is no general agreement that a bracket should be inserted to remove the ambiguity.

$4\div\frac{1}{4}$ of $\frac{2}{3}$ should be written $4\div(\frac{1}{4}$ of $\frac{2}{3})$,

and $4\div\frac{1}{4}\times\frac{2}{3}$ should be written $(4\div\frac{1}{4})\times\frac{2}{3}$.

But this procedure is often not adopted.

Example.—Simplify $\frac{2}{3}+\cfrac{4}{1+\cfrac{1}{3+\frac{1}{4}}}$.

$$3+\frac{1}{4}=\frac{13}{4};\quad \frac{1}{3+\frac{1}{4}}=\frac{4}{13};\quad 1+\frac{1}{3+\frac{1}{4}}=1+\frac{4}{13}=\frac{17}{13},$$

$\therefore$ the expression

$$=\frac{2}{3}+4\times\frac{13}{17}=\frac{2}{3}+\frac{52}{17}=\frac{34}{51}+\frac{156}{51}=\frac{190}{51}=3\frac{37}{51}.$$

The solution should be written thus:

$$\frac{2}{3}+\cfrac{4}{1+\cfrac{1}{3+\frac{1}{4}}}=\frac{2}{3}+\cfrac{4}{1+\cfrac{4}{13}}=\frac{2}{3}+\cfrac{4}{\cfrac{17}{13}}$$

$$=\frac{2}{3}+4\times\frac{13}{17}=\frac{34}{51}+\frac{156}{51}=\frac{190}{51}=3\frac{37}{51}.$$

Example.—Simplify $\dfrac{10\frac{2}{3}-7\frac{1}{4}+2\frac{1}{2}}{2\frac{1}{3}+\frac{1}{4}\text{ of }\frac{2}{9}}$.

$$\frac{10\frac{2}{3}-7\frac{1}{4}+2\frac{1}{2}}{2\frac{1}{3}+\frac{1}{4}\text{ of }\frac{2}{9}}=\frac{5\frac{8}{12}-\frac{3}{12}+\frac{6}{12}}{\frac{7}{3}+\frac{1}{18}}=\frac{5\frac{11}{12}}{\frac{42}{18}+\frac{1}{18}}$$

$$=\frac{71}{\underset{2}{\cancel{12}}}\times\frac{\overset{3}{\cancel{18}}}{43}=\frac{213}{86}=2\frac{41}{86}.$$

N.B.—Do *not* write : $\dfrac{5\frac{8}{12}-\frac{3}{12}+\frac{6}{12}(=5\frac{11}{12})}{\frac{7}{3}+\frac{1}{4}\text{ of }\frac{2}{9}(=\frac{1}{18})}$.

EXERCISE XIV (a)

Which of the following Ex. 1–16 are correct, and which incorrect ?

1. $\frac{2\times3}{10}=\frac{2}{10}\times\frac{3}{10}$. 2. $\frac{2+3}{10}=\frac{2}{10}+\frac{3}{10}$. 3. $(4\times3)5=20\times15$.

4. $(4+3)5=20+15$. 5. $(5\times6)\div10=\frac{5\times6}{10}$. 6. $5+6\div10=\frac{11}{10}$.

7. $(5\times6)\div10=\frac{5}{10}\times\frac{6}{10}$.
8. $(5+6)\div10=\frac{11}{10}$.
9. $\frac{1}{2}\times\frac{1}{3}\times5=\frac{5}{2}\times\frac{5}{3}$.
10. $\frac{1}{2}+\frac{1}{3}\times5=\frac{5}{2}+\frac{5}{3}$.
11. $(\frac{1}{3}\times\frac{1}{4})\frac{1}{2}=\frac{1}{6}\times\frac{1}{4}$.
12. $(\frac{1}{3}+\frac{1}{4})\frac{1}{2}=\frac{1}{6}+\frac{1}{8}$.
13. $(\frac{1}{2}\times\frac{1}{3})\div\frac{1}{4}=\frac{4}{2}\times\frac{4}{3}$.
14. $(\frac{1}{2}+\frac{1}{3})\div\frac{1}{4}=\frac{5}{6}\times4$.
15. $\frac{1}{2}+\frac{1}{3}\div\frac{1}{4}=\frac{5}{6}\times4$.
16. $(\frac{1}{2}\times\frac{1}{3})\frac{1}{4}=\frac{1}{8}\times\frac{1}{12}$.
17. Multiply 13×14 by 10 ; multiply $(13+14)$ by 10.
18. What multiple of 15×13 is 150×130 ?
19. What multiple of $15+13$ is $150+130$?
20. What multiple of $\frac{1}{2}+\frac{1}{3}$ is $\frac{5}{2}+\frac{5}{3}$?
21. What multiple of $\frac{1}{2}\times\frac{1}{3}$ is $\frac{5}{2}\times\frac{5}{3}$?

Write, by using brackets, expressions which represent the following, and then simplify them :

22. The fraction of a pole which is above water, if $\frac{1}{4}$ is in the water and $\frac{1}{8}$ in the mud.

23. The amount left when the sum of $\frac{2}{3}$ and $\frac{5}{8}$ is subtracted from 2.

24. The remainder when $\frac{1}{3}$ the difference between $\frac{2}{3}$ and $\frac{1}{4}$ is subtracted from $\frac{5}{8}$.

25. The remainder when $\frac{3}{4}$ of the sum of $\frac{5}{12}$ and $\frac{1}{8}$ is subtracted from $\frac{6}{7}$.

26. The result of dividing $\frac{1}{2}$ the sum of $\frac{6}{7}$ and $\frac{2}{3}$ by $\frac{3}{4}$ the difference between $1\frac{1}{2}$ and $\frac{4}{5}$.

27. The remainder when $\frac{2}{3}$ of $3\frac{1}{4}$ is subtracted from $\frac{3}{4}$ the difference between $2\frac{1}{4}$ and $1\frac{2}{3}$.

28. The fraction of a journey to be finished if $\frac{1}{3}$ has been done by train and $\frac{2}{3}$ of the rest by motor.

Simplify

29. $\frac{1}{2}-\frac{2}{3}$ of $\frac{1}{4}$.

30. $\frac{2}{3}+\frac{1}{4}\times\frac{8}{9}$.

31. $\frac{1}{2}$ of $\frac{3}{4}+\frac{1}{8}$.

32. $\frac{1}{2}\div 3+\frac{1}{4}$.

33. $\frac{3}{4}$ of $\frac{1}{8}-\frac{1}{16}$.

34. $\frac{3}{4}-\frac{1}{8}\div\frac{5}{16}$.

35. $\frac{2}{3}(\frac{1}{2}+\frac{1}{4})+\frac{4}{9}$.

36. $1-\dfrac{3-1}{4}$.

37. $1-\frac{1}{3}\times(\frac{1}{2}+\frac{2}{5})$.

38. $2\frac{2}{3}+\frac{1}{3}(\frac{1}{2}+\frac{1}{4})$.

39. $1\frac{1}{4}+\frac{3}{5}(\frac{2}{3}+\frac{1}{2})$.

40. $(\frac{7}{18}-\frac{5}{27})\div 6\frac{1}{9}$.

41. $(\frac{1}{3}+\frac{3}{11})\times(\frac{1}{5}+\frac{3}{7})$.

42. $\frac{1}{3}(4\frac{1}{2}-3\frac{3}{4})+1\frac{5}{6}$.

43. $\frac{3}{5}\div(\frac{2}{3}$ of $\frac{7}{10})$.

44. $\dfrac{\frac{1}{2}\text{ of }\frac{3}{4}}{\frac{9}{16}}$.

45. $\dfrac{\frac{3}{4}+\frac{1}{2}}{\frac{2}{3}}$.

46. $\dfrac{2\frac{1}{15}-1\frac{1}{3}}{2\frac{1}{4}\times 3\frac{1}{5}}$.

47. $\dfrac{\frac{2}{3}\text{ of }\frac{1}{2}-\frac{1}{4}}{\frac{2}{3}(\frac{1}{2}-\frac{1}{4})}$.

48. $\dfrac{\frac{2}{3}\times(\frac{1}{2}-\frac{1}{4})}{\frac{2}{3}\times\frac{1}{2}-\frac{1}{4}}$.

49. $\dfrac{\frac{1}{2}+\frac{1}{3}\text{ of }\frac{3}{5}}{(\frac{1}{2}+\frac{1}{3})\frac{3}{5}}$.

50. $\dfrac{1}{\frac{1}{2}+\frac{1}{3}}$.

51. $\dfrac{1}{\frac{1}{4}-\frac{1}{5}}$.

52. $\dfrac{\frac{1}{4}+\frac{1}{9}}{\frac{2}{3}-\frac{1}{8}}$.

53. $\dfrac{\frac{1}{2}+\frac{1}{3}\text{ of }\frac{1}{4}}{(\frac{1}{2}+\frac{1}{3})\times\frac{1}{4}}$.

54. $\dfrac{\frac{1}{2}\text{ of }\frac{3}{4}+\frac{1}{16}}{\frac{1}{2}\times(\frac{3}{4}+\frac{1}{16})}$.

55. $4+\dfrac{5}{3+\frac{2}{5}}$.

56. $\dfrac{1}{2}+\dfrac{\frac{1}{3}}{2+\frac{1}{2}}$.

57. $1\frac{1}{3}+\dfrac{1\frac{2}{3}}{1\frac{1}{3}+\frac{2}{9}}$.

58. $\dfrac{\frac{1}{4}-\frac{1}{9}}{\frac{1}{2}+\frac{1}{4}\div\frac{1}{3}}$.

59. $\dfrac{1\frac{1}{4}\div 1\frac{1}{2}}{\frac{1}{15}+(1-\frac{9}{10})}$.

60. $\dfrac{\frac{3}{4}+\frac{1}{2}+\frac{5}{6}}{\frac{2}{3}+1\frac{1}{6}+\frac{1}{4}}$.

61. $\dfrac{x}{2}+\dfrac{x}{3}$.

62. $\dfrac{x}{3}+\dfrac{x}{4}$.

63. $\dfrac{x}{2}-\dfrac{x}{3}$.

64. $\dfrac{1}{a}+\dfrac{1}{b}$.

65. $\dfrac{1}{a}-\dfrac{1}{b}$.

66. $a\left(\dfrac{x}{a}+\dfrac{x}{2a}\right)$.

67. $a\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$.

68. $\dfrac{a}{2}\left(\dfrac{2}{a}+\dfrac{3}{a}\right)$.

69. $\dfrac{1}{ab}+\dfrac{1}{b^2}$.

70. $1+\dfrac{a}{b}$.

71. $1+\dfrac{1}{b}$.

72. $2-\dfrac{a}{b}$.

73. $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}$.

74. $\dfrac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a}-\dfrac{1}{b}}$.

75. $\dfrac{\dfrac{x}{2}+\dfrac{x}{3}}{\dfrac{x}{2}-\dfrac{x}{3}}$.

76. $\dfrac{\frac{3}{4}+\frac{2}{5}}{\frac{3}{4}-\frac{2}{5}}\times\dfrac{2\frac{2}{3}}{\frac{2}{3}+\frac{3}{7}}$.

77. $\dfrac{1\frac{1}{2}-(\frac{1}{3}-\frac{1}{4})}{1\frac{1}{2}-\frac{1}{3}-\frac{1}{4}}\div\dfrac{1\frac{1}{2}\text{ of }(\frac{1}{3}-\frac{1}{4})}{1\frac{1}{2}\text{ of }\frac{1}{3}-\frac{1}{4}}$.

78. $1\frac{1}{3}+\dfrac{4\frac{2}{5}}{1\frac{3}{5}+\frac{2}{7}}$.

79. $\dfrac{\frac{1}{8}-\frac{1}{9}}{\frac{1}{9}-\frac{1}{10}}$ of $\dfrac{\frac{1}{6}-\frac{1}{7}}{\frac{1}{7}-\frac{1}{8}}$.

80. $\dfrac{(1-\frac{1}{3})(\frac{1}{5}-\frac{1}{7})}{1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}}$.

Problems involving Fractions

If $\frac{3}{4}$ of a stick measures 12 ft., find the length of $\frac{5}{6}$ of it.

The methods of ratio can often be usefully employed in questions such as this.

$\frac{3}{4}$ of a stick measures 12 ft.
∴ the whole stick ,, $12\times\frac{4}{3}$ ft.
∴ $\frac{5}{6}$ of the stick ,, $\frac{5}{\cancel{6}}\times\overset{2}{\cancel{12}}\times\frac{4}{3}$ ft.
$=13\frac{1}{3}$ ft.

EXERCISE XIV (b)

1. If $\frac{2}{3}$ of a line is 4 in., what is the length of the whole line?

2. If $\frac{3}{7}$ of a journey cost 15s., what should $\frac{2}{7}$ of it cost?

3. I pay 2s. 3d. for $\frac{3}{5}$ yd., what should I pay for $\frac{2}{3}$ yd.?

4. I saved an additional £40 by spending $\frac{3}{4}$ of my income instead of $\frac{7}{8}$. What was my income?

5. One boy takes $\frac{1}{3}$ of a cake, another takes $\frac{1}{4}$ of it. What fraction of the cake is left?

6. A man lost $\frac{1}{6}$ of his money and spent $\frac{4}{5}$ of what was left. He then had £30. What had he originally?

7. A carpenter cuts off $\frac{2}{5}$ of a plank and planes away $\frac{1}{10}$ from what is left. The planed piece weighs $13\frac{1}{2}$ lb. What was the weight of the original plank?

8. One-eighth of a regiment are killed, $\frac{3}{16}$ are wounded, and $\frac{1}{11}$ of the rest are sick. 240 men are left on parade. What was the original strength?

9. A owns $\frac{3}{5}$ of an estate, B owns $\frac{2}{7}$ of it, and C the rest. If C's share is worth £84, what are the other shares worth?

10. How many times must I use a rod $5\frac{1}{2}$ ft. long to measure a length of 264 ft.?

11. A bar of iron 5 ft. 4 in. long is cut into lengths of $2\frac{1}{4}$ in. How many pieces will there be, and how much will be left over?

12. How many times is $7\frac{1}{2}$d. contained in 5s. $10\frac{1}{2}$d., and how much remains?

13. If a ball bounces to a height $\frac{2}{7}$ of that from which it drops, from what height was it dropped, if, after the third bounce, it rose $1\frac{2}{7}$ ft.?

14. If a bicycle loses each year $\frac{1}{4}$ of its value at the beginning of the year, what was its original cost, if, after 4 years, it was worth £4, 1s.? Would it ever be worth nothing at this rate?

15. If a statute mile is $\frac{13}{15}$ of a nautical mile, how long is a nautical mile in yards?

16. A litre being $\frac{1}{30}$ of a cu. ft., find how many litres there are in $1\frac{3}{4}$ cu. ft.

17. If $\frac{9}{10}$ of gun metal is copper, find how much gun metal contains 2 cwt. of copper.

18. One man takes $\frac{1}{3}$ of a sum of money, another takes $\frac{4}{5}$ of what was left, the remainder amounted to £12. How much was there originally?

19. Two-sevenths of a card is coloured red, $\frac{1}{3}$ of the remainder is blue, and $\frac{1}{4}$ of the rest is green. If the part uncoloured amounts to $7\frac{1}{2}$ sq. in., what was the area of the whole card?

20. A man goes $\frac{1}{4}$ of his journey by train, half as far again by coach, motors $\frac{2}{3}$ of the rest of the way, and still has 6 miles to walk. How far was the journey?

21. By how much must the sum of $\frac{13}{84}$ and $\frac{3}{35}$ be increased so that when the result is subtracted from unity the remainder will be $\frac{1}{5}$?

22. By what fraction must $\frac{3}{7}$ be multiplied (1) to increase it by $\frac{1}{5}$; (2) to decrease it by $\frac{1}{5}$? By what fraction must it be divided to produce the same results?

23. Is the fraction $\frac{3}{7}$ increased or diminished by adding the same quantity to both numerator and denominator? Does a similar result hold good for the fraction $\frac{7}{3}$?

24. A boy and his bicycle together weigh 200 lb. The boy's weight is $\frac{7}{3}$ that of the bicycle. Find the weight of each.

25. Two-fifteenths of the water in a tank is drawn off; then $\frac{3}{7}$ of what is left, and then 7 gallons. If 6 gallons still remain, what did the tank hold to begin with?

26. A man increases his bank balance by $\frac{1}{3}$. What fraction of his final balance will represent this increase?

27. Concrete is made from gravel, lime, and sand, in the ratio $4 : 1\frac{1}{2} : 2\frac{1}{2}$. How much of each will be required to make 4 cwt. of concrete?

28. The volumes of two blocks of ice are in the ratio 6 : 5. When they melt, the volume increases by $\frac{1}{9}$. If the volume of water produced is $36\frac{2}{3}$ cu. ft., find the volumes of the two blocks of ice.

29. Find two fractions in the ratio of $3 : 2\frac{1}{2}$ whose sum is $\frac{11}{17}$.

30. A piece of lead whose volume is 87·5 cu. in. is melted and recast into two cubes whose sides are in the ratio 2 : 3. Find the volume of each cube.

The focal length (f) of a mirror is given by the formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, where u is the distance from the mirror of the object and v the distance of the image. Use this formula in questions 31–35.

31. Find f when $u=2\frac{1}{2}$, $v=3\frac{1}{3}$.
32. Find f when $v=12$, $u=4$.
33. Find v when $f=12\frac{1}{2}$, $u=20$.
34. Find f when $u=20\frac{1}{4}$, $v=9$.
35. Find v when $u=11$, $f=10\frac{1}{12}$

CHAPTER XV

SIGNIFICANT FIGURES AND APPROXIMATION

Some mention has already been made (Part II, pp. 67–69) of the meaning of significant figures, and it is suggested that the present chapter should be prefaced by a revision of Ex. VI (*b*).

No operations involving measurement can be performed exactly; the accuracy of the results will depend upon the delicacy of the instruments and the skill of the operator. A boy measures the length of the diagonal of a square whose side is 2 in. His first estimate is 2·8 in., but by using a diagonal scale he is able to measure the length more accurately as 2·83 in. Neither of these measures is exact; the first is correct to two significant figures, and the second is correct to three significant figures; if a greater degree of accuracy is required, calculation must be substituted for measurement: by Pythagoras, the length is $\sqrt{8}=2{\cdot}8284$ . . . in., and we say that the length is 2·828 in. correct to four significant figures. The number of significant figures in the result, therefore, indicates the degree of accuracy attained. *No result should ever be given to a greater degree of accuracy than is justified by the data and the method of working.*

If we are told that the length of a rod is 2·7 metres correct to two significant figures, we may write the length as 270 cm. or 2700 mm. This cannot make any difference in the degree of accuracy of the length as given; it still remains correct to *two*, and, as far as we know, *only two* significant figures: the zero digits at the end must not, therefore, be counted as significant figures unless this has been specially indicated: if it is stated that the length is 2·70 metres correct to three significant figures, then of course the final zero digit is counted, but not otherwise: *e.g.* $\sqrt{60}=7{\cdot}7460$ correct to *five* significant figures.

Similarly, we may write the length of the rod as 0·0027 km., but this makes no difference in the degree of accuracy: it still remains correct only to *two* significant figures; the zero digits at the beginning must, therefore, not be counted as significant figures. If the length of the rod is 2·70 m. correct to three significant figures, then we may say it is ·00270 km. correct to three significant figures, where the two initial zero digits are not counted, but the final zero digit is counted, owing to the data.

Any ambiguity as to the number of figures to be treated as significant may be removed by using the index notation: thus the length of a degree of latitude at the equator is 6000 ft. correct to two significant figures; in order to show that two of the digits are significant we express it in the form $6{\cdot}0\times10^3$ ft. The length of a degree of latitude at Greenwich is 6080 ft. correct to four significant figures: this could be indicated by stating it in the form $6{\cdot}080\times10^3$ ft.

Errors.—If a measurement is given as 2·7 m. correct to two significant figures, the true length cannot exceed 2·75 m., and cannot be less than 2·65 m.; in representing the length as 2·7 m., the error may be as great as 0·05 m., and we may say the true length is $(2{\cdot}7\pm0{\cdot}05)$ m. The difference between the true length and the approximate length is called the *absolute error.* The ratio of the maximum absolute error to the true length is called the *relative error.* If the exact true length is unknown, we must take the nearest ascertainable value. Here the relative error is $\frac{0{\cdot}05}{2{\cdot}7}$, which equals 0·02 correct to one significant figure. It is usually unnecessary to take more than one significant figure in evaluating the relative error. If the relative error is expressed as a percentage, the result is called the *percentage error* or the *error per cent.* For practical purposes, the absolute error is far less important than the relative error or the error per cent. Thus, if it is stated that the number of pupils in a school is 1000, when it is actually 950, the absolute error s 50, and the error per cent. is 5 per cent.; but if it is stated that the population of a town is 50,000 when it is really 49,950, the absolute error is still 50, while the error per cent. is now only 0·1 per cent. The former error is therefore much more serious than the latter.

Example.—The length of a rod is $12{\cdot}35 \pm 0{\cdot}02$ *cm. Between what limits must its length lie? What is the maximum error per cent. in taking the length as* 12·35 *cm.?*

The length does not exceed $12{\cdot}35 + 0{\cdot}02 = 12{\cdot}37$ cm.

The length is not less than $12{\cdot}35 - 0{\cdot}02 = 12{\cdot}33$ cm.

∴ the true length lies between 12·37 cm. and 12·33 cm.

The maximum *percentage error*, due to taking the length as 12·35 cm., is $\frac{0{\cdot}02}{12{\cdot}33} \times 100 \simeq \frac{2}{12} \simeq 0{\cdot}2$.

Example.—The length and breadth of a rectangular field are measured as 124 *yd. and* 87 *yd., correct to the nearest yard. Between what limits must the perimeter lie?*

The length cannot exceed 124·5 yd., and the breadth cannot exceed 87·5 yd. Therefore the perimeter cannot exceed

$$2(124{\cdot}5 + 87{\cdot}5) = 2 \times 212 = 424 \text{ yd.}$$

The length cannot be less than 123·5 yd. nor the breadth less than 86·5 yd. Therefore the perimeter cannot be less than

$$2(123{\cdot}5 + 86{\cdot}5) = 2 \times 210 = 420 \text{ yd.}$$

∴ the perimeter lies between 424 yd. and 420 yd.

Note.—(i) This argument may be expressed more shortly as follows: the length $= 124 \pm 0{\cdot}5$ yd. and the breadth $= 87 \pm 0{\cdot}5$ yd.

$$\begin{aligned} \therefore \text{ the perimeter} &= 2(124 \pm 0{\cdot}5) + 2(87 \pm 0{\cdot}5) \text{ yd.} \\ &= 2(124 + 87) \pm 4 \times 0{\cdot}5 \text{ yd.} = 422 \pm 2 \text{ yd.} \end{aligned}$$

(ii) The *maximum* difference between the true length and the true breadth is $(124 + 0{\cdot}5) - (87 - 0{\cdot}5)$ yd. $= (124 - 87) + 2 \times 0{\cdot}5$ yd.
$= 38$ yd.

Example.—A boy measures a line as 1·48 *in.; by calculation it can be shown that the line is* 1·4142 . . . *in. What is his error per cent.?*

Correct to $\frac{1}{100}$ in., the absolute error is 0·07 in., and the true length is approx. 1·41 in.

∴ the error per cent. is $\frac{0{\cdot}07}{1{\cdot}41} \times 100 = \frac{7}{1{\cdot}41} \simeq 5$.

∴ the error is 5 per cent. correct to one significant figure.

Note.—If the error is not large, the same result, correct to one significant figure, is obtained if the error is reckoned as a percentage of the *incorrect* result, for $\frac{0\cdot07}{1\cdot48}\times 100 \bumpeq 4\cdot7 \bumpeq 5$ per cent. correct to one significant figure.

EXERCISE XV (a)

1. The length of a bar is expressed in feet with increasing degrees of accuracy as follows:

(i) 10, (ii) 11, (iii) 11·1, (iv) 11·08, (v) 11·080, (vi) 11·0804.

State the number of significant figures in each case.

2. Repeat No. 1 for the set of successive measurements:

(i) 100, (ii) 110, (iii) 108, (iv) 108·0, (v) 108·04.

3. Repeat No. 1 for the set of successive measurements:

(i) 0·03, (ii) 0·032, (iii) 0·0320, (iv) 0·032046.

4. How many figures do you think should be counted as significant in the following statements?

(i) There are 20 shillings in a pound.
(ii) The cube root of 5 is 1·7100.
(iii) The mean distance of the Earth from the Sun is 93,000,000 miles.
(iv) The weight of a given block is 0·074000 Kg.
(v) A man buys 300 penny stamps.
(vi) A man's weight is 70·08 Kg.
(vii) A boy measures a line and says its length is 2·333333 . . . in.
(viii) As the result of an experiment which has involved weighing and measurement, a boy says 1 cu. cm. of lead weighs 11·327 gr.
(ix) The mean radius of the Earth is $4\cdot0\times10^3$ miles.
(x) 1 kilometre$=3\cdot9370\times10^4$ in.

5. Calculation shows that a certain number is 0·0230258. How nany significant figures does this appear to have? What would be he value of the number correct to (i) 3, (ii) 4, (iii) 5 significant figures?

6. The weight of a body correct to the nearest centigram is 96·73 rams. Write down its weight in grams correct to the nearest i) dekagram, (ii) gram, (iii) decigram. Between what limits must ts true weight lie?

7. The weight of a body is stated to be 3·074±0·002 grams. Vhat does this mean? State its true weight correct to as many gnificant figures as the data justify.

8. The volume of a body is 12·93 cu. cm. correct to four significant gures. Between what limits must its true volume lie? What is ne maximum error?

9. The mean distance of the Sun from the Earth is 93,000,000 miles, correct to two significant figures. Why is the distance not given with greater accuracy?

10. A boy's height is measured in inches correct to three significant figures. What is the maximum error?

11. The population of a town is stated correct to the nearest thousand. What is the maximum error?

12. A sum of money is expressed in pounds correct to three places of decimals. Is the maximum error more or less than a farthing?

13. In laying out a racing track 120 yd. long, there may be an error of as much as 6 in. Express in yards the length of the track to five significant figures (*cf.* No. 7).

14. The time of oscillation of a pendulum is stated to be 1·136 ± 0·002 seconds. What can you say about the time of 1000 oscillations?

Find (i) the maximum absolute error, (ii) the maximum relative error, (iii) the maximum percentage error, giving (ii) and (iii) correct to two significant figures in Nos. 15–20.

15. The length of a rod is 20 cm. correct to the nearest cm.

16. The weight of a body is 132 mgr. correct to the nearest mgr.

17. The distance between two houses is 3400 yd. correct to two significant figures.

18. The volume of a body is 154·23 ± 0·03 cu. cm.

19. A journey takes 1 hour 40 minutes correct to the nearest five minutes.

20. A man weighs 11 stone 8 lb. correct to the nearest lb.

Find correct to one significant figure the error per cent. in the results in Nos. 21–26.

21. One-third is taken as equivalent to 0·3.

22. A metre, which is 39·3701 in., is taken as $3\frac{1}{4}$ ft.

23. The value of $\pi = 3{\cdot}14159\ldots$ is taken as $3\frac{1}{7}$.

24. A boy measures an angle as $30\frac{1}{2}°$; calculation gives the value 30° 18′.

25. A boy, after constructing a triangle, measures its longest side as 12·3 cm.; calculation gives 12·65 cm.

26. A kilometre is taken as $\frac{5}{8}$ mile: actually 1 Km. = 0·62137 . . . mile.

27. Three boys find the value of π experimentally and give as their results 3·144, 3·139, 3·136. To how many significant figures do all three results agree with each other? What is the error per cent. if the average is taken?

28. The lengths of the sides of a triangle are found to be 4·7, 5·8, 6·3 cm. correct to the nearest mm. What is the maximum error if these are used to find the perimeter?

29. With the data of No. 28, find the difference between the longest and shortest side.

30. Rubber is quoted at 3s. $7\frac{1}{4}$d. per lb. correct to the nearest farthing. What error can this cause in the price of a ton?

31. Five bodies weighed separately are found to be 3·824, 2·179, 4·356, 1·088, 2·773 grams; each result is correct to the nearest ·001 gram. Between what limits must the combined weight of the five bodies lie? What is the maximum absolute error in taking the total weight as the sum of the five separate results?

32. A cyclometer records distances in miles and tenths of a mile. One journey is recorded as 21·6 miles and a second as 19·9 miles. What is the difference of length of the two journeys? What is the total length of the two journeys? Is it true to say that the reading is correct to the nearest tenth of a mile?

Multiplication

Suppose the dimensions of a rectangle are measured as 25·3 in. by 64·7 in. correct to $\frac{1}{10}$ in.; then the area lies between 25·25 × 64·65 sq. in. and 25·35 × 64·75 sq. in., *i.e.* between 1632·4125 sq. in. and 1641·4125 sq. in.

But 25·3 × 64·7 = 1636·91, so that the area obtained from the measurements is 1636·91 sq. in.; it would, however, be absurd to say that the area is 1636·91 sq. in., because in reality it may be less than 1633 sq. in. and more than 1641 sq. in. Correct to *three* significant figures, the limits for the area are 1630 sq. in. and 1640 sq. in., and the computed area is 1640 sq. in. It is not, therefore, even accurate to state the computed area correct to three significant figures.

Correct to *two* significant figures, the limits for the area are 1600 sq. in. and 1600 sq. in., and the computed area is also 1600 sq. in. Therefore the data only justify the statement that the area is 1600 sq. in., correct to two significant figures. A closer indication of the area is, however, given by the expression 1637 ± 5 sq. in.

This example illustrates the fact that if two numbers are each given correct to three significant figures, we cannot rely on the accuracy of more than two significant figures in the product: and there are exceptional border-line cases where a small error occurs even in the second figure.

But in all cases where the data are approximate, the number of significant figures in the answer must never exceed the number in the data. In the above example the answer for the area would often be given as 1640 sq. in. (*i.e.* 1636·91 expressed correct to three significant figures), although the digit in the third place is not really reliable. But it cannot be stated too emphatically that, if only three figures are supplied in the data, the presence of more than three figures in the answer is meaningless, and, in the case given above, the statement that the area is 1636·91 sq. in. should never be allowed. No answer should appear to suggest a higher degree of accuracy than the data allow.

Consider the following examples of multiplication:

$$\begin{array}{r} 7{\cdot}8552 \\ 6{\cdot}7524 \\ \hline 47\ 1312 \\ 5\ 4986\ 4 \\ 3927\ 60 \\ 157\ 104 \\ 31\ 4208 \\ \hline 53{\cdot}0414\ 5248 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 7{\cdot}86 \\ 6{\cdot}75 \\ \hline 47\ 16 \\ 5\ 50\ 2 \\ 39\ 30 \\ \hline 53{\cdot}05\ 50 \\ \hline \end{array}$$

The two products correct to three significant figures are respectively 53·0 and 53·1, so that if we work with only *three* significant figures, as on the right, we can obtain a result correct to *two* significant figures. And, similarly, if we work correct to five significant figures in a simple product, our result will usually be correct to four significant figures.

Division

Similar results are true of division.

Consider the following examples of division:

$$\begin{array}{r|r} & 0{\cdot}7676 \\ \cline{2-2} 6{\cdot}4283 & 4{\cdot}93448 \\ & 4{\cdot}49981 \\ \cline{2-2} & 43467 \\ & 38570 \\ \cline{2-2} & 4897\times \\ & 4500\times \\ \cline{2-2} & 397\times\times \\ \cline{2-2} \end{array} \qquad\qquad \begin{array}{r|r} & 0{\cdot}7673 \\ \cline{2-2} 6{\cdot}43 & 4{\cdot}934 \\ & 4{\cdot}501 \\ \cline{2-2} & 433 \\ & 386\times \\ \cline{2-2} & 47\times \\ & 45\times\times \\ \cline{2-2} & 2\times\times \\ \cline{2-2} \end{array}$$

The two quotients correct to three significant figures are respectively 0·768 and 0·767, so that if we work with only *three* significant figures, as on the right, we can obtain a result correct to *two* significant figures. And similarly, if we work correct to four significant figures in a simple division, the quotient will usually be correct to three significant figures.

Before making an accurate computation, it is necessary to obtain a rough approximation to the answer. This may be done by taking each term in the data correct to one significant figure only.

Example.—Find the approximate value of

$$\frac{62{\cdot}4\times{\cdot}0873}{\sqrt{15{\cdot}27}}.$$

The expression is roughly $\dfrac{60\times{\cdot}09}{4}\simeq 1{\cdot}0$.

EXERCISE XV (b)

Rough Approximations

Find, to **one** significant figure only, approximate values of the following:

1. $26{\cdot}3\times{\cdot}0048$.
2. $({\cdot}024)^2$.
3. ${\cdot}063\times{\cdot}0084$.
4. $1{\cdot}06\div{\cdot}038$.
5. ${\cdot}0067\div21{\cdot}4$.
6. ${\cdot}063\div{\cdot}0084$.
7. $\sqrt{{\cdot}936}$.
8. $\sqrt{{\cdot}0124}$.
9. $624\div83{\cdot}2$.
10. ${\cdot}61\times{\cdot}08\times{\cdot}12$.
11. ${\cdot}42\times{\cdot}86\div{\cdot}003$.
12. $\sqrt[3]{862{\cdot}4}$.
13. $\sqrt{2163\times1948}$.
14. $({\cdot}163)^2$.
15. $\dfrac{13{\cdot}4\times16{\cdot}2}{{\cdot}083}$.
16. $62{\cdot}4+17{\cdot}2({\cdot}08+1{\cdot}6)$.
17. $\dfrac{1}{6{\cdot}2}+\dfrac{1}{4{\cdot}8}$.
18. $\dfrac{1}{613{,}218}$.
19. $1{\cdot}62+2{\cdot}8\times{\cdot}07$.
20. 19s. 8d. × 206.
21. £1, 19s. 3d. ÷ 479.
22. £4, 18s. 7d. ÷ 198.
23. Number of seconds in 24 hours.
24. Number of inches in 1 mile.
25. Number of £'s in 10^6 pence.
26. Number of ounces in 1 ton.
27. Number of square yards in 1 square mile.
28. $574{,}000\times{\cdot}000046\times0{\cdot}17$.
29. $\dfrac{348{\cdot}6}{24\times20\times12}\times0{\cdot}058$.

30. £47, 16s. 2d. $\times \frac{43}{365} \times \frac{7}{100}$.

31. $\frac{3\cdot1416 \times 426 \times (0\cdot24)^2}{8\cdot43}$.

32. $\sqrt{\cdot064} \times \frac{1}{26\cdot2}$.

33. $\frac{2\cdot615}{\frac{1}{5}+\frac{2}{7}} \times 0\cdot06$.

34. $\frac{6\cdot2 \times (21\cdot2)^2}{56,382}$.

35. $\frac{\text{£}51, 13\text{s. } 4\text{d.} \times \cdot062}{15\cdot7}$.

36. $6\cdot23 \times 14\cdot8 + 120\cdot8$.

37. $51\cdot2 + 2\cdot06 \div \cdot0043$.

38. $\frac{14}{62\cdot3} + \frac{\cdot23}{2\cdot082}$.

39. How many miles per hour is the equivalent of 100 yd. in $9\frac{4}{5}$ seconds ?

40. How many tons in 12,263 Kg. ? 1 Kg. = 2·2 lb.

41. How many Kg. is 542 tons ? (See Question 40.)

42. How many cu. cm. in ·362 cu. yd. ? [1 c.c. = ·0610 cu. in.]

43. How many feet per second is a speed of 64·2 Km. per hour ? [1 metre = 1·094 yd.]

44. How many Kg. of rain fall on an acre during a fall of 1·24 in. ? 1 cu. ft. of water weighs 62·3 lb., 1 Kg. = 2·2 lb.

45. Blackwood's magazine published its 1313th number in March 1925. In what year was its first number published ?

46. Evaluate $\frac{\frac{1}{x}}{\frac{1}{x} - \frac{1}{y}}$ when $x = 26\cdot32$, $y = 45\cdot72$.

47. If $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$, find p when $a = 23\cdot2$, $b = 36\cdot8$.

48. Evaluate H, given $v = 50\sqrt{\frac{dH}{L + 54d}}$, when $v = 4\cdot83$, $d = \cdot74$, $L = 1043$.

49. Evaluate d, given $\frac{3\cdot14}{144}wt(d+t) = W$, when $w = 465$, $t = 0\cdot82$, $W = 7\cdot6$.

50. Evaluate H, given $G = \sqrt{\frac{(3d)^5 \times H}{L}}$, when $d = 4\cdot36$, $L = 1760$, $G = 83\cdot68$.

Example.—Measurement gives the circumference of a cylinder as 20·2 *cm. and its diameter as* 6·42 *cm.; find the ratio* $\frac{\textit{circumference}}{\textit{diameter}}$.

$$\text{The ratio} = \frac{20{\cdot}2}{6{\cdot}42} \simeq \frac{20}{6} \simeq 3$$

$$\begin{array}{r|l} & 31 \\ \hline 642 & 202 \\ & \underline{193\times} \\ & 9\times \\ & \underline{6\times\times} \end{array}$$

Since only three figures are used in the divisor, the quotient can only be found to two figures; therefore the result cannot be given more accurately than 3·1.

Note.—Since $\frac{22}{7}=3{\cdot}1428\ldots$ and $\pi=3{\cdot}14159\ldots$, it is clear that products or quotients involving π cannot be given to more than three significant figures if π is taken as $\frac{22}{7}$ or 3·142.

Example.—The value of 6,218,427 *tons of imported goods being* £5,938,264, 17*s.* 9*d., find the average value per ton, to the nearest penny.*

Rough check $£\frac{6}{6} \simeq £1$.

Since the third decimal place will give the pence correctly ($\frac{1}{4}$d. $= £{\cdot}001\frac{1}{24}$), we need only four figures in the divisor.

$$\begin{array}{r|l} & 955 \\ \hline 6218 & 59383 \\ & \underline{55962} \\ & 3421 \\ & \underline{3109\times} \\ & 312\times\times \\ & \underline{311\times\times} \end{array}$$

$$\begin{aligned} \text{Value} &= £0{\cdot}955 \\ &= \underline{19\text{s. } 1\text{d.}} \end{aligned}$$

Example.—In 10 *years the turnover of a company increases from* £1,263,478, 14*s. od.* to £2,718,419, 18*s. od. Find the increase per cent.*

Rough Check.—The increase is £1·5 millions on £1·3 millions, rather more than 100 per cent., so that three figures are required in the answer, and we work with four.

$$\begin{array}{rl} & 2718 \\ & \underline{1263} \\ \text{Increase} & \underline{1455} \text{ thousands on } 1263 \text{ thousands.} \end{array}$$

$$\therefore \text{increase per cent.} = \frac{1455}{1263} \times 100 = 115 \text{ per cent.}$$

$$\begin{array}{r|l} & 115 \\ \hline 1263 & 1455 \\ & \underline{1263} \\ & 192\times \\ & \underline{1263} \\ & 66\times\times \end{array}$$

or ratio of $\frac{\text{new turnover}}{\text{old turnover}}=\frac{2718}{1263}$

$$=\frac{2{\cdot}15}{1}$$

$=215$ per cent.

$\therefore$ increase $=115$ per cent.

$$\begin{array}{r|l} & 215 \\ \hline 1263 & 2718 \\ & \underline{2526} \\ & 192\times \\ & \underline{1263} \\ & 66\times\times \end{array}$$

If the evaluation of an expression involves two operations (*e.g.* both a multiplication and a division), the first operation must be carried out with *two* more figures than are required in the final result.

Example.—Evaluate $\frac{62{\cdot}343\times{\cdot}078624}{124{\cdot}83}$ *to two significant figures.*

Rough check $\frac{60\times{\cdot}08}{120}={\cdot}04.$

Since two significant figures are needed in the answer, the second operation requires three significant figures, and therefore the first operation requires four significant figures.

$$\begin{array}{r} 6234 \\ \underline{7862\cdot} \\ 43638 \\ 49872 \\ 37404 \\ 12468 \\ \hline \underline{49011708} \end{array}$$

$$\begin{array}{r|l} & 39 \\ \hline 125 & 490 \\ & \underline{375} \\ & 115 \\ & \underline{112\times} \\ & \underline{3\times} \end{array}$$

Answer $\underline{0{\cdot}039.}$

Example.— A debtor pays 14s. 10d. *in the* £ *: how much is received by a creditor to whom* £4526 *is due ?*

This should be worked by Practice. If 14s. 10d. is written as a decimal of £1, any error will be multiplied by 4526. The answer in £'s to the nearest penny will contain seven figures, and it would be necessary to work with eight.

EXERCISE XV (c)

(The contracted methods explained in the next section can be used for these examples if desired.)

1. Find the area of a rectangle whose sides are given as 2·42 in. and 3·67 in.

2. Find the circumference of a circle whose radius is 4·36 cm. [Circumference $= 2\pi r$.]

3. Find the area of a circle whose radius is 6·21 in. [Area $= \pi r^2$.]

4. The velocity of light being 186,330 ml./sec., find the time to the nearest second light takes to travel from the sun, 92,860,000 miles away.

5. In 1912 the coal output was:

	Tons.	Number of Persons engaged.
British Isles	264,595,395	1,072,393
Germany	255,810,100	718,673
U.S.A.	484,864,901	722,622

Find to the nearest ton the average output per person in each country.

6. Find the average number of people per square mile in India and England, given:

	Area in Square Miles.	Population.
India	1,802,577	315,086,372
England	50,866	34,038,537

7. Find the number of births and deaths in the following towns, given:

	Population.	Rate per 1000.	
		Births.	Deaths.
Birmingham	919,438	27·6	12·6
Exeter	59,608	18·89	11·66
London	7,476,168	26·4	12·6
Winchester	23,791	17·3	13·4

8. The following table gives the receipts from income tax:

1920.	1921.	1922.
317,029,000	340,665,000	337,027,000

Calculate the percentage change between consecutive years and also between 1920–1922. (Correct to two significant figures.)

9. The National Debt in 1921 was £7,831,744,300, which was an increase of 5·34 per cent. on 1920. Find the 1920 debt.

10. The area of a rectangle is 4840 sq. yd., and one side is 92·78 yd. Find the other side.

11. After deducting 34·6 per cent. for expenses, the net receipts of an exhibition were £2,634,817, 15s. What were the gross receipts?

12. 4,362,817 inhabitants pay rates amounting to £12,416,837, 8s. 10d. What is the average payment by each, to the nearest shilling?

13. 6,413,825 tons of goods are taxed at £9,863,714, 18s. 2d. What is the tax per ton, to the nearest penny?

14. 264,817,513 lb. are worth £97,863,418, 7s. 4d. Find the value per lb. (to a shilling).

15. 1,370,116 cwt. of sugar imported was valued at £3,562,718, 15s. Find the average value per cwt. (to a penny).

16. The takings of a railway company drop from £362,817, 12s. 9d. to £319,463, 18s. 7d. Find the percentage drop to one decimal place.

17. What integer is most nearly equal to $22·76 \times 37·48$?

18. The value of 13,426,004 tons of iron ore mined in 1 year is given as £6,288,100. From it 4,399,814 tons of metal were obtained worth £14,244,937. Find (1) the percentage of metal in the ore; (2) the average value per ton of ore and metal correct to a penny.

19. Find by Practice the total value of 64,328 tons of goods for which it is stated that the average value per ton is £1, 13s. 8d. Why is it undesirable to calculate this by decimalising £1, 13s. 8d. and multiplying by 64,328?

20. The population per square mile of China, whose area is 1,298,000 square miles, is given as 246. What is the total population? By how much may your answer be incorrect?

21. A bar, after expanding ·00427 of its original length, is then 273·84 ft. What was its original length?

22. A country exported 111,646,640 lb. of tea to Britain, the value of which was £3,411,921, and 21,015,220 lb. to Australia, where it fetched £642,225. Find the difference in the receipts per lb. from the two markets to the nearest $\frac{1}{10}$th of a penny.

23. A factory employs 2648 men, who receive in wages per week £9142, 17s. 8d. What is the average wage to nearest penny?

24. It is calculated that the average deposit in a savings bank was £62, 12s. 7d. If there were 163,287 depositors, what was the total amount they had saved? If the average is correct to the nearest penny, to what degree of accuracy can the answer be obtained?

25. The population of England and Wales was 29,002,527 in 1891 and 32,577,843 in 1901: that of Scotland was 4,025,647 in 1891 and 4,472,103 in 1901. Find which population was increasing at the greater proportionate rate during 1891–1901, and by how much per cent.

26. In 1907–8 the expenditure on the Navy was £31,141,000. In 1911–12 it was £44,392,500. The value of the trade in these years was

1907–8,	imports	£645,807,940;	exports	£517,977,167;
1911–12,	,,	£680,157,527;	,,	£556,878,432.

Find the increase per cent. in the cost of the Navy; the increase per cent. in the total trade; and the ratio between these two percentages.

27. The populations of two towns are 572,614 and 783,527. Their death-rates are given as 16·8 and 15·4 per thousand respectively. Find the death-rate for the two towns taken together.

28. If the average price per cwt. for butter was £5·36 when the total of butter imported was £23,614,000, what would have been the value of this butter if the average price had been £5·21 per cwt.?

29. A bar of metal when heated expanded by ·00368 of its original length and then measured 127·36 ft. What was its original length?

30. The amount cleared by the Birmingham banks in the thirty-sixth week of 1925 was £2,063,992, compared with £1,867,632 for the corresponding week of the previous year. The total to date for the year was £92,656,759, compared with £85,742,142 in the previous year. By how much did the percentage increase in the week differ from the percentage increase in the total to date for the year? By how much per cent. did the amount in the thirty-sixth week of each year differ from the average weekly amount cleared in 1925 and in 1924 respectively?

31. The average price of wheat per quarter increased from 26s. 5¼d. to 27s. 2¼d. How much was the difference in value of 2,386,020 quarters?

32. If a metal ball weighs x gram in air and y gram when weighed in water, its specific gravity is $\frac{x}{x-y}$. If $x=12{\cdot}483$ and $y=11{\cdot}217$, find the specific gravity of the metal as accurately as possible.

33. The value of g is found from the formula $g=\frac{4\pi^2 l}{t^2}$. Find its value when $l=8{\cdot}24$ and $t=3{\cdot}18$.

34. If $\frac{355}{113}$ is taken for π (3·14159265, . . .), what is the percentage error? What percentage error will be made by using this value in finding the circumference of a circle whose diameter is 23·64 inches?

35. Find to the nearest $\frac{1}{10}$ the number of avoirdupois pounds in a Kg., given that 36 cu. ft. of water weighs 2250 lb., 1 cu. metre of water weighs 1000 Kg., and 1 metre$=3{\cdot}28$ ft.

36. The Earth's circumference measured along a meridian is 40 million metres or 21,600 nautical miles. Assuming 1 metre $=39{\cdot}370$ in., find the number of feet in a nautical mile.

37. The tonnage of freight conveyed by English railways in June 1925 was 24,049,335, a decrease of 4·8 per cent. on that of the previous June. Find the tonnage carried in June 1924.

38. The gross receipts of a business fell 13·26 per cent. in 1925. The difference in the receipts for 1925 and 1924 was £62,147, 14s. Find the receipts in 1925.

Formal Contracted Methods. Multiplication and Division

Expressions involving much numerical work are usually evaluated by logarithms, and it is doubtful whether time spent on learning formal contracted methods is always profitably employed. It is, however, desirable that calculations should not be commenced with an unnecessary number of figures, whether the subsequent work is contracted or not.

Example.—Evaluate 623·423×0·0284732 *correct to one place of decimals.*

Rough check $600\times0{\cdot}03=18$.

Therefore the answer must contain 3 significant figures. If we retain 4 figures, the calculation is as follows:

$$\begin{array}{r} 6234\hphantom{0000} \\ \underline{2847} \\ 12468\hphantom{000} \\ 49872\hphantom{00} \\ 24936\hphantom{0} \\ \underline{43638} \\ \underline{\underline{17748198}} \end{array}$$

This suggests that the answer is 17·7.

If, however, we retain 5 figures, we obtain 1775063766, which shows that the answer is nearer 17·8 than 17·7.

When using contracted methods, work with two *more figures than are required in the answer.*

Since 3 figures are required in the answer, draw a line down the page after the 5th (=3+2) figure.

a. Before multiplying by 8, cross off the 2 in the top line; also carry 2, since 8×2=16, and 16 is nearer 20 than 10.

b. Before multiplying by 4, cross off the 4 in the top line and carry 2.

$$\begin{array}{r|l}
6\cancel{2}\cancel{3}\cancel{4}\cancel{2} & \\
2 & \underline{8473} \\ \hline
124684 & \\
49874 & a \\
2494 & b \\
436 & \\
19 & \\ \hline
\underline{\underline{177507}} &
\end{array}$$

Answer $\underline{17 \cdot 8}$.

Example.—Evaluate 0·062943 ÷ 17·642 *correct to* 4 *decimal places.*

Rough check $\dfrac{0 \cdot 06}{20} = 0 \cdot 003$.

Therefore the answer must contain 2 significant figures. Therefore we retain 4 (=2+2) figures in the divisor. Draw a line after the 4th figure.

a. Instead of bringing down a figure, cross off the 4 in the divisor; when multiplying by 5, carry 2 to allow for 5×4=20.

$$\begin{array}{r|r|l}
 & 356 & \\ \cline{2-2}
17\cancel{6}\cancel{4} & 6294 & \\
 & \underline{5292} & \\
 & 1002 & a \\
 & 882 & \\ \cline{2-2}
 & 120 & \\
 & \underline{\underline{106}} &
\end{array}$$

Answer $\underline{0 \cdot 0036}$.

The previous Exercises XIV (*b*) and XIV (*c*) may now be used, if desired, to give practice in formal contracted methods.

EXERCISE XV (d)

1. Find the value of π^2 correct to 4 significant figures. [See XV (*c*), No. 34.]

Calculate Nos. 2–11 correct to 3 significant figures:

2. $\frac{1}{23}$.

3. $\dfrac{1}{\cdot 076}$.

4. $216 \cdot 83 \times \cdot 0074832$.

5. $6 \cdot 9854 \div \cdot 08217$.

6. $\cdot 016482 \times 6 \cdot 7438 \times 17 \cdot 53$.

7. 2^{13}.

8. $\dfrac{\pi \times (2 \cdot 643)^2 \times 0 \cdot 0625}{18 \cdot 637}$.

9. $\dfrac{(3 \cdot 027)^2 - 4 \cdot 32 \times 0 \cdot 074}{4 \cdot 382}$.

10. $6 \cdot 243 \times (1 \cdot 03)^3$.

11. Express a barometric height of 758·4 mm. in inches, given 1 metre = 39·370 in.

12. A year is 365·2422 days and a lunar month is 29·5306 days. Express a lunar month as a decimal of a year.

13. Express $\dfrac{1}{3{\cdot}1416}$ as a decimal correct to ·0001. Use the result to find the diameter of a circle whose circumference is 68·78 in.

14. A metre was originally taken to be 39·37079 inches. Afterwards it was found to be 39·370113 inches. Find to 3 significant figures to how many inches in a mile this alteration corresponds.

15. Find to 4 significant figures the value of $\dfrac{1}{{\cdot}026607}+\dfrac{1}{1{\cdot}493721}$.

16. Evaluate to the nearest 100,000 the value of $(8476)^2$.

17. Calculate to 3 significant figures the value of $\dfrac{{\cdot}06842\times17{\cdot}536}{162{\cdot}78}$.

18. Find to the nearest unit the value of $\dfrac{62{\cdot}476\times0{\cdot}85934}{{\cdot}0076247}$.

19. Find to the nearest penny the value of £33·728945 ÷ 0·7892364.

20. The Czechoslovak expenditure for 1926 was estimated at 10,070,271,410 crowns or £61,780,806. Calculate the value of a Czechoslovak crown to $\frac{1}{100}$ of a penny. This is stated to be an increase of 496,685,609 crowns above the expenditure for 1925. What is this increase in pounds sterling ?

21. Find to 3 decimal places the square of 2·71828.

22. 1 metre = 39·370113 inches. Find the number of square inches in a square metre correct to 6 significant figures.

23. 1 Kg. = 2·2046223 lb. Express 1 lb. in Kg. correct to 5 significant figures.

24. Evaluate $34,857\times1,567,843$ correct to the nearest million.

25. The quantity of coffee imported in 1903 was 1,143,526 cwt., valued at £3,134,924. In 1904 the amount was 1,055,866 cwt. What was the 1904 import worth at the same average price per cwt. as in 1903 ?

CHAPTER XVI

SQUARES AND SQUARE ROOTS

Example.—Given that $37^2=1369$, *write down the squares of* 370, ·7, 0·37.

$$370^2=37^2\times10^2=136900$$

$$3{\cdot}7^2=\frac{37^2}{10^2}=13{\cdot}69$$

$$0{\cdot}37^2=\frac{37^2}{100^2}=0{\cdot}1369.$$

EXERCISE XVI (a)

1. Write down the squares of 12, 1·2, 120, 0·12, 0·012.

2. Write down the squares of 70, 0·8, 300, 0·06, 1·1.

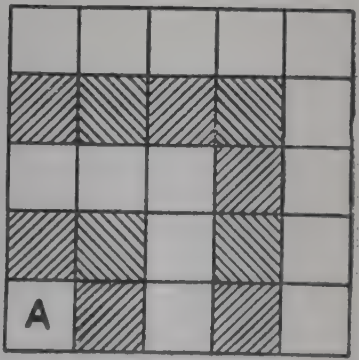
3. Which of the following numbers are perfect squares, and of hat are they the squares: 49, 4·9, 0·49, 1·44, 9000, 2·5, 0·0004?

4. What is the *final* digit in each of the following: 1^2, 2^2, 3^2, 4^2, 5^2, ^{2}, 7^2, 8^2, 9^2, 10^2? Is the final digit ever 2?

Which of the following numbers cannot possibly be a perfect quare: 5776, 8468, 2917, 6561, 4483, 9025, 8412, 6084, 3969?

5. If any perfect square is divided by 5, the remainder is either or 1 or 4. Give an example of each case.

6. $(n+1)^2$ exceeds n^2 by $2n+1$. Use this fact to write down succession the values of 21^2, 22^2, 23^2, ^{2}, 25^2.



7. The diagram represents a floor of square les, the shaded tiles being black and the hers white. The tile A is laid first; it is ien bordered with 3 black tiles as shown, to ake a square figure. This figure is then rdered with white tiles to make a larger uare, and so on. What can you say about $+3+5$ and $1+3+5+7$ and $1+3+5+7+9$? hat is the sum of the first ten odd numbers? hat is the sum of the first n odd numbers?

8. What is the smallest whole number by which 360 must be ultiplied to make the result a perfect square?

9. Write down the squares of $\frac{1}{3}$, $\frac{1}{10}$, $\frac{3}{4}$, 0·1, $1\frac{1}{2}$, 1·3, 2·5. What can u say about x if (i) $x<x^2$, (ii) $x>x^2$?

10. Simplify (i) $\frac{(3\cdot1)^2}{31^2}$; (ii) $\frac{(270)^2}{(2\cdot7)^2}$; (iii) $\frac{15^2}{(1\frac{1}{2})^2}$.

11. Write down the square roots of 81; 0·81; 810,000; 12,100; 1·21; 0·000121.

12. Write down the square roots of $\frac{4}{9}$, $\frac{1}{100}$, 0·01, 0·0001, $\frac{64}{81}$, $1\frac{7}{9}$, $1\frac{11}{25}$, $12\frac{1}{4}$, 1·69.
What can you say about x if (i) $x > \sqrt{x}$, (ii) $x < \sqrt{x}$?

13. Which is the greater: 12 or $\sqrt{140}$; 17 or $\sqrt{290}$; $3\frac{1}{3}$ or $\sqrt{11}$; 2·5 or $\sqrt{6}$; $7\frac{3}{4}$ or $\sqrt{60}$?

14. Square 1·4, 1·5, 1·41, 1·42. Show that $\sqrt{2}$ lies between 1·41 and 1·42.

15. Show that $\sqrt{3}$ lies between 1·73 and 1·74.

16. Show that $\sqrt{0\cdot1}$ lies between 0·3 and 0·4.

17. What are the values of $\sqrt{100}$ and $\sqrt{10,000}$? How many figures are there to the left of the decimal point in the square roots of 469, 9409, 13,724, 19, 940·9?

18. How many figures are there to the left of the decimal point in the square root of a number containing (i) 6, (ii) 7, (iii) 8 digits to the left of the decimal point?

19. Simplify (i) $\frac{\sqrt{16}}{\sqrt{0\cdot16}}$; (ii) $\frac{\sqrt{1\cdot44}}{\sqrt{144}}$; (iii) $\frac{\sqrt{173}}{\sqrt{1\cdot73}}$; (iv) $\frac{\sqrt{6234}}{\sqrt{0\cdot6234}}$.

20. Show that $\sqrt{10}$ lies between 3·1 and 3·2.

State rough limits for the values of $\sqrt{40}$, $\sqrt{90}$, $\sqrt{250}$.

State approximate values for $\frac{\sqrt{16}}{\sqrt{1\cdot6}}$, $\frac{\sqrt{2500}}{\sqrt{2\cdot5}}$, $\sqrt{100,000}$.

Rule for Extracting Square Root

$$\begin{array}{r|ll}
 & 3\cdot\ \ 7\ \ 2 & \\
\hline
3 & 13\cdot\ 83\ 84 & \\
 & 9 & \\
\hline
6\cdot7 & 4\cdot8384 & (1) \\
 & 4\cdot69 & (2) \\
\hline
7\cdot42 & \cdot1484 & (3) \\
 & \cdot1484 & \\
\hline
\end{array}$$

Find the square root of 13·8384.

The greatest square of a whole number below 13 is $9=3^2$, which, when subtracted from 13·8384, leaves 4·8384 [see (1)]. A nearer approximation is obtained by forming a larger square than 3^2: this may be done by adding a shaded border, of area slightly greater than a rectangle of length $3+3=2\times3=6$, and width approximately

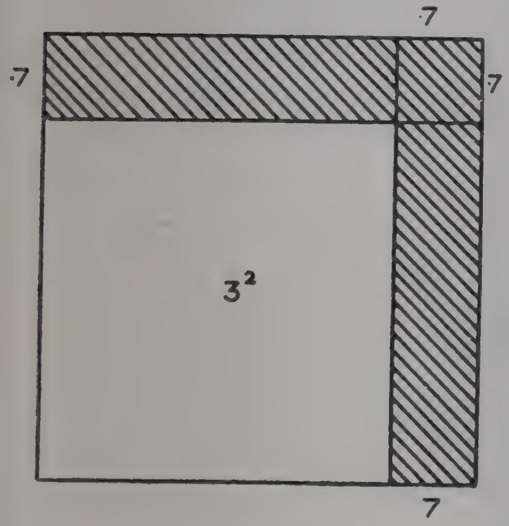
$\frac{4\cdot8384}{6}$. Trial shows ·8 would be too large, for the border area would then be

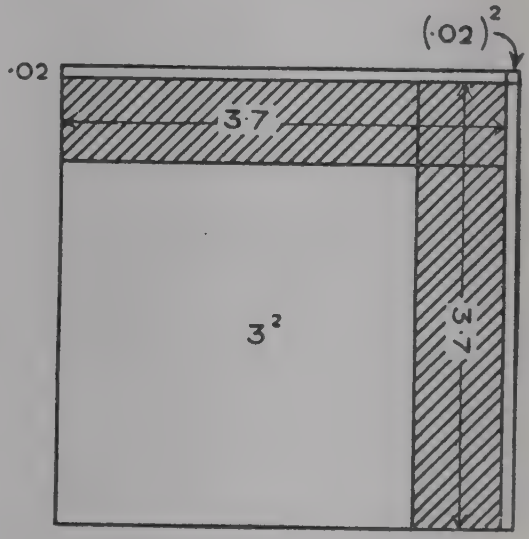
$$6\times\cdot8+(\cdot8)^2=(\cdot8)(6+\cdot8)=\cdot8\times6\cdot8=5\cdot44\,;$$

but ·7 gives a border of area $6\times\cdot7+(\cdot7)^2=\cdot7(6+\cdot7)=4\cdot69$ [see (2)]. The remainder is $13\cdot8384-[3^2+(\cdot7)(6\cdot7)]=\cdot1484$ [see (3)].

Another border will have an area slightly greater than a rectangle of length 2(3·7) and width approximately $\frac{\cdot1484}{7\cdot4}\overset{\frown}{=}\cdot02$. We thus obtain a square whose side is 3·72 and area

$$3^2+\cdot7(6\cdot7)+\cdot02(7\cdot4)+(\cdot02)^2=13\cdot8384.$$





The calculation is as follows:

Divide the figures into pairs in both directions from the decimal point. By trial $3^2=9$. Double the 3 and use 6 ✻ as a divisor: by trial find the quotient 7 and then write the divisor as 67.

Again double the figures 37 and use 74 ✻ as a divisor: by trial find 2 as the quotient and use 742 as the divisor.

$$\begin{array}{r|l}
 & \underline{3\cdot}\ \ \underline{7}\ \ \underline{2} \\
3 & 13\cdot\ 83\ 84 \\
 & \underline{9} \\
67 & 4\,83 \\
 & \underline{4\,69} \\
742 & 14\,84 \\
 & \underline{14\,84}
\end{array}$$

Answer **3·72.**

Example.—Find the square root of 104,976.

Rough check $300^2 = 90,000$
$400^2 = 160,000$

$$\begin{array}{r|r|r|r|}
 & 3 & 2 & 4\cdot \\ \hline
 & 10 & 49 & 76\cdot \\
 & 9 & & \\ \hline
62 & & 149 & \\
 & & 124 & \\ \hline
644 & & & 2576 \\
 & & & 2576 \\ \hline
\end{array}$$

Mark off the digits in pairs, *starting from the decimal point.*

Answer $\underline{324}$.

Example.—Find the square root of 0·000,104,976 *correct to three significant figures.*

Rough check $(0{\cdot}01)^2 = 0{\cdot}0001$.

$$\begin{array}{r|c|r|r|r|r|r|}
 & \cdot & 0 & 1 & 0 & 2 & 4 \\ \hline
 & \cdot & 00 & 01 & 04 & 97 & 60 \\
 & & & 1 & & & \\ \hline
202 & & & & & 497 & \\
 & & & & & 404 & \\ \hline
2044 & & & & & & 9360 \\
 & & & & & & 8176 \\ \hline
\end{array}$$

Answer $\underline{0{\cdot}0102}$.

Note.—$\sqrt{104976} = 324$ and $\sqrt{1049{\cdot}76} = \sqrt{\dfrac{104976}{100}} = \dfrac{324}{10} = 32{\cdot}4$,

but $\sqrt{10497{\cdot}6} = 102{\cdot}4$

and $\sqrt{\cdot000104976} = \sqrt{\dfrac{10497{\cdot}6}{100,000,000}} = \dfrac{102{\cdot}4}{10000} = {\cdot}01024$.

This shows that if the decimal point is moved an *even* number of places in the given number, the significant figures in the square root remain unaltered, but if it is moved an odd number of places, the figures in the square root are entirely changed.

Example.—Given $\sqrt{2} = 1{\cdot}414 \ldots$, *find the value of* $\dfrac{1}{\sqrt{2}}$.

We can avoid division of decimals in evaluating $\dfrac{1}{\sqrt{2}}$ by the following device:

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1{\cdot}414}{2} \ldots = 0{\cdot}707 \ldots$$

EXERCISE XVI (b)

Find the square roots of the following:

1. 1444. **2.** 3025. **3.** 3481. **4.** 8649.
5. 11664. **6.** 16129. **7.** 64009. **8.** 546121.
9. 817216. **10.** 129600.

Find the square roots of

11. 1·4161. **12.** 136·89. **13.** 331·24.
14. 60·9961. **15.** 100·2001. **16.** 0·0196.
17. 0·017161. **18.** 0·1024. **19.** 0·00001849.
20. 0·00063001. **21.** 0·288369. **22.** 0·034225.

Find to three significant figures the square roots of

23. 30. **24.** 300. **25.** 3. **26.** 13.
27. 130. **28.** 16·2. **29.** 12·74. **30.** 1·274.
31. 14·43. **32.** $\frac{1}{3}$. **33.** $3\frac{1}{3}$. **34.** 0·263.
35. ·006217. **36.** 83·619. **37.** 0·000372.
38. 0·523. **39.** ·0523. **40.** 1·93204.

For further exercises see XVI (*c*).

Table of Squares

Squares of numbers may be found by using tables, the degree of pproximation depending on the number of significant figures given n the tables.

A rough approximation should be first found, in order to fix the umber of figures to the left of the decimal point in the answer.

Example.—Use the tables to square 62·35.

Rough approximation, $60^2 = 3600$.

Look for 62 in the left-hand column of the tables: in the same row s 62, and underneath 3, we find 3881. And in the same row as 62, n the *difference columns* at the end underneath 5, we find 6. This gure 6 is to be added to the last figure of 3881.

$\therefore$ $(62{\cdot}35)^2 = 3887$ to four significant figures.

Note.—(i) Since the difference columns can only give average differences, e fourth figure is not always reliable, but the error, if any, is small.

(ii) Since 6·235, 62·35, 623·5, etc., have the same significant figures, their quares will only differ in the position of the decimal point;

thus $(6{\cdot}235)^2 = 38{\cdot}87$ to four figures,
and $(0{\cdot}6235)^2 = 0{\cdot}3887$ to four figures.

Table of Square Roots

The general method of reading off square roots from the tables is the same as that already explained for reading off squares. But there is one important difference.

$$\sqrt{7}=2{\cdot}646,\ \sqrt{70}=8{\cdot}367,\ \sqrt{700}=26{\cdot}46,\ \sqrt{7000}=83{\cdot}67,$$

and so on. Consequently, corresponding to any set of figures in a number, there are two distinct sets of figures in the square root, according to the position of the decimal point.

Before using the tables, mark off the digits in pairs from the decimal point and determine the first significant figure in the square root, by inspection. Then use the tables to find the remaining significant figures.

Example.—Find $\sqrt{34120}$ *correct to three significant figures.*

Write the number as 3 | 41 | 20 | . ; this shows the first significant figure is 1, and as a rough check we call it 100.

The tables give the figures 1848 ;

$$\therefore\ \sqrt{34120} \simeq 184{\cdot}8.$$
$$= \underline{185} \text{ to three significant figures.}$$

Example.—Find $\sqrt{3412}$ *correct to three significant figures.*

Write the number as 34 | 12 | . ; this shows the first significant figure is 5, and as a rough check we call it 50.

The tables give the figures 5842 ;

$$\therefore\ \sqrt{3412} \simeq 58{\cdot}42$$
$$= \underline{58{\cdot}4} \text{ to three significant figures.}$$

Table of Reciprocals

If the product of two numbers is 1, either number is said to be the *reciprocal* of the other.

E.g. the reciprocal of 2 is $\frac{1}{2}$ or 0·5 ; the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ or $1\frac{1}{2}$.

The reciprocals of numbers containing *four* or fewer significant figures are read off from the table by precisely the same method as used in reading off squares, except that the number in the difference column is subtracted, instead of being added (see Note below).

The position of the decimal point is fixed by a rough check.

Example.—Find the reciprocals of 23·7 *and* 23·8.

Corresponding to 237 we find 4219;
and corresponding to 238 we find 4202.

Now $$\frac{1}{23{\cdot}7} \bumpeq \frac{1}{20} \text{ or } 0{\cdot}05.$$

$$\therefore\ \frac{1}{23{\cdot}7} = \underline{0{\cdot}04219} \text{ and } \frac{1}{23{\cdot}8} = \underline{0{\cdot}04202}.$$

Note.—As a number increases, its reciprocal *decreases*. Thus the reciprocal of 23·76 is *less* than the reciprocal of 23·7.

We therefore find the reciprocal of a number containing four significant figures by *subtracting* the number given in the difference column.

∴ corresponding to 23·76, we take 4219−11=4208, and we have $\frac{1}{23{\cdot}76} = \underline{0{\cdot}04208}$.

EXERCISE XVI (c)

Find as accurately as the tables permit the squares of

1. 173. 2. 1536. 3. 41·25. 4. 1·826. 5. 3·142.
6. 0·1269. 7. 0·02137. 8. 171·2. 9. 1·013.

Find from the tables the square roots of

10. 37. 11. 0·5. 12. 0·005. 13. 13·78. 14. 1·378.
15. 50. 16. $6\frac{3}{4}$. 17. $49\frac{1}{9}$. 18. $4\frac{2}{9}$. 19. $25\frac{9}{16}$.
20. $3\frac{2}{19}$. 21. $11\frac{2}{3}$.

Find from the tables the values of

22. $\frac{1}{17}$. 23. $\frac{1}{131}$. 24. $\frac{1}{23}$. 25. $\frac{1}{3{\cdot}46}$. 26. $\frac{1}{21{\cdot}7}$.
27. $\frac{1}{112{\cdot}2}$. 28. $\frac{1}{97{\cdot}38}$. 29. $\frac{1}{0{\cdot}01666}$. 30. $\frac{1}{100{\cdot}4}$. 31. $\frac{1}{2{\cdot}703}$.
32. $\frac{1}{31{\cdot}48}$. 33. $\frac{2}{15{\cdot}3}$. 34. $\frac{3}{17{\cdot}2}$. 35. $\frac{5}{16{\cdot}08}$. 36. $\frac{3}{612{\cdot}2}$.
37. $\frac{1}{\sqrt{62{\cdot}3}}$. 38. $\frac{1}{\sqrt{167{\cdot}8}}$. 39. $\sqrt{\frac{1}{15{\cdot}4}}$. 40. $\frac{1}{\sqrt{{\cdot}0432}}$.
41. $\frac{1}{15{\cdot}3}+\frac{1}{17{\cdot}2}$. 42. $\frac{2}{10{\cdot}2}+\frac{3}{16{\cdot}6}$. 43. $\frac{1}{{\cdot}253}+\frac{1}{{\cdot}367}$.
44. $\frac{1}{(16{\cdot}1)^2}+\frac{1}{(12{\cdot}9)^2}$. 45. $\frac{1}{\sqrt{17{\cdot}3}}+\frac{1}{\sqrt{1{\cdot}85}}$.
46. $\frac{1}{\sqrt{{\cdot}0326}}-\frac{1}{\sqrt{{\cdot}326}}$. 47. $\frac{1}{\sqrt{21{\cdot}34}}-\frac{1}{\sqrt{321{\cdot}7}}$.

Given $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$, find p when

48. $a=11{\cdot}2$, $b=10{\cdot}7$. 49. $a=0{\cdot}52$, $b=0{\cdot}73$. 50. $a={\cdot}0626$, $b={\cdot}0387$.

Pythagoras' Theorem.—Place four sheets of white paper one above the other, and cut out simultaneously four right-angled triangles, which are therefore congruent. Suppose the hypotenuse is c in., and the lengths of the other two sides are a in., b in. Draw a square of side $a+b$ in. Shade it, and then arrange the four triangles on it as in fig. (i).

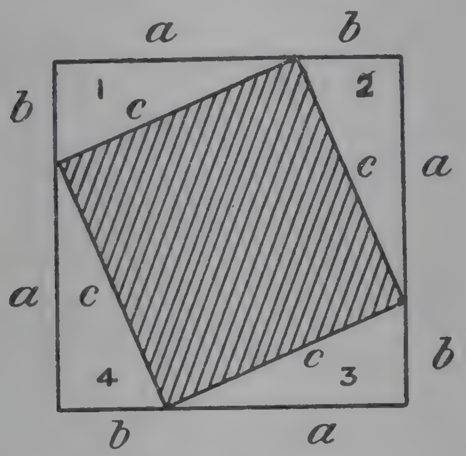


Fig. (i).

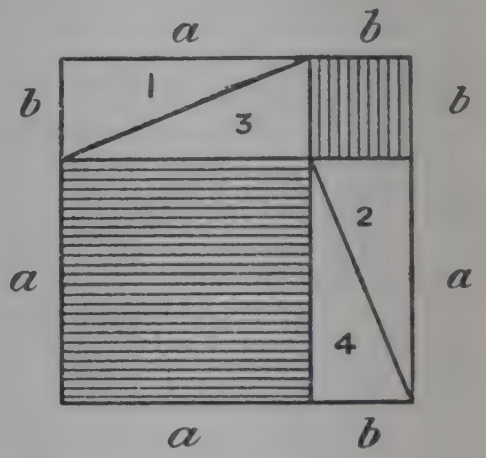


Fig. (ii).

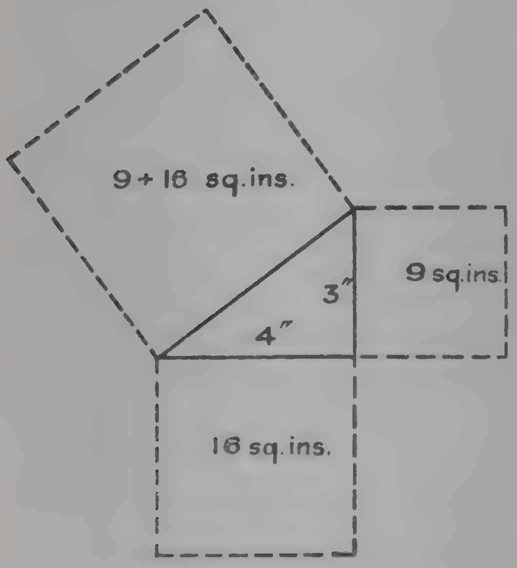
What is the shape and area of the shaded area in fig. (i) ?

Next slide the triangles into the positions shown in fig. (ii).

What are the shapes and areas of the shaded areas in fig. (ii) ?

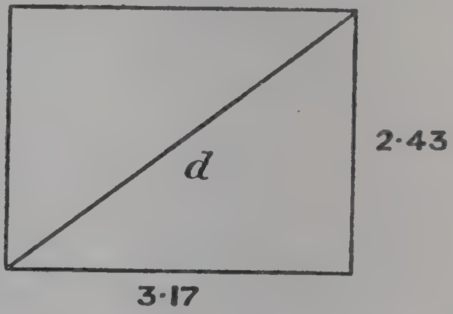
What conclusion can you draw from this experiment ?

It seems probable that Pythagoras, who lived in the sixth century B.C., used the method indicated above to prove his theorem that the area of the square on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares on the other two sides.



Thus if the sides containing the right angle are 3 in. and 4 in., the area of the square on the hypotenuse equals $9+16=25$ sq. in., and therefore the length of the hypotenuse is 5 in.

Example.—The sides of a rectangle are 2·43 *in. and* 3·17 *in. Find the length of a diagonal.*



If d is the diagonal, we have

$$\begin{aligned} d^2 &= (3{\cdot}17)^2+(2{\cdot}43)^2 \\ &= 10{\cdot}05+5{\cdot}905 \\ &= 15{\cdot}955 \\ \therefore\ d &= \sqrt{15{\cdot}955}=\underline{3\ 995}\text{ in.} \end{aligned}$$

Example.—The diagonal of a rectangle is 2·84 *in., and one side is* 1·66 *in. Find the other.*

If the required side is x *in.*, we have

$$\begin{aligned} (2{\cdot}84)^2 &= (1{\cdot}66)^2+x^2 \\ \therefore\ x^2 &= (2{\cdot}84)^2-(1{\cdot}66)^2=(2{\cdot}84+1{\cdot}66)(2{\cdot}84-1{\cdot}66) \\ &= 4{\cdot}5\times 1{\cdot}18 \\ &= 5{\cdot}31 \\ \therefore\ x &= \sqrt{5{\cdot}31}=2{\cdot}30\text{ in.} \end{aligned}$$

EXERCISE XVI (d)

Find to three significant figures the side of a square whose area is

1. 124 sq. ft. 2. 12 sq. in. 3. 1 acre (in yards).
4. 10 acres. 5. 640 sq. yd. 6. 10 sq. chains.

7. A beam of timber is 12·4 ft. long and its volume 14·2 cu. ft. If its section is a square, find its edge.

8. Find the diagonal of a square whose side is 4·23 ft.

9. Find by factors the square root of 27,225.

10. Find the diagonal of a square field whose area is 20 acres.

11. How long will it take to walk round a square field of 40 acres at the rate of 4 miles per hour ?

12. Find the hypotenuse of a right-angled triangle whose other sides are 3·23 in. and 5·37 in.

13. A right-angled isosceles triangle has a hypotenuse of 14·14 in. Find the equal sides.

14. An electric-lamp holder hangs from a ceiling 15 ft. high by a cord 8 ft. long. The nearest wall is 6 ft. from the holder. Find to

the nearest inch how far above the floor the holder, if set swinging, will strike the wall.

15. The total surface of a cube is of area 10 sq. in. Find to $\frac{1}{100}$ in. the length of an edge.

16. A man now living will be x years old in the year x^2 A.D. When was he born ?

17. Archimedes proved that $\sqrt{3}$ lies between $\dfrac{1351}{780}$ and $\dfrac{265}{153}$. Verify this.

18. A square field contains 10 acres. Find the cost of fencing it with wire at 8½d. per foot length.

19. A map is drawn to a scale of 25 in. to 1 mile. Find to 0·1 in. the side of a square on the map which represents an area of 15 acres.

20. Ramanujan gave $\frac{9}{5}+\sqrt{\frac{9}{5}}$ as an approximation for π(3·14159265 . . .). To how many places of decimals was he correct ?

21. Find the diagonal of a cube whose edge is 4 in.

22. The slant side of a circular cone is 12 in. and the height is 8 in. Find the diameter of the base.

23. What is the longest rod that can be placed in a box whose internal dimensions are 4 in. by 5 in. by 6 in. ?

24. A room is 18 ft. long, 15 ft. wide, 11 ft. high. What is the length of a diagonal ?

25. A pyramid of height 9 in. stands on a square base, side 4 in. Find the length of a slant edge.

26. Prove that the triangle whose sides are x^2+y^2, x^2-y^2, $2xy$ is right-angled. What do you get if (i) $x=3$, $y=2$; (ii) $x=5$, $y=2$?

Use the formula $\dfrac{1}{f}=\dfrac{1}{u}+\dfrac{1}{v}$ in Questions 27–30.

27. Find f when $u=-20{\cdot}4$, $v=10{\cdot}6$.

28. Find v when $u=14{\cdot}6$, $f=25{\cdot}5$.

29. Find v when $u=10{\cdot}2$, $f=-4{\cdot}38$.

30. Find u when $f=-16{\cdot}2$, $v=19{\cdot}4$.

31. Find the value of $\dfrac{1}{\sqrt{6{\cdot}13}}+\sqrt{\dfrac{1}{10{\cdot}5}+\dfrac{1}{8{\cdot}73}}$.

32. Evaluate $\dfrac{\sqrt{17}-\sqrt{13}}{\sqrt{17}+\sqrt{13}}$ to three significant figures.

33. The length of a rectangle is four times its breadth and the area is 1264·78 sq. yd. Find its perimeter.

34. Ten feet of iron tubing have an internal volume of 142·64 cu. in. Find the bore. [Area of circle πr^2.]

35. A hemispherical bowl of radius 8·7 in. is filled with water to a depth of 6·2 in. Find the radius of the water surface.

36. A hall is 60 ft. long, 20 ft. wide, 15 ft. high. What is the distance from a corner of the ceiling to the opposite corner of the floor?

MISCELLANEOUS EXAMPLES

M (V)

1. On a certain day there were 5 hours 37 minutes of sunshine, which was 82·3 per cent. of the average for the week. Find this average.

2. If 8250 cu. ft. of asphalt are spread over a quarter of a mile of road, 30 ft. wide, what will be the thickness of the asphalt?

3. A season ticket for a quarter (13 weeks) to a place a few miles out of London is £4, 1s. 6d., but the fare for a double journey is 5s. 4½d. How much will a traveller save in a quarter by taking a season ticket, if he has to travel to and fro twice a week?

4. What is the error per cent. caused by using the rule: "The speed of a train in ft./sec. can be found by multiplying the speed in miles/hr. by 1½"?

5. Find the square root of (1) $64\frac{64}{81}$, (2) $\frac{1{\cdot}28}{{\cdot}125}$.

6. A square chain is pegged out on a field of roots, and the weight of roots gathered within it was 581 lb. How many tons would you expect to get from 1 acre?

7. If a cubic yard of coal weighs 1 ton, find its specific gravity.

8. Three bells toll at intervals of 12, 15, 28 seconds respectively. A person at a distance from them can hear their sound only when two at least toll at the same time. If they begin together, how many times will he hear them in the first 7½ minutes?

9. One gallon of water contains 277·274 cu. in. Find in feet to two decimal places what must be the length of a square tank the depth of which is 5 ft. and which holds 2000 gallons.

10. A bankrupt's debts amount to £4875, 5s. 2d. and his assets to £3250, 3s. 7d. What will a creditor receive whose claim is £574, 10s.?

11. If goods are marked for sale at 20 per cent. above cost price and 5 per cent. discount is allowed for cash, what is the actual profit on an outlay of £700?

12. Find the rate per cent. per annum at which £525 will produce £183, 15s. in 7 years at simple interest.

13. If it costs £56, 12s. to keep a family of 7 persons in food for 11 weeks, what will be the expense of keeping 15 persons for 9 weeks at the same rate?

14. A village of 1235 inhabitants has 85 more females than males. How many are there of each sex?

15. A drill $3\frac{3}{8}$ in. long tapers gradually from one end, where the diameter is 0·45 in., to the other, where it is 0·325 in. What will be its diameter 1 in. from the thin end?

What must be the diameter of the broad end if the length and the diameter of the thin end are unaltered, but it is to taper 0·04 in. per inch of length?

16. The following table gives the population of Australia:

Year	1871	1881	1891	1901	1911	1921
Population in millions	1·66	2·25	3·17	3·77	4·46	5·44

Represent these on a graph. Find also by another graph on the same paper what would have been the population in 1921, if the rate of growth from 1871–1881 had been maintained continuously.

17. An estate whose area is 362 acres is found to cover 22·7 sq. in. on a map. Find the scale of the map.

18. If the shares of a company cost £$3\frac{11}{32}$ each, how many can a man buy with £100, if he has to pay £1, 2s. 3d. for expenses?

19. Taking the Earth as a sphere of radius 3960 miles, what is the length of the Equator? How much longer would a string require to be if it were kept 6 ft. away from the globe all round?

20. In an examination the top boy scored 84 per cent., and the bottom boy 16 per cent., of the maximum. Their marks differed by 47: find what they actually were.

21. A square of cardboard of side 1 ft. 2 in. weighs 14 oz. If it is cut down to a square of side $10\frac{1}{2}$ in., what does it then weigh?

22. Find the difference between $\frac{1}{x}+\frac{1}{y}$ and $\frac{1}{x+y}$ when $x=\frac{2}{3}$, $y=\frac{5}{7}$.

23. A rectangular field 121 yd. long and 80 yd. wide is to be drained with pipes 12 in. long laid in trenches 15 ft. apart. If the trenches begin 2 yd. from the longer side, how many pipes will be required?

24. What is the difference in tax on £450 when the rate is altered from $3\frac{1}{2}$ per cent. to 8d. in the £?

25. A wagon and its load weigh $4\frac{1}{3}$ tons, and the wagon weighs $\frac{4}{9}$ of the load. Find the weight of each.

26. In 1891 the population of a town was 12 per cent. more than in 1881. The increase was 12,420: find the population in 1891.

27. Simplify $\dfrac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}+2\frac{1}{7}} \div \dfrac{\frac{2}{7}+3\frac{1}{2}}{3\frac{1}{2}-2\frac{5}{7}}$.

28. A boat steaming at 15 knots is 21 sea miles behind another which is going at 12½ knots. How far will the faster boat have to travel before it overtakes the one in front?

29. A boy multiplies a sum of money by 2·3, instead of multiplying by 2⅓. His answer is £77, 16s. 4d. How much is the error?

30. Simplify $[(0{\cdot}4)^2-0{\cdot}3\times0{\cdot}5]\div[\frac{3}{4}\div0{\cdot}25]$.

31. Evaluate $[(14{\cdot}27)^2-(13{\cdot}27)^2]\div0{\cdot}09$.

32. What multiplying factor converts a charge of pence per ounce into a charge of £ per cwt.?

33. How many tons, to the nearest hundred, of water fall on a square mile of land during a rainfall of 0·3 in.?

34. In an examination, 1200 candidates were boys and 700 were girls; 40 per cent. of the boys and 30 per cent. of the girls passed. What percentage of the total failed?

35. The net income of the Southern Railway was £6,960,000 in 1922 and £6,090,000 in 1923. Find the decrease per cent. correct to three significant figures.

36. In 1924 the total proceeds of the coal-mining industry in Great Britain were £162,386,330; of this, 91·81 per cent. was received by the miners in wages. What was left over, correct to the nearest £100?

37. A motorist uses 389 gallons of petrol in driving 8251 miles: this is equivalent to a cost of 5d. every 9 miles. What is the average cost of petrol per gallon to the nearest halfpenny?

38. The charge for an announcement on the front page of *The Times* is one guinea for three lines and five shillings for each additional line. How many lines can be inserted for £2, 16s.?

39. In 1921 the population of County Tyrone was 141,786, of whom 77.982 were Roman Catholics; and the population of County Fermanagh was 61,214, of whom 34,619 were Roman Catholics. Find the percentage of Roman Catholics in each case correct to one place of decimals.

40. Which travels the faster, a train at 51 miles per hour, or a car at 73 feet per second?

41. In an examination a candidate must get 35 per cent. of the total to pass. On the first paper he gets 56 out of 150, and on the second he gets 75 out of 150. Full marks for the remaining paper are 200. What must he get on it to pass?

42. A photograph 4 in. by 3 in. is enlarged. The length of the shorter side in the enlargement is 10 in. What is the area of the enlargement?

43. When a boy has spent x per cent. of his pocket money he has y shillings left. How much has he spent?

44. A sea mile is 1·854 Km., and a land mile is 1·609 Km. What is the velocity of a train in miles per hour which travels 40 per cent. faster than a ship which is doing 25 knots?

45. Will a 3-inch shell fit a 75-millimetre gun? [Take 1 metre =39·37 inches.]

46. At an election, when only $\frac{7}{8}$ of the voters went to the poll, one candidate received $\frac{1}{4}$ as many votes again as the other and secured a majority of 896. How many voters were on the register?

47. Water boils at 212° F. or 100° C., and freezes at 32° F. or 0° C. Express 60° C. in Fahrenheit and 50° F. in Centigrade.

48. In a stove 89 per cent. of the products of combustion escape in the form of gas. In a month the cinder and ashes left weigh 12 cwt., and the soot in the flues and chimney weighs 100 lb. What weight of fuel to the nearest cwt. was used?

49. What is the length of paling required for a square field whose area is 6 acres 2 roods?

50. The face of an eight-day clock is pierced with two circular holes on opposite sides of the centre of the face for winding purposes. Each hole subtends an angle of 30° at the centre of the face. For what percentage of every 12 hours is at least one of the holes partially blocked by a hand of the clock?

REVISION PAPERS

(R 31–40)

R 31

1. Simplify $\frac{2\frac{1}{2}+4\frac{1}{4}}{9}\times3\frac{1}{3}-3\frac{1}{3}$.

2. Find correct to one significant figure the value of

$$\frac{67{\cdot}824\times0{\cdot}0892}{3{\cdot}1416}.$$

3. Ceylon tea is quoted wholesale (November 1925) at 1s. 10·65d. per lb. correct to $\frac{1}{100}$th penny. What is the difference between the greatest and least possible prices per ton ?

4. Use tables to find correct to two significant figures the value of

$$(0\cdot6837)^2+\frac{1}{6\cdot354}-\sqrt{0\cdot0962}.$$

5. An analysis of 0·8572 gr. of a Lee-Enfield bullet casing showed that it contained 0·6695 gr. of copper. Express this as a percentage correct to the nearest whole number.

6. If lead is 11·3 times as heavy as water, and if a cubic foot of water weighs $62\frac{1}{2}$ lb., what is the weight of 72 cu. in. of lead, correct to $\frac{1}{10}$ lb. ?

R 32

1. Find the value of £2, 11s. 9d. $\div 3\frac{2}{7}$.

2. If $\frac{2}{3}$ of a tradesman's stock is sold, by what percentage must the rest be increased to bring the stock to its original size ?

3. Simplify $\dfrac{\frac{1}{2}+\frac{1}{3}\times\frac{1}{5}}{\frac{1}{2}-\frac{1}{3}\times\frac{1}{5}}$.

4. Write the formula πr^2 (the area of a circle) in terms of d, the diameter. Find the area of a circle of diameter 3·72 cm.

5. A debtor pays 7s. 6d. in the £. What is the ratio of what he has paid to what he still owes ? If he has paid altogether £248, what does he still owe ?

6. Telegraph wire weighs 450 lb. to the mile. If 1 cu. ft. weighs 484 lb., find the cross-section of the wire.

R 33

1. Simplify

$$\text{(i)}\quad \left(\frac{1}{4\frac{1}{2}}+\frac{1}{2\frac{4}{7}}\right)\div\left(\frac{1}{3\frac{2}{3}}+\frac{1}{2\frac{1}{5}}\right);$$

$$\text{(ii)}\quad (a+b)\div\left(\frac{1}{a}+\frac{1}{b}\right).$$

2. Find correct to two significant figures the difference between the squares of $\sqrt{5}+\sqrt{2}$ and $5+2$.

3. How many payments of 2s. 7d. each can be made from a sum of £5, and how much remains over ?

4. After spending $\frac{2}{7}$ of his pocket-money for the term, and losing $\frac{1}{5}$ of it, a boy finds he has 18s. left. How much does he have for the term ?

5. Find the difference correct to the nearest penny between $3\frac{1}{2}$ per cent. of £765, 4s. and 4 per cent. of £800.

6. A brick is $3x$ in. long, $2x$ in. broad, x in. high. What is (i) the sum of the lengths of its edges ; (ii) its superficial area ?

R 34

1. Find the cost of 176 lb. at 9s. 8d. per lb.

2. The price of a metal increases from £126, 12s. 4d. to £147, 13s. 9d. per ton. What is the extra cost for 2634 tons ?

3. Find the square roots of 56169 and ·056169.

4. Find the total percentage increase in the population of a town which grows from 376,482 to 427,519 in 10 years.

5. Simplify (i) $\dfrac{2\frac{2}{3}+1\frac{1}{3}\times 3\frac{1}{6}}{4\frac{3}{7}(2\frac{1}{3}-1\frac{3}{4})}$; (ii) $\dfrac{1}{\cdot 02}+\dfrac{5}{\cdot 01}$.

6. A wire increases by ·0000117 of its length for every degree it is heated. When it is heated 210°, its length is 62·48 ft. What was the original length ?

R 35

1. Arrange in ascending order of magnitude $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{11}$.

2. A man, riding at 9 miles an hour, takes $7\frac{1}{2}$ seconds to cross a bridge. What is its length ?

3. Forty-eight tons of rubber are sold at 3s. $3\frac{7}{8}$d. per lb. Find the proceeds of the sale.

4. A rectangular sheet of paper is 5·7 in. long and 3·8 in. wide. A strip 0·3 in. wide parallel to the length is first cut away and then a strip 0·2 in. wide parallel to the breadth is cut away. What area is left ?

5. The productive acreage of England in 1921 was stated to be as follows : Arable, 10,844,000 ; Pasture, 12,698,000 ; Rough Grazing, 3,253,000. Express each as a percentage of the whole, correct to one place of decimals.

6. A rectangular park is 400 yd. long, 250 yd. broad, and has a circular pond 80 yd. in diameter exactly in the centre. How far can one walk from one corner, going in a straight line, to the opposite corner before coming to the rim of the pond ?

R 36

1. Find the length in Km. of a road 13 miles 5 fur. 176 yd. long, given 1 Km. = 0·62137 miles.

2. If $\frac{1}{8}$ of a mixture is sand, $\frac{3}{7}$ of the rest is ashes, and the remainder weighs 48 lb., find the weight of the whole.

3. The value of 2,213,487 tons of fish was estimated at £13,428,517. What was the average value per cwt. ?

4. The capital of a company, after being reduced by 15 per cent., then amounts to £208,250. What was it before reduction ?

5. Find the weight of a solid iron rod 17 ft. 6 in. long, whose cross-section is a square of side $\frac{3}{4}$ in., taking iron to weigh 450 lb. per cu. ft.

6. By what fraction must $\frac{7}{9}$ be multiplied (i) to increase it by $\frac{2}{5}$, (ii) to diminish it by $\frac{3}{7}$?

R 37

1. (i) Divide $\frac{3}{11}-\frac{1}{7}$ by $\frac{3}{22}-\frac{1}{14}$. (ii) Multiply $\frac{3}{7}+\frac{1}{5}$ by $\frac{7}{3}+5$.

2. Use tables to find correct to two significant figures the value of
$$1 \div \left[\frac{1}{(0{\cdot}1736)^2}-\frac{1}{(0{\cdot}6529)^2}\right].$$

3. Express half a kilometre in yards correct to the nearest yard, given 1 yd. = 0·9144 metres.

4. A speedometer records the pace of a car as 30 miles per hour when it actually covers 800 yd. in a minute. What will be the reading when the car is running at 30 miles per hour ?

5. A man who cannot pay his debts in full pays 16s. 8d. for every £1 that he owes. How much does he pay to a creditor to whom he owes £27, 11s. 3d. ?

6. The ends A, B of a straight piece of elastic 7 in. long are fixed ; the mid-point of the elastic is now pulled a distance of 1 in. at right angles to A, B. What additional extension is caused in the elastic ?

R 38

1. Express 10,829 in prime factors.

2. A man rents a house for the holidays at $6\frac{1}{2}$ guineas per week. What should he pay for $5\frac{1}{2}$ weeks ?

3. Express 0·0317 in the form $\frac{1}{n}$.

4. A debtor whose assets are £2697, 14s. owes £5218, 17s. 6d. How much can he pay in the £?

5. The value of an estate of 2164 acres 3 roods is estimated at £125,000. How much is that per acre, to the nearest shilling?

6. Find to the nearest $\frac{1}{10}$th of an inch the edge of a cube whose total surface area is 62·4 sq. in.

R 39

1. Simplify (i) $(5\frac{1}{6}-3\frac{1}{9}+2\frac{1}{4})\div(4\frac{3}{7}\div3\frac{1}{5})$;
(ii) $0{\cdot}2\times0{\cdot}31\times3{\cdot}68\div2{\cdot}3$.

2. Find the square root of 453·69.

3. Which is the greater ratio, 2 yd. to 1 ft. 8 in., or 2 miles to 4 fur. 5 ch.?

4. The population of England and Wales was 32,527,843 in 1901 and 36,070,492 in 1911. Find the increase per cent. correct to three significant figures.

5. To see a procession, one-third of the population of a city are seated in 250 grand-stands; 280,000 people stand in the streets; and the rest, amounting to one-fifth of the population, stay at home. What is the total population?

6. I walk for $\frac{2}{3}$ mile along a road; I then turn to the right at right angles and walk for $\frac{1}{2}$ mile; I then turn to the left at right angles and walk for 1 mile. How far am I from my starting-point in a straight line?

R 40

1. Divide 2·7648 by ·007963 correct to three significant figures.

2. I paid a bill in France for 2462 fr. 40 c. How much was that in English money when the exchange was 105 fr. 20 c. for £1?

3. An estate of 876 acres 1 rood 20 poles fetched an average price of £64, 3s. 8d. per acre. What was the price paid for it, to the nearest £?

4. Find the value of $\frac{1}{19{\cdot}6}+\frac{1}{23{\cdot}4}$.

5. The cross-section of a pipe is 9·6 sq. in., and 56 gallons pass through every minute. What is the speed of flow?

6. The external dimensions of a closed metal box are 26·2 cm., 18·4 cm., and 13·3 cm. The thickness of the metal is 2·8 mm. Find the weight of the box if 1 c.c. of the metal weighs 9·2 grams.

PROBLEM PAPERS

(P 31–40)

P 31

1. Find the value of $758^2 - 757 \times 759$.

2. A house is built by three brothers; the first pays $\frac{2}{5}$ of the cost, the second pays $\frac{7}{15}$ of the cost, and the third pays £966. What did the house cost?

3. Find two numbers which do not differ by more than 0·01, such that the square of the larger is greater than 5 and the square of the smaller is less than 5.

4. A train is timed to run 52 miles in 1 hr. 48 min. If it starts 12 min. late, at what extra average speed in m.p.h. must it run to arrive up to time?

5. The length of a fence is stated to be 68 yd. correct to the nearest yard and 62 metres correct to the nearest metre. Between what limits (in yards) must its true length lie? [1 metre = 1·0936 yd.]

P 32

1. Find the altitude of an equilateral triangle whose side is 6·4 in.

2. The average gradient of the course for a cycling contest was 1 in 8. For the first $\frac{1}{4}$ mile it was 1 in 20, and for the last 800 yd. it was 1 in 6. If the course was 2 miles long, find the average gradient of the rest of it.

3. A gas fire burns 1 cu. ft. of gas in 73 sec. If 1000 cu. ft. of gas are equivalent to 4·8 therms and the gas costs $9\frac{1}{4}$d. per therm, find the cost to the nearest £ of keeping the fire burning continuously for a year of 365 days.

4. The distance from the North Pole to the Equator, measured along a meridian, is 10^7 metres. Assuming the Earth a sphere of radius 3960 miles, find the number of cm. in 1 yd. [Circumference of circle $= 2\pi r$.]

5. A tea blender mixes 3 grades of tea, the wholesale prices of which are 2s. 1d., 2s. 3d., and 2s. 7d. per lb., in the proportions 7 : 8 : 5. What percentage profit will he make if he sells the mixture at 2s. 11d. per lb.?

P 33

1. The airship R34 travelled 3130 sea miles in 108 hours. When nearing New York it had enough petrol on board to continue for

40 minutes. How many more miles (to the nearest mile) could it travel ?

2. An examiner wishes to mark three questions in a paper in the proportion 2 : 3·7 : 4·3. The sum of the marks is to be 100. How many should he allot to each question ?

3. If 9999 is multiplied by any number less than a hundred, prove that at least two digits in the product are nines.

4. A man walks 2 miles due south, then 5 miles due east, then 4 miles due south, then 1 mile due west. How far is he from his starting-point as the crow flies ?

5. A Bryant & May's match-box is 6 cm. long, 4 cm. wide, 2 cm. deep. What total area of matchwood is needed to make the box, and the cover in which the box slides ? What is the volume of an ordinary closed cubical box which requires the same area of matchwood for its manufacture ? [Neglect the thickness of the wood.]

P 34

1. A man can spend up to £8, 9s. on coal at £2, 5s. 10d. per ton. If the dealer does not sell smaller quantities than ½ cwt., find the greatest quantity of coal the man can buy.

2. I buy a return railway ticket for £10, 8s. 9d. If the return fare is calculated as 1¾ times a single fare, find the single fare.

3. Buses start simultaneously every 15 minutes from both ends of a route AB 10 miles long, and run at an average rate of 8 m.p.h. If a man travels the whole route in one of the buses, how many does he meet *between* A and B ?

4. A family spends on an average £2, 15s. a week on food out of a total expenditure of £3, 17s. 6d. When the cost of food goes up 15 per cent., by how much per cent. will the total expenses increase if the other expenses are kept unaltered ?

5. The ratio of land to water on the whole globe is 1 : 2, but in the Northern Hemisphere it is as 2 : 3. What is it for the Southern Hemisphere ?

P 35

1. A headmaster's income is made up as follows : (i) a fixed salary of £300 per annum ; (ii) a capitation fee of £2, 10s. on each boy in the school after the first 50 (there are 82 boys) ; (iii) profits on boarders, reckoned at a total of £135 per annum. He has to pay income tax at the rate of 4s. 6d. in the £, except on the following sums: (i) the first £120 of his income; (ii) £25 for each of his two children; (iii) £14 for life insurance. What is his net income after deducting the tax?

2. One-lb. tins of galantine are sold at two prices: (i) all meat, 1s. 6d.; (ii) meat and egg, 1s. 5d. The value of the egg in a tin is 4d., what is its weight?

3. A besieged town of 10,000 inhabitants has rations enough to last for 35 days. After 5 days another 2500 refugees are admitted. If the town is now put on half rations, how long will the food last?

4. The perimeter of an isosceles right-angled triangle is 30 cm. What is the length of the hypotenuse, correct to the nearest mm.?

5. A rectangle of cardboard is 31·9 cm. long, 15·8 cm. wide. From this, rectangular strips 15·5 cm. by 2 cm. are to be cut. How many strips will be obtained if the strips are cut (i) with their long sides parallel to the long sides of the rectangle; (ii) in a perpendicular direction to (i). Find in each case the amount of cardboard left over in square cm.

P 36

1. After paying 2d. in the 1s. entertainment tax, the proceeds of a concert were £78, 16s. 8d. What were the gross takings?

2. Railway fares, after being raised 75 per cent. above pre-war level, were afterwards reduced by making a cut of $\frac{1}{7}$th of the increased fares. It was said that this was equivalent to a reduction of 25 per cent., and that the fares would then be 50 per cent. above pre-war rates. Criticise these statements.

3. In a competitive examination $\frac{1}{17}$ of the candidates failed to qualify, 45 per cent. of the rest were unsuccessful, and 176 passed. How many were examined?

4. Two hundred and forty persons guarantee the same amount to pay the expenses of a pageant. Fifteen of them fail to pay their share, twenty-five of the rest refuse to increase their guarantee, and consequently the others each pay £3, 15s. more to make up the deficiency. What was the cost of the pageant?

5. Two men run a race of 100 yd. The race is won by $9\frac{1}{11}$ yd. in distance and by 1 second in time. Find the speed of each runner.

P 37

1. A cog-wheel which has 24 teeth engages with another which has 45 teeth. The former is turning 5 times in 4 seconds. How many revolutions does the latter make in 12 seconds?

2. In a certain innings the average total scored by each member of an XI was 30; the average of the first six batsmen was 40. What was the average of the totals of each of the last five?

3. An irregular figure is drawn on a rectangular sheet of paper, 10 in. by 7 in. The paper weighs 3·29 gr. The irregular area is then cut out and the remainder of the paper weighs 2·75 gr. Find the number of square inches in the irregular area, correct to two significant figures.

4. Gas mantles cost 3½d. each, or 3s. 3d. if bought by the dozen. It is found that when bought in bulk 10 per cent. are broken, but when bought singly none are broken. If nine dozen are needed, is it better to buy them singly or in bulk? What is the difference in the total cost?

5. A legacy of £4500 is left to three people, so that after a duty of 20 per cent. has been paid their shares are in the ratio 1 : 1½ : 2. Find their shares.

P 38

1. From Liverpool to London is 201 miles, to Penzance 376 miles; from London to Penzance is 325 miles. Ordinary fares are reckoned as 1½d. per mile, but return tourist tickets are issued from Liverpool to Penzance at $\frac{11}{12}$ of the ordinary direct fare available *via* London if desired. How much will be saved by taking a tourist ticket by a man who goes *via* London on both journeys?

2. A tradesman who calculated his gain on the selling price said he gained 25 per cent. Find his true percentage gain.

3. A salvo of guns is fired from a battery and the guns are then fired at intervals of 12, 16, 21, and 70 seconds respectively. What interval will elapse before they are all fired together again?

4. By adopting a new lamp costing 3s. 4d. instead of my old ones costing 2s. 6d., I reckon to reduce my bill for electric current by 40 per cent. and thus save £15. I find, however, that (owing to breakages) I have to buy 20 per cent. more lamps than before and that I only save £10. Find the original cost of the current I used.

5. An aeroplane travelling 80 miles an hour in still air goes from A to B with the wind and then returns. If the speed of the wind is 20 m.p.h., find the average speed for the whole journey.

P 39

1. How many whole numbers are there whose cube roots lie *between* 99 and 100?

2. At the end of August 1925 it is known that Kawie Rubber Company has sold forward, for 1926, 24 tons of rubber at 2s. 3d. per lb., 24 tons at 2s. 5d. per lb., 24 tons at 2s. $7\frac{1}{4}$d. per lb. At the end of November it is stated that the total forward sales amount to 108 tons at an average price of 2s. $8\frac{1}{4}$d. per lb. What was the average price at which the rubber was sold between the two dates?

3. The perimeter of a square enclosure is given as 122 ft., and the area is calculated from this statement. Show that the maximum error in the result obtained for the area is less than $7\frac{2}{3}$ sq. ft. if the given measurement is correct to the nearest foot.

4. A chain is composed of 80 circular links. Each link is 5 in. in diameter externally and is made of material 1 in. thick. What is the total length of the chain when taut?

5. The incomes of A and B are in the ratio 4 : 3; their expenditures are in the ratio 25 : 16; and their savings in the ratio 7 : 8. Find the ratio of A's income to B's expenditure.

P 40

1. Express as a decimal the ratio of the altitude of an equilateral triangle to one of the sides.

2. A publisher prints 5000 copies of a book to be sold at 7s. 6d. a copy net: the cost per copy of paper and printing is 1s. 3d., and of binding 8d.: carriage, etc., costs £55 in all. He sells them to a bookseller, charging 25 copies as 24, at 30 per cent. less than the published price, and takes 25 per cent. of the total receipts as his commission. Find the respective gains of the author, publisher, and bookseller.

3. A man's capital on January 1, 1922, was £4375, and on January 1, 1924, it was £4368. The increase per cent. during 1922 was equal to the decrease per cent. in 1923. What was his capital on January 1, 1923?

4. How much water must be added to a cask containing 70 gallons of spirit worth 13s. 4d. a gallon to reduce its value to 11s. 8d. a gallon?

5. In a regiment consisting of English, Irish, and Scotch, 40 per cent. were Irish and 30 per cent. Scotch. After 200 Irish and 320 Scotch were added, then 22 per cent. were English. What was the original strength of the regiment?

CHAPTER XVII

INDICES AND LOGARITHMS

Historical Note.—Logarithms were invented by John Napier of Merchiston, who was born in 1550. In his day the art of computation was in such a backward state that his invention of Napier's bones was hailed as a wonderful achievement, although they merely dispensed with the necessity of learning the multiplication table! Tables of the trigonometrical ratios were, however, widely used by scientific men, and the labour involved in using them led Napier to invent some method by which the operations of multiplication and division could be replaced by addition and subtraction. The method he devised was totally different from that now in use, for it must be remembered that in his day the idea of writing a^2 for $a \times a$ had not been suggested. He imagines a point P to move along one line AB with uniform speed, while a point Q moves along another line CD, starting with the same speed as P, but reducing its speed proportionally to its distance from D; QD represented the number whose logarithm was given by AP when both P and Q had travelled for the same time.

The theory (as opposed to the practice) of logarithms is usually regarded as belonging to the algebra course; the present chapter contains no more than is necessary to make the process intelligible. It is suggested that as a preliminary the work on pp. 86–87 (Part II) should be revised and some of the following examples taken orally, Exercise VII (*b*), Nos. 1–27, 48–53, 57–59.

The following results exemplify the rules of indices:

$$a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a^5.$$

$$\frac{a^3}{a^2} = \frac{a \times a \times a}{a \times a} = a.$$

$$(a^3)^2 = (a \times a \times a) \times (a \times a \times a) = a^6.$$

These are generalised into the following statements:

$$a^m \times a^n = a^{m+n}.$$

$$\frac{a^m}{a^n} = a^{m-n}.$$

$$(a^m)^n = a^{mn}.$$

Whatever meaning we attach to such expressions as $a^{\frac{1}{2}}$, $a^{\frac{1}{3}}$, $a^{\frac{2}{3}}$, it is obviously convenient that they shall obey the same rules as when the indices are whole numbers. We therefore agree that in *all* cases $a^m \times a^n = a^{m+n}$.

Then $a^{\frac{1}{2}}\times a^{\frac{1}{2}}=a^1$. $\therefore a^{\frac{1}{2}}=\sqrt{a}$.

Similarly $a^{\frac{1}{3}}=\sqrt[3]{a}$ and $a^{\frac{1}{4}}=\sqrt[4]{a}$.

Also $a^{\frac{2}{3}}\times a^{\frac{2}{3}}\times a^{\frac{2}{3}}=a^2$ (add the indices).

$$\therefore a^{\frac{2}{3}}=\sqrt[3]{a^2}.$$

Thus $\sqrt{x^8}=(x^8)^{\frac{1}{2}}=x^4$ (multiply the indices).

EXERCISE XVII (a) (Oral)

1. State in full the proof that (i) $x^2\times x^3=x^5$; (ii) $y^5\div y^3=y^2$; (iii) $(x^2)^3=x^6$; (iv) a cube root of y^6 is y^2.

2. State in their simplest forms:

(i) $x^3\times x^4$. (ii) $x^6\div x^2$. (iii) $x\times x^5$. (iv) $x^8\div x$.
(v) $(x^3)^2$. (vi) $x^{29}\times x^4$. (vii) $x^{20}\div x^4$. (viii) $(x^4)^3$.

3. In what general statements are the following special cases included?

(i) $y^3\times y^7=y^{10}$; $y^2\times y^4=y^6$; $y^{10}\times y^{20}=y^{30}$.
(ii) $y^7\div y^3=y^4$; $y^6\div y^3=y^3$; $y^{40}\div y^{10}=y^{30}$.
(iii) $(x^2)^3=x^6$; $(x^5)^4=x^{20}$; $(x^{12})^7=x^{84}$.

4. State in their simplest forms:

(i) $\sqrt{x^{16}}$. (ii) $\sqrt{x^{12}}$. (iii) $\sqrt[3]{x^{27}}$. (iv) $\sqrt[3]{x^{12}}$.
(v) $\sqrt[4]{x^{16}}$. (vi) $\sqrt[4]{x^{12}}$. (vii) $\sqrt{(16x^{16}y^8)}$. (viii) $\sqrt[3]{(27a^{27}b^9)}$.

5. Express as powers of 10, *i.e.* in the form 10^n:

(i) $10^4\div 10$; (ii) $10^4\div 10^2$; (iii) $10^4\div 10^3$; (iv) $10^4\div 10^4$.

What are the values of these expressions?

6. Use the relation $x^m\times x^n=x^{m+n}$ to simplify

(i) $10^{-2}\times 10^3$; (ii) $2^{-3}\times 2^5$; (iii) $10^{-3}\times 10^3$; (iv) $9^{\frac{1}{2}}\times 9^{\frac{1}{2}}$; (v) $8^{\frac{1}{3}}\times 8^{\frac{1}{3}}\times 8^{\frac{1}{3}}$.

What are the values of $9^{\frac{1}{2}}$, $8^{\frac{1}{3}}$?

7. What is the value of n if

(i) $127=10^n\times 1{\cdot}27$; (ii) $6090=10^n\times 6{\cdot}09$; (iii) $986200=10^n\times 9{\cdot}862$.

8. Simplify $10^{\frac{1}{2}}\times 10^{\frac{1}{2}}$ and show that $10^{\frac{1}{2}}\simeq 3{\cdot}16$.

9. Simplify $10^{\frac{1}{4}}\times 10^{\frac{1}{4}}\times 10^{\frac{1}{4}}\times 10^{\frac{1}{4}}$ and show that $10^{\frac{1}{4}}\simeq 1{\cdot}78$.

10. Show that $10^{\frac{3}{4}}\simeq 5{\cdot}62$.

11. Express as a power of 10:

(i) $\sqrt{10}$. (ii) $\sqrt[3]{10}$. (iii) $\sqrt[3]{100}$. (iv) $\sqrt{1000}$.
(v) $\sqrt[4]{10}$. (vi) $\sqrt[4]{100}$. (vii) $\sqrt[4]{1000}$. (viii) $\sqrt[5]{1000}$.
(ix) $100\sqrt{10}$. (x) $\sqrt{10}\times\sqrt[3]{10}$.

12. Express as a power of 64:

(i) 8. (ii) 2. (iii) 4. (iv) 128. (v) 1. (vi) 256.

13. Show how to calculate the approximate values of 10^x given in the following table:

$x=0$	0·125	0·25	0·375	0·5	0·625	0·75	0·875	1
$10^x=1$	1·33	1·78	2·37	3·16	4·22	5·62	7·50	10

Draw the graph of 10^x from $x=0$ to $x=1$, and use it to read off the values

(i) of $10^{\cdot 2}$, $10^{\cdot 4}$, $10^{\cdot 7}$, $10^{\cdot 9}$;
(ii) of x for which $10^x=3$, $10^x=5$, $10^x=8{\cdot}1$;
(iii) of $\sqrt[5]{10}$, $\sqrt[3]{100}$, $\sqrt[5]{1000}$.

Use the graph to express as powers of 10: (*a*) 2, (*b*) 3·5, (*c*) 7.

14. Given that $2 \bumpeq 10^{\cdot 301}$ and $3 \bumpeq 10^{\cdot 477}$,

(*a*) Express as powers of 10:

(i) 20. (ii) 2000. (iii) 300. (iv) 2 × 3. (v) 12. (vi) $\sqrt{2}$.

(*b*) Write down the approximate values of

(i) $10^{2\cdot 301}$. (ii) $10^{3\cdot 477}$. (iii) $10^{\cdot 301+\cdot 477}$. (iv) $10^4\times 10^{\cdot 301}$.
(v) $10^{\cdot 301+\cdot 477}$. (vi) $10^{1-\cdot 301}$. (vii) $10^{\frac{\cdot 477}{2}}$.

15. What is the whole number in the index when each of the following numbers is expressed as a power of 10?

(i) 25·17. (ii) 973·4. (iii) 861072. (iv) 34920.
(v) 7·82. (vi) $6{\cdot}91\times 10^4$. (vii) 99·9.

16. Which is the greater,

(i) $10^{1\cdot 01}$ or 9·9; (ii) $10^{\cdot 7}$ or 11; (iii) $10^{4\cdot 8}$ or 8210?
(iv) $10^{1\cdot 4}$ or $\sqrt{1000}$; (v) $10^{\cdot 7}$ or $\sqrt[3]{100}$?

17. Express as powers of 10:

(i) $10^2\times 10^{\cdot 2}$. (ii) $10^2\div 10^{\cdot 2}$. (iii) $10^{\cdot 2}\times 10^{\cdot 3}$.
(iv) $10^{\cdot 35}\times 10^{\frac{1}{2}}$. (v) $\sqrt[3]{1000}$. (vi) $\sqrt{10^{\cdot 6}}$.
(vii) $\sqrt{10^{\cdot 16}}$. (viii) $\sqrt[3]{10^{\cdot 6}}$. (ix) $10^{1\cdot 3}\times 10^{2\cdot 7}$.
(x) $10^{2\cdot 4}\div 10^{1\cdot 2}$. (xi) $10^{\cdot 6}\times 10^{\cdot 42}\div 10^{\cdot 53}$. (xii) $10\sqrt{10}\div\sqrt[3]{10}$.

If a number is expressed as a power of 10, the index is called the *logarithm* of the number to *base* 10; *e.g.* since $178=10^{2\cdot25}$, the logarithm of 178 to base 10 is 2·25, and this is written $\log_{10} 178=2\cdot25$, or more shortly $\log 178=2\cdot25$, when there is no doubt what the base is. In arithmetical computation, the use of 10 as base is invariable.

The integral part of a logarithm is called its *characteristic*, and the figures in a logarithm after the decimal point are called its *mantissa*: thus for $\log 178=2\cdot25$, the characteristic is 2 and the mantissa is ·25. The tables contain merely the mantissa.

The characteristic or integral part of a logarithm may be found by an inspection of the following group of numbers: $10{,}000=10^4$, $1000=10^3$, $100=10^2$, $10=10^1$. Since 624 is >100 and <1000, its logarithm is greater than 2 and less than 3. The mantissa will be found in the tables to be ·7952. $\therefore\ 624=10^{2\cdot7952}$.

Now $6\cdot24=\dfrac{624}{100}=\dfrac{10^{2\cdot7952}}{10^2}=10^{0\cdot7952}$. Similarly, any number between 0 and 10 will have 0 for the characteristic of the logarithm.

It is also clear that moving the decimal point in a number will change the characteristic, but will make no difference to the mantissa, *e.g.* $6240=624\times10=10^{2\cdot7952}\times10^1=10^{3\cdot7952}$. If four-figure tables are used, it is impossible to rely on the accuracy of the fourth figure, because the difference columns can only give average differences; moreover, as the result of a series of additions, subtractions, etc., it is possible that the neglect of the figure in the fifth place in the working may affect the figure in the fourth place in the answer.

EXERCISE XVII (b)

Use four-figure tables to express as powers of 10:

1. 4. **2.** 40. **3.** 400. **4.** 40000. **5.** 720.
6. 72. **7.** 7·2. **8.** 72000. **9.** 723. **10.** 7·23.
11. 72·3. **12.** 7230. **13.** 72 35. **14.** 7·235. **15.** 7235.
16. 72350. **17.** 60. **18.** 382. **19.** 93·4. **20.** 2·97.
21. 3010. **22.** 27·88. **23.** 160·8. **24.** 400·8. **25.** 4·362.
26. 397300. **27.** 6053. **28.** 78·04. **29.** 1062. **30.** 100·6.
31. 52070. **32.** 80·07. **33.** 900·7. **34.** 20050. **35.** 7·008.

Use four-figure tables (*not* the anti-logarithm tables) to find the values of

36. $10^{0\cdot3424}$. 37. $10^{1\cdot3424}$. 38. $10^{2\cdot3424}$. 39. $10^{4\cdot3424}$. 40. $10^{1\cdot7782}$.
41. $10^{0\cdot7782}$. 42. $10^{3\cdot7782}$. 43. $10^{4\cdot7782}$. 44. $10^{0\cdot3118}$. 45. $10^{1\cdot9074}$.
46. $10^{1\cdot549}$. 47. $10^{\cdot549}$. 48. $10^{\cdot0549}$. 49. $10^{2\cdot882}$. 50. $10^{1\cdot882}$.
51. $10^{0\cdot882}$. 52. $10^{0\cdot0882}$. 53. $10^{3\cdot7582}$. 54. $10^{1\cdot705}$. 55. $10^{0\cdot0864}$.
56. $10^{2\cdot936}$. 57. $10^{1\cdot6561}$. 58. $10^{0\cdot8}$. 59. $10^{1\cdot989}$. 60. $10^{5\cdot8261}$.

61. $10^{2{\cdot}6021}$. 62. $10^{4{\cdot}959}$. 63. $10^{0{\cdot}4853}$. 64. $10^{0{\cdot}0485}$. 65. $10^{2{\cdot}6079}$.
66. $10^{1{\cdot}2911}$. 67. $10^{{\cdot}4772}$. 68. $10^{1{\cdot}2657}$. 69. $10^{4{\cdot}4781}$. 70. $10^{2{\cdot}3108}$.
71. $10^{0{\cdot}0876}$. 72. $10^{0{\cdot}3}$. 73. $10^{0{\cdot}03}$. 74. $10^{3{\cdot}0}$. 75. $10^{0{\cdot}003}$.

The pupil is advised to follow the method set out in the illustrative examples given below, at any rate at the start. Any necessary simplification should be done in special columns at the side of the page, the process being explained by the use of the index notation in the middle of the page. When sufficient skill and experience have been acquired, these explanatory steps may be omitted, and the middle of the page will then contain little more than the question and the answer. But the work in the side columns must be kept neat and logarithms must not be allowed to straggle unattended into the middle of the page.

Example.—Evaluate correct to 3 significant figures $3{\cdot}142\times41{\cdot}27$.

From the tables, $3{\cdot}142\times41{\cdot}27=10^{0{\cdot}4972}\times10^{1{\cdot}6156}$

$=10^{0{\cdot}4972+1{\cdot}6156}=10^{2{\cdot}1128}$

$=129{\cdot}6$

$=130$ to 3 significant figures.

Rough Check.
$3\times40=120$.

$$\begin{array}{r}1{\cdot}6156\\ \underline{{\cdot}4972}\\ \underline{\underline{2{\cdot}1128}}\end{array}$$

Example.—Evaluate correct to 3 significant figures $407{\cdot}9\div14{\cdot}56$.

From the tables, $407{\cdot}9\div14{\cdot}56=10^{2{\cdot}6106}\div10^{1{\cdot}1632}$

$=10^{2{\cdot}6106-1{\cdot}1632}=10^{1{\cdot}4474}$

$=28{\cdot}02$

$=28{\cdot}0$ to 3 significant figures.

Rough Check.
$400\div14\risingdotseq30$.

$$\begin{array}{r}2{\cdot}6106\\ \underline{1{\cdot}1632}\\ \underline{\underline{1{\cdot}4474}}\end{array}$$

Example.—Evaluate $(4{\cdot}172)^5$.

$(4{\cdot}172)^5=10^{0{\cdot}6203\times5}=10^{3{\cdot}1015}$

$=1263$

$=1260$ to 3 significant figures.

$$\begin{array}{r}0{\cdot}6203\\ \underline{5}\\ \underline{\underline{3{\cdot}1015}}\end{array}$$

Rough Check.—$4^5=1024$.

Example.—Evaluate $\sqrt[3]{152{\cdot}7}$.

$\sqrt[3]{152{\cdot}7}=(152{\cdot}7)^{\frac{1}{3}}=10^{2{\cdot}1838\times\frac{1}{3}}$

$=10^{0{\cdot}7279}=5{\cdot}344$

$=5{\cdot}34$ to 3 significant figures.

$$\begin{array}{r|l}3 & 2{\cdot}1838\\ \hline & \underline{\underline{0{\cdot}7279}}\end{array}$$

Rough Check.—$5^3=125$.

Note.—The practice of testing the answer by a rough check should be encouraged. And in the first few examples of the next exercise the result should be compared with that obtained by ordinary multiplication or division.

EXERCISE XVII (c) (Positive Characteristics)

Find correct to 3 significant figures the values of the following:

1. $6{\cdot}17 \times 3{\cdot}42$.
2. $74{\cdot}6 \times 29{\cdot}3$.
3. $378 \div 4{\cdot}82$.
4. $849 \div 97{\cdot}5$.
5. 684×719.
6. $98200 \div 64{\cdot}7$.
7. $35{\cdot}62 \times 7{\cdot}934$.
8. $1607 \times 5{\cdot}638$.
9. $8074 \div 29{\cdot}08$.
10. $70{\cdot}08 \div 11{\cdot}06$.
11. $(86{\cdot}37)^2$.
12. $(7{\cdot}836)^3$.
13. $8439 \times 39{\cdot}62$.
14. $5{\cdot}738 \times 116$.
15. $153{\cdot}8 \div 99{\cdot}06$.
16. 74720×1650.
17. $88360 \div 68{\cdot}62$.
18. $19{\cdot}82 \div 18{\cdot}93$.
19. $3{\cdot}009 \times 87{\cdot}64$.
20. $559{\cdot}6 \div 97{\cdot}24$.
21. $84390 \div 600{\cdot}4$.
22. $852{\cdot}9 \div 9{\cdot}7036$.
23. $300{\cdot}7 \div 30{\cdot}708$.
24. $3909 \times 7150{\cdot}32$.
25. $(60{\cdot}07)^3$.
26. $(8{\cdot}104)^4$.
27. $\sqrt[3]{189}$.
28. $\sqrt[3]{18{\cdot}9}$.
29. $\sqrt[3]{1{\cdot}89}$.
30. $(1{\cdot}075)^6$.
31. $\sqrt[4]{16{\cdot}87}$.
32. $\sqrt[3]{(64{\cdot}73)^2}$.
33. $\sqrt[5]{1000}$.
34. $(73{\cdot}192)^2$.
35. $(731{\cdot}92)^2$.
36. $\sqrt[4]{(10{\cdot}07)^3}$.

Example.—Evaluate $\dfrac{31{\cdot}7 \times 920000}{415{\cdot}1 \times 1{\cdot}6}$.

The expression equals

$$\frac{10^{1\cdot5011} \times 10^{5\cdot9638}}{10^{2\cdot6181} \times 10^{0\cdot2041}} = \frac{10^{7\cdot4649}}{10^{2\cdot8222}}$$

$$= 10^{4\cdot6427} = 43{,}920$$

$= 43{,}900$ to 3 significant figures.

$$\begin{array}{r|r} 1{\cdot}5011 & 2{\cdot}6181 \\ 5{\cdot}9638 & 0{\cdot}2041 \\ \hline 7{\cdot}4649 & 2{\cdot}8222 \\ 2{\cdot}8222 & \\ \hline \underline{4{\cdot}6427} & \end{array}$$

Rough check.

$$\frac{30 \times 900{,}000}{400 \times 2} = \frac{27 \times 10^6}{8 \times 10^2}$$

$$\simeq 30{,}000.$$

Note.—In dealing with complicated expressions it is best to make two side-columns, as above, one for logarithms of factors of the numerator and the other for those of the denominator.

EXERCISE XVII (d) (Positive Characteristics)

Find correct to 3 significant figures the values of the following:

1. $11{\cdot}83 \times 4{\cdot}072 \times 3{\cdot}681$.
2. $7{\cdot}935 \times 6{\cdot}804 \times 3{\cdot}07$.
3. $2{\cdot}947 \times 37{\cdot}04 \times 5{\cdot}906$.
4. $17{\cdot}7 \times 20{\cdot}06 \times 3{\cdot}028$.
5. $6{\cdot}726 \times 12{\cdot}914 \times 3{\cdot}227$.
6. $800{\cdot}7 \times 2{\cdot}935 \times 73{\cdot}04$.
7. $1{\cdot}07 \times 6{\cdot}93 \times 12{\cdot}8 \times 139$.
8. $16{\cdot}03 \times 7{\cdot}081 \times 39 \times 60{\cdot}02$.
9. $\dfrac{23{\cdot}7 \times 158}{84{\cdot}3}$.
10. $\dfrac{10{\cdot}08 \times 8329}{764{\cdot}3}$.
11. $\dfrac{393{\cdot}1 \times 6{\cdot}084}{23{\cdot}22 \times 47{\cdot}71}$.

12. $\dfrac{8392 \times 2974}{5681 \times 3722}$.
13. $\dfrac{687}{19\cdot06 \times 7\cdot992}$.
14. $\dfrac{100}{6\cdot724 \times 8\cdot073}$.

15. $\dfrac{10}{3\cdot126 \times 1\cdot872 \times 1\cdot107}$.
16. $\dfrac{(61\cdot09)^2}{38\cdot15 \times 57\cdot06}$.
17. $\dfrac{(3\cdot072)^3}{(4\cdot61)^2}$.

18. $\dfrac{300\cdot7 \times 70\cdot06}{(10\cdot92)^3}$.
19. $\dfrac{(7\cdot064)^2 \times 118}{(3\cdot63)^3 \times (6\cdot106)^2}$.
20. $\dfrac{(3\cdot71)^2 \times 1000}{(6\cdot072)^4}$.

21. $\left(\dfrac{3\cdot082 \times 270}{41\cdot03 \times 16}\right)^2$.
22. $\dfrac{13\frac{1}{4} \times 17\frac{2}{3}}{11\cdot27 \times 1\frac{3}{7}}$.
23. $\sqrt{\left(\dfrac{86\cdot04}{7\cdot63}\right)}$.

24. $\left(\dfrac{61\cdot42 \times 185}{3\frac{1}{4} \times 811}\right)^3$.
25. $(13\frac{2}{3})^2 \div (4\cdot372)^3$.
26. $\sqrt[3]{\left(\dfrac{9\cdot076}{7\cdot962}\right)}$.

27. $\dfrac{5\sqrt{171}}{6\cdot734 \times 7\cdot098}$.
28. $\sqrt{\left(\dfrac{11\cdot07 \times 6\cdot944}{5\cdot137 \times 8\cdot096}\right)}$.
29. $\dfrac{3\cdot07 \times 10^5 \times \sqrt{187}}{62\cdot34 \times 78\cdot88}$.

30. $\sqrt[3]{\left(\dfrac{7\cdot321 \times 1\cdot062}{2\cdot451 \times 2\cdot084}\right)}$.
31. $\dfrac{\sqrt[3]{100}}{1\cdot097 \times \sqrt{5\cdot16}}$.
32. $\dfrac{73\frac{1}{4}}{2\frac{1}{2} \times \sqrt[3]{100}}$.

33. $3\cdot82 \times 7\cdot16 + 11\cdot05 \times 2\cdot11$.
34. $173 \div 2\cdot81 - 639 \div 13\cdot7$.

35. $60\cdot72 \times 11\cdot43 - 7\cdot804 \times 9\cdot137$.
36. $(8\cdot074)^2 + 13\cdot37 \times 2\cdot871 \times 1\cdot473$.

Negative Indices.

10,000	10^4	This Table shows that the index is diminished by 1 each time the number on the left is divided by 10: it is therefore reasonable to assume we may continue in the same way, as follows:—
1,000	10^3	
100	10^2	
10	10^1	
1	—	$1 = 10^0$
$\frac{1}{10}$	—	$0\cdot1 = \frac{1}{10} = 10^{-1}$
$\frac{1}{100}$	—	$\cdot01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$
$\frac{1}{1000}$	—	$\cdot001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$.

If we agree that when the index is zero or negative, the relation $a^m \times a^n = a^{m+n}$ is still to hold good, then $a^0 \times a^2 = a^{0+2} = a^2$.

$$\therefore\ a^0 = 1.$$

Also $$a^{-2} \times a^{2} = a^{-2+2} = a^{0} = 1.$$

$$\therefore\ a^{-2} = \frac{1}{a^{2}}.$$

This justifies the statement above that $10^{0} = 1$ and $10^{-2} = \frac{1}{10^{2}}$, for a may stand for any number.

Tabulating our results, we have

$$10^{4} = 10{,}000\ ;\ 10^{3} = 1000\ ;\ 10^{2} = 100\ ;\ 10^{1} = 10\ ;\ 10^{0} = 1\ ;$$

$$10^{-1} = \frac{1}{10} = \cdot 1\ ;\ 10^{-2} = \frac{1}{10^{2}} = \cdot 01\ ;\ 10^{-3} = \frac{1}{10^{3}} = \cdot 001.$$

From the tables we have $6{\cdot}243 = 10^{0{\cdot}7954}$.

$$\therefore\ 0{\cdot}6243 = \frac{6{\cdot}243}{10} = \frac{10^{0{\cdot}7954}}{10^{1}}.$$

If we now subtract the indices we get $10^{-0{\cdot}2046}$ and so obtain a negative mantissa with different digits. This is avoided by writing $\frac{10^{0{\cdot}7954}}{10^{1}} = 10^{\bar{1}{\cdot}7954}$ where the logarithm is $-1 + {\cdot}7954$, but is written with the minus sign *over* the integer, to which it alone refers. The logarithm is read "bar one point 7954."

Similarly $$\cdot 06243 = \frac{6{\cdot}243}{100} = \frac{10^{0{\cdot}795}}{10^{2}} = 10^{\bar{2}{\cdot}7954}.$$

An inspection of the table on p. 218 shows that ·06243, which is between ·01, and ·1, will lie between 10^{-2} and 10^{-1}; its logarithm is therefore on the positive side of -2 and will be represented by -2 + a positive decimal.

Example.—Evaluate $\frac{18{\cdot}61}{0{\cdot}0451}$.

$$\frac{18{\cdot}61}{0{\cdot}0451} = \frac{10^{1{\cdot}2697}}{10^{\bar{2}{\cdot}6542}} = 10^{2{\cdot}6155}$$
$$= 412{\cdot}6.$$

$$\begin{array}{r} 1 + {\cdot}2697 \\ -2 + {\cdot}6542 \\ \hline 2 + {\cdot}6155 \\ \hline \end{array}$$

Note.—(i) At first it is best to write out the work in full, as shown in the columns at the side.

(ii) If there is any difficulty in subtracting the integers, use the rule for subtraction: change the sign of the lower line and add.

Example.—Evaluate $\sqrt[5]{0{\cdot}0736}$.

$$\begin{aligned}\sqrt[5]{0{\cdot}0736}&=(\ 0736)^{\frac{1}{5}}\\&=10^{\bar{2}{\cdot}8669\times\frac{1}{5}}\\&=10^{\bar{1}{\cdot}7734}\\&=0{\cdot}593.\end{aligned}$$

$$\begin{array}{r|r}&-2+\ {\cdot}8669\\5&\underline{-5+3{\cdot}8669}\\&\underline{-1+\ {\cdot}7734}\end{array}$$

Note.—In order to divide $\bar{2}{\cdot}8669$ by 5, we write the logarithm so that the negative characteristic is the simplest multiple of 5, thus $\bar{2}{\cdot}8669=\bar{5}+3{\cdot}8669$.

Example.—Evaluate $(0{\cdot}08726)^3$.

$$\begin{aligned}(0{\cdot}08726)^3&=10^{\bar{2}{\cdot}9408\times3}=10^{\bar{4}{\cdot}8224}\\&=0{\cdot}000664.\end{aligned}$$

$$\begin{array}{r}-2+\ {\cdot}9408\\\underline{3}\\-6+2{\cdot}8224\\-4+\ {\cdot}8224\end{array}$$

Note.—In the early stages of the work, the characteristic should be determined by inspection; later on, the process may be summarised in the following rule: count the number of figures from the first significant figure to the digit in the unit place, positive for numbers greater than 1 and negative for numbers less than 1; the result is the characteristic.

EXERCISE XVII (e) (Oral)

Write the answer so that the fractional part is positive, thus write $-2{\cdot}3$ as $-3+{\cdot}7$ or $\bar{3}{\cdot}7$.

Simplify

1. $2{\cdot}4+\bar{1}{\cdot}3$.
2. $3{\cdot}7+\bar{1}{\cdot}8$.
3. $2{\cdot}6+\bar{2}{\cdot}4$.
4. $\bar{2}{\cdot}3+\bar{3}{\cdot}5$.
5. $\bar{2}{\cdot}7+\bar{3}{\cdot}8$.
6. $\bar{5}{\cdot}7+2{\cdot}8$.
7. $2{\cdot}6-\bar{1}{\cdot}2$.
8. $2{\cdot}4-\bar{1}{\cdot}7$.
9. $\bar{3}{\cdot}6-1{\cdot}2$.
10. $\bar{3}{\cdot}3-1{\cdot}7$.
11. $2{\cdot}6-2{\cdot}8$.
12. $2{\cdot}4-4{\cdot}1$.
13. $2{\cdot}4-5{\cdot}7$.
14. $\bar{3}{\cdot}5-\bar{4}{\cdot}8$.
15. $\bar{2}{\cdot}3\times2$.
16. $\bar{2}{\cdot}7\times2$.
17. $\bar{2}{\cdot}9\times4$.
18. $2{\cdot}4\times(-3)$.
19. $1{\cdot}7\times(-5)$.
20. $\bar{1}{\cdot}7\times(-4)$.
21. $\bar{2}{\cdot}8\div2$.
22. $\bar{3}{\cdot}8\div2$.
23. $\bar{1}{\cdot}4\div3$.
24. $\bar{5}{\cdot}8\div3$.
25. $\bar{2}{\cdot}4\div5$.
26. $\bar{3}\div10$.
27. $\bar{2}{\cdot}2\div6$.
28. $1{\cdot}2\div(-2)$.
29. $\bar{1}{\cdot}6\div4$.
30. $\frac{2}{3}$ of $\bar{4}{\cdot}7$.

EXERCISE XVII (f)

Find correct to three significant figures the values of the following:

1. $0{\cdot}861\times1{\cdot}243$.
2. $0{\cdot}786\times6{\cdot}934$.
3. $52{\cdot}87\times0{\cdot}0962$.
4. $0{\cdot}437\times0{\cdot}2071$.
5. $0{\cdot}868\times0{\cdot}927$.
6. $0{\cdot}0862\times0{\cdot}00934$.

7. $0{\cdot}008263 \times 69{\cdot}31$. 8. $635 \div 0{\cdot}286$. 9. $635 \div 0{\cdot}886$.
10. $0{\cdot}004172 \div 78{\cdot}6$. 11. $129{\cdot}3 \div 362{\cdot}7$. 12. $86{\cdot}09 \div 9872$.
13. ${\cdot}0524 \div {\cdot}00863$. 14. $({\cdot}0721)^2$. 15. $({\cdot}9316)^3$.
16. $\sqrt{{\cdot}8623}$. 17. $\sqrt{{\cdot}08623}$. 18. $\sqrt[3]{{\cdot}4771}$. 19. $\sqrt[3]{{\cdot}04771}$.
20. $\sqrt[3]{{\cdot}004771}$. 21. $({\cdot}8)^5$. 22. $\sqrt{{\cdot}007324}$. 23. $\sqrt[4]{{\cdot}09627}$.
24. $\sqrt[3]{{\cdot}03215}$. 25. $({\cdot}8817)^{\frac{3}{4}}$. 26. $\sqrt[3]{({\cdot}7269)^2}$. 27. $(0{\cdot}8791)^{\frac{3}{2}}$.
28. $({\cdot}00571)^{\frac{5}{3}}$. 29. $({\cdot}0826)^{\frac{1}{5}}$. 30. $({\cdot}000084)^{\frac{2}{7}}$. 31. $\sqrt{\left(\frac{{\cdot}872}{31{\cdot}65}\right)}$.
32. $\sqrt[3]{\left(\frac{47{\cdot}32}{{\cdot}6918}\right)}$. 33. $\frac{1}{\sqrt{18{\cdot}93}}$. 34. $\frac{10}{\sqrt{{\cdot}7623}}$. 35. $\left(\frac{{\cdot}007254}{{\cdot}1083}\right)^{\frac{1}{2}}$.
36. $\sqrt[3]{\left(\frac{1}{197{\cdot}4}\right)}$.

Example.—Find the value of $({\cdot}014)^{1{\cdot}4}$.

$$\begin{aligned}({\cdot}014)^{1\cdot4} &= (10^{\bar{2}\cdot1461})^{1\cdot4}\\ &= 10^{\bar{3}\cdot4045}\\ &= \underline{{\cdot}00254}.\end{aligned}$$

$$\begin{aligned}\bar{2} \times 1{\cdot}4 &= -2{\cdot}8 = \bar{3}{\cdot}2\\ {\cdot}1461 \times 1{\cdot}4 &= +{\cdot}2045.\end{aligned}$$

Example.—Given $(0{\cdot}62)^x = 0{\cdot}312$, *find* x.

Now $0{\cdot}62^x = (10^{\bar{1}\cdot7924})^x$; $0{\cdot}312 = 10^{\bar{1}\cdot4942}$.

$$\begin{aligned}\therefore\ 10^{\bar{1}\cdot7924x} &= 10^{\bar{1}\cdot4942}.\\ \therefore\ \bar{1}{\cdot}7924x &= \bar{1}{\cdot}4942.\\ \therefore\ x &= \frac{-{\cdot}5058}{-{\cdot}2076} = \frac{{\cdot}5058}{{\cdot}2076} = \frac{10^{\bar{1}\cdot7040}}{10^{\bar{1}\cdot3173}}\\ &= 10^{\cdot3867}\\ &= \underline{2{\cdot}44}.\end{aligned}$$

EXERCISE XVII (g)

Find correct to three significant figures the values of the following :

1. $\frac{3{\cdot}972}{27{\cdot}24 \times 16{\cdot}38}$. 2. $\frac{4{\cdot}861 \times 17{\cdot}93}{6374}$. 3. $\frac{51{\cdot}94 \times {\cdot}8723}{129{\cdot}6}$.
4. $\frac{16{\cdot}05}{8379 \times {\cdot}0183}$. 5. $\frac{({\cdot}8629)^2}{163 \times {\cdot}94}$. 6. $\frac{{\cdot}005271 \times {\cdot}04896}{{\cdot}000087}$.
7. $\frac{11{\cdot}07 \times 90{\cdot}08}{127{\cdot}3 \times 66{\cdot}46}$. 8. $\frac{(18{\cdot}37)^3}{999 \times 18{\cdot}07}$. 9. $\frac{(3{\cdot}612)^3}{861 \times {\cdot}07245}$.

10. $\dfrac{63{\cdot}9\times47{\cdot}4\times75{\cdot}8}{869\times{\cdot}084\times{\cdot}77}$.

11. $\dfrac{86{\cdot}37\times{\cdot}08241\times{\cdot}0069}{527{\cdot}1\times4{\cdot}63\times8{\cdot}008}$.

12. $\dfrac{({\cdot}671)^3\times({\cdot}046)^2}{19{\cdot}37\times12{\cdot}73}$.

13. $\dfrac{72{\cdot}94\times\sqrt{{\cdot}1873}}{{\cdot}8644\times{\cdot}03251}$.

14. $\dfrac{1}{({\cdot}0885)^2\times({\cdot}777)^3}$.

15. $\dfrac{6\frac{3}{4}}{5\frac{1}{7}\times\sqrt{{\cdot}3714}}$.

16. $\sqrt{\left(\dfrac{{\cdot}8666\times{\cdot}0977}{35{\cdot}54\times4{\cdot}082}\right)}$.

17. $\sqrt{\left(\dfrac{18{\cdot}33\times{\cdot}6211}{{\cdot}0838\times{\cdot}7724}\right)}$

18. $\sqrt[3]{\left(\dfrac{4{\cdot}156\times{\cdot}00612}{{\cdot}0891}\right)}$.

19. $\dfrac{(16{\cdot}7)^{\frac{1}{2}}\times({\cdot}872)^{\frac{2}{3}}}{1007}$

20. $\dfrac{({\cdot}6624)^{\frac{3}{4}}\times{\cdot}0872}{({\cdot}3725)^{\frac{1}{3}}}$.

21. $\dfrac{({\cdot}076)^3\times\sqrt{{\cdot}8837}}{8\frac{2}{3}\times\sqrt[3]{{\cdot}7872}}$.

22. $\dfrac{3{\cdot}142\times({\cdot}8723)^2\times70{\cdot}06}{({\cdot}0561)^3\times994}$.

23. $\dfrac{\sqrt{1{\cdot}1}\times\sqrt{{\cdot}14}\times\sqrt{{\cdot}062}}{\sqrt[3]{{\cdot}087}\times\frac{1}{2}\times\sqrt{{\cdot}962}}$.

24. $\dfrac{({\cdot}86)^{\frac{3}{2}}\times({\cdot}09)^{\frac{4}{3}}}{(187)^{\frac{1}{4}}\times({\cdot}1)^{\frac{1}{2}}}$.

25. $3{\cdot}142[({\cdot}872)^2-({\cdot}621)^2]\times1{\cdot}07$.

26. $4\frac{1}{3}\times\sqrt{{\cdot}1872}-1\frac{3}{4}\times\sqrt{{\cdot}0935}$.

27. $\dfrac{{\cdot}384+({\cdot}384)^2}{3+\sqrt[3]{{\cdot}817}}$.

28. $6{\cdot}283[({\cdot}871)^2+({\cdot}871)(3{\cdot}06)]$.

29. $6{\cdot}1\times(2{\cdot}718)^{1{\cdot}3}$.

30. $8{\cdot}34\times({\cdot}924)^{2{\cdot}4}$.

31. If $10^x=7$, find x.

32. Evaluate (i) $(0{\cdot}1)^{0{\cdot}1}$; (ii) $({\cdot}0326)^{0{\cdot}43}$; (iii) $({\cdot}0214)^{1{\cdot}42}$; (iv) $({\cdot}024)^{0{\cdot}16}$.

33. Find the value of $213(10^{0{\cdot}8})(10^{0{\cdot}1}-3^{0{\cdot}1})$.

34. Find x if $(\frac{6}{11})^x=\frac{9}{16}$.

35. Find x, given $3^{3x-1}=2^{2x+1}$.

36. Evaluate (i) $(31{\cdot}6)^{-0{\cdot}23}$; (ii) $({\cdot}023)^{-1{\cdot}4}$.

37. Evaluate (i) $\dfrac{\log{\cdot}062}{\log1{\cdot}43}$; (ii) $\dfrac{-{\cdot}6241}{\log0{\cdot}036}$.

38. In steam engines the mean pressure is $\dfrac{1+\dfrac{\log r}{\log e}}{r}$ times the initial pressure. Calculate this ratio when $r=2{\cdot}5$, $e=2{\cdot}718$.

EXERCISE XVII (h)

[Take $\pi=3{\cdot}142$; give answers correct to 3 sig. figures.]

1. A car travelled 972 yd. in 1 min. 13 sec. Express its average speed in miles per hour.

2. Find the weight of a slab 1·53 m. long, 1·41 m. wide, 0·325 m. thick, if the material weighs 2730 Kg. per cubic metre.

3. The Representative Fraction of a 1-inch map is $\frac{1}{63360}$. Express his as a decimal.

4. A car was reported in November 1925 to establish a world ecord by travelling 5 kilometres in 1 min. 32·87 sec. Express this n miles per hour, taking 1 mile as equal to 1·609 Km.

5. Find the volume of 1 ton of water, assuming that 1 cu. ft. of vater weighs 62·4 lb.

6. The horse-power H transmitted by a belt is given by $H=\frac{\pi d n T}{33000}$. Find H, given $d=3{\cdot}7$, $n=148$, $T=310$.

7. Find correct to the nearest shilling the value of £4·75 × $(1{\cdot}055)^6$.

8. π radians are equal to two right angles. Express 1 radian in legrees.

9. A fence 578 yds. long is represented on a map by a line 1·97 in. ong. What distance is represented by a line on the map 3·08 in. ong ?

10. The area of a circle of radius r feet is πr^2 sq. ft. Find the area f a circle of radius 0·632 ft.

11. If wind is blowing at v ft. per sec., it exerts a pressure of ·00229 v^2 lb. per square foot. Find the pressure in a hurricane vhen $v=128$.

12. The volume of a sphere of radius r inches is $\frac{4}{3}\pi r^3$ cu. in. Find the volume of a sphere of radius 7·08 in.

13. In a town of population 67,841 there are 29,074 males. Express the number of males as a percentage of the total population.

14. Taking 1 gallon = 0·1605 cu. ft. and 1 litre = 61·03 cu. in., nd the number of litres in a gallon.

15. From the pendulum formula $t=2\pi\sqrt{\left(\frac{l}{32{\cdot}2}\right)}$, find t, given that $=2{\cdot}08$.

16. The diameter of a circle of area A sq. ft. is $2\sqrt{\left(\frac{A}{\pi}\right)}$ ft. Find the diameter of a circle of area 1 sq. ft.

17. The diameter of a sphere of volume V cu. ft. is $\sqrt[3]{\left(\frac{6V}{\pi}\right)}$ ft. ind the diameter of a sphere of volume 1 cu. ft.

18. If $f=\frac{uv}{u+v}$, find f, given that $u=3{\cdot}07$, $v=0{\cdot}89$.

19. If water stands to a depth h in. in a spherical vessel of radius in., its volume is $\pi h^2\left(r-\frac{h}{3}\right)$ cu. ft. Evaluate this if $r=11{\cdot}08$, $h=8{\cdot}14$.

20. The area of an annulus of outer radius x in. and inner radius y in. is $\pi x^2-\pi y^2$ sq. in. Find the area if $x=8{\cdot}73$, $y=3{\cdot}51$.

21. Find V from the formula $V=\pi h(R+r)(R-r)$, given that $R=8{\cdot}73$, $r=7{\cdot}89$, $h=2{\cdot}06$.

22. The time t seconds for pneumatic transmission through a tube l ft. long, d ft. in diameter, under a pressure of P lb. per square inch, is given by $t=0{\cdot}000482\sqrt{\left(\dfrac{l^3}{Pd}\right)}$. Find t, given that $l=53{\cdot}7$, $d=0{\cdot}28$, $P=6{\cdot}45$.

23. A cylinder $1\frac{1}{3}$ ft. high holds, when full, 100 gallons. Evaluate the expression $\sqrt{\left\{\dfrac{4\times16{\cdot}05}{1{\cdot}333\pi}\right\}}$, which represents its diameter in feet.

24. The area of a triangle whose sides are of lengths a, b, c is $\sqrt{\{s(s-a)(s-b)(s-c)\}}$ where $s=\frac{1}{2}(a+b+c)$. Find the area of a triangle whose sides are 8·62, 5·91, 4·87.

25. The rate of flow in a circular sewer of diameter d inches is $\dfrac{9{\cdot}4\sqrt[3]{d^2}}{\sqrt{i}}$ ft. per second, where i equals length of sewer ÷ fall. Find the rate if $d=39$, $i=1000$.

26. In order to raise a load of W lb., the diameter d ft. of a balloon, filled with a certain gas, is given by $d=\sqrt[3]{\left\{\dfrac{W}{0{\cdot}52\times0{\cdot}071}\right\}}$. What diameter is needed to raise a load of 1 ton?

27. With the data of No. 26, find what load can be raised if the diameter is 21 ft.

28. Find the value of $A\,.\,e^{kt}$ when $A=5{\cdot}32$, $e=2{\cdot}718$, $k=2{\cdot}8$, $t=\frac{3}{4}$.

29. The weight of rail, W lb. per yard, necessary for safety when the maximum velocity is v miles per hour and the maximum load on the driving wheels is L tons, is given by $W=17\sqrt[3]{\{L+0{\cdot}0001Lv^2\}^2}$. Find W if $L=7{\cdot}9$, $v=51$.

30. From the formula $T_1=T_2\left(\dfrac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$, find T_1 when $T_2=293$, $p_1=6{\cdot}47$, $p_2=5{\cdot}38$, $\gamma=1{\cdot}4$.

31. The thrust P lb. for driving through mild steel a drill of diameter D in. with a feed of T in. per revolution is given by $P=35500D^{0\cdot7}T^{0\cdot6}$. Find P if $D=\frac{3}{4}$, $T=\frac{1}{100}$.

32. If a solid cast-iron pillar is of diameter d in. and height h ft., it can carry a load of w tons where $w=\dfrac{15d^{3\cdot7}}{h^{1\cdot8}}$. Find the load which can be carried by this pillar if its diameter is 1 ft. and its height $6\frac{3}{4}$ ft.

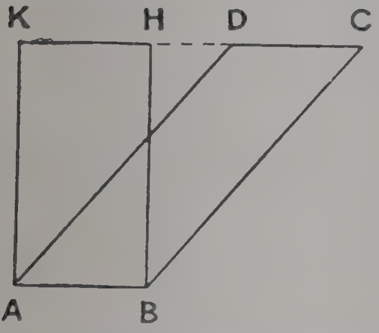
CHAPTER XVIII

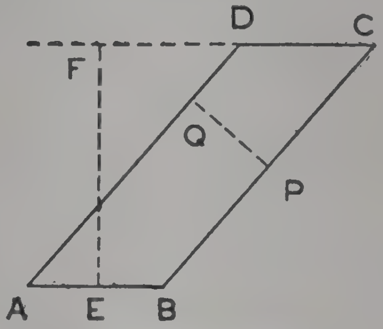
MENSURATION

(i) *The Parallelogram*

ABCD is a parallelogram. Draw the rectangle ABHK on the same base AB and between the same parallels AB, KHDC.

Since $\Delta ADK \equiv \Delta BCH$, parallelogram ABCD = rectangle ABHK.



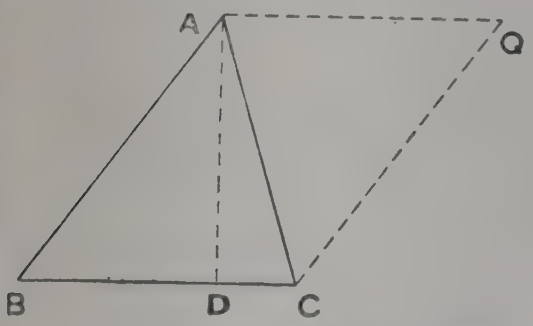


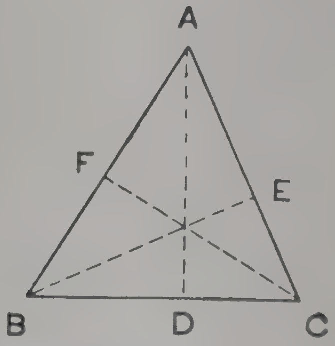
$\therefore$ area of parallelogram ABCD = AB . BH = **base × height.**

Note that BC may be taken as the base of the parallelogram nd PQ as its height. Then area of parallelogram ABCD = AB . EF = BC . PQ.

(ii) *The Triangle*

ABC is a triangle. Complete the parallelogram ABCQ. Then ΔABC $= \frac{1}{2}$ parallelogram ABCQ.





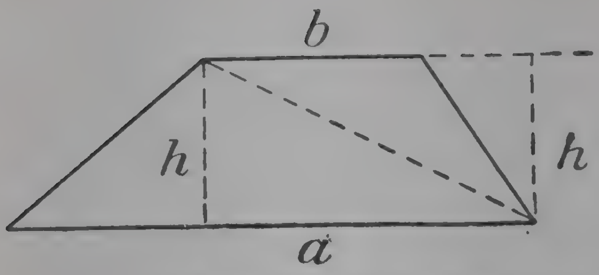
$\therefore$ area of $\Delta ABC = \frac{1}{2}BC \,.\, AD = \frac{1}{2}$ **base × height.**

A triangle may be regarded as having three distinct bases and ree correspondingly distinct heights.

$\therefore$ area of $\Delta ABC = \frac{1}{2}BC \,.\, AD = \frac{1}{2}CA \,.\, BE = \frac{1}{2}AB \,.\, CF.$

Note.—If the lengths of the sides of a triangle are a, b, c in., and if $=\frac{1}{2}(a+b+c)$, the area of the triangle $=\sqrt{\{s(s-a)(s-b)(s-c)\}}$ sq. in.

(iii) *The Trapezium*

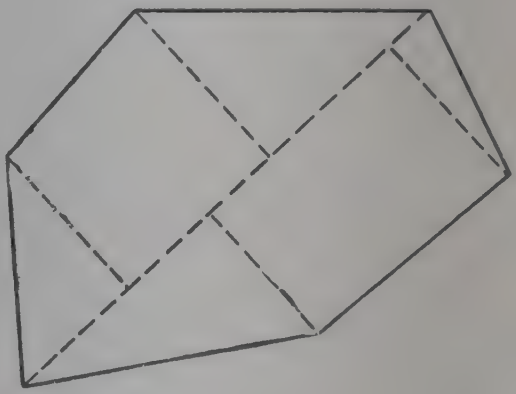


Draw a diagonal dividing the trapezium into two triangles. Then the area of the trapezium $= \frac{1}{2}ah + \frac{1}{2}bh$ $= \frac{1}{2}(a+b)h$

$= \frac{1}{2}$ (sum of parallel sides) × distance between them.

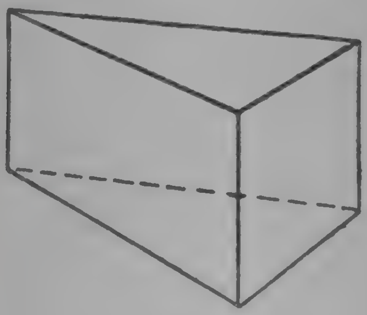
(iv) *Any Polygon*

Join two opposite corners of the polygon and drop perpendiculars to this line from the remaining corners of the figure. Then the polygon is divided into a number of compartments, each of which is either a right-angled triangle or a trapezium.

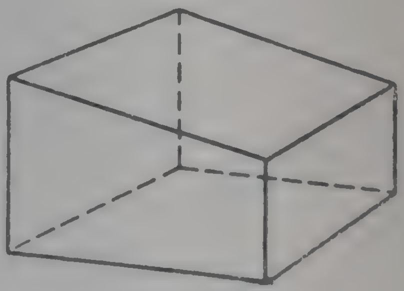


(v) *Prism*

If a prism is cut by any plane parallel to the base, the cross-section so obtained is congruent to the base.



(i)



(ii)

A cuboid is a prism on a rectangular base: a Kohinoor pencil is a prism on a hexagonal base: a wedge is a prism whose base is a triangle. [Fig. (i).]

The ordinary lean-to shed (*e.g.* a conservatory) is a prism whose base (the side face) is a trapezium. [Fig. (ii).]

Volume of a prism = **base area × height**

= **area of cross-section × distance between the end faces.**

(vi) *Any Pyramid*

If we join the centre O of a cube of edge a in. to each of the eight corners, the cube is divided into six equal pyramids, each having its vertex at the centre of the cube and one face of the cube as base.

∴ the volume of each pyramid $=\frac{1}{6}a^3$ cu. in.

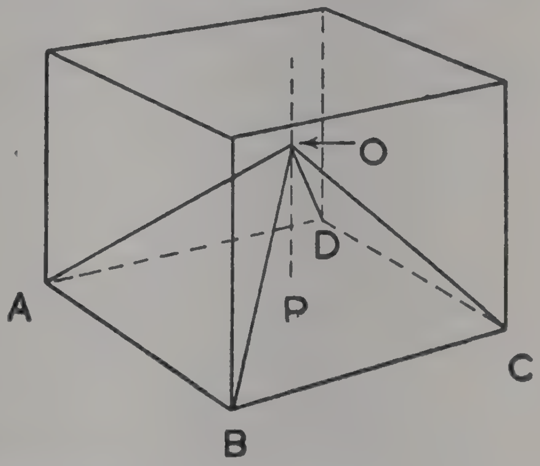
Now the height of each pyramid is $\frac{a}{2}$ in., and the area of the base is a^2 sq. in.

∴ the volume of each pyramid

$=\frac{1}{3}\times a^2\times\frac{a}{2}$ cu. in.

$=\frac{1}{3}\times$ base area × height.

The result proved in this special case is true in general.



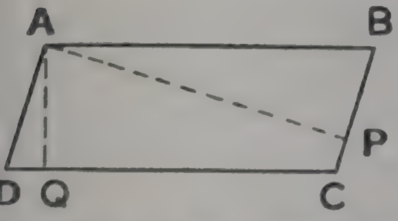
Volume of pyramid = $\frac{1}{3}$ × **base area × height**

= $\frac{1}{3}$ × volume of prism on the same base and of equal height.

EXERCISE XVIII (a)

1. ABCD is a parallelogram. AP, AQ are the perpendiculars from A to BC, CD.

(i) If AB=8·4 in., AQ=2·5 in., find the area of ABCD.



(ii) If AB=5 in., AP=4 in., AD=6 in., find AQ.

(iii) If the area of ABCD is 6·3 sq. in. and if AP=3·5 in., find BC.

(iv) If AB=8 in., AP=6 in., AQ=4·5 in., find AD.

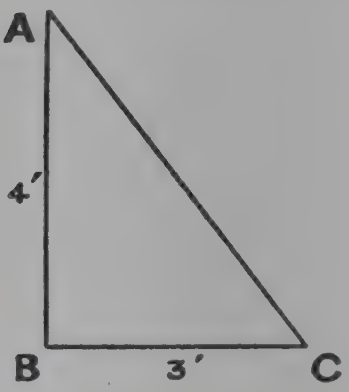
2. Draw a parallelogram ABCD with AB=BC=2 in. and ∠ABC=60°. Measure or calculate its altitude and find its area.

3. If $\angle ABC = 90°$, find the area of ΔABC and the length of the perpendicular from B to AC. [Fig. (i).]

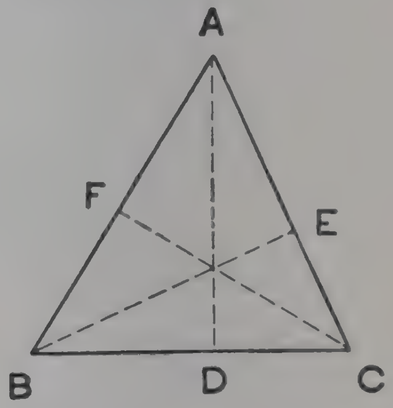
4. AD, BE, CF are altitudes of the triangle ABC. [Fig. (ii).]

(i) If AD = 7 in., BC = 5 in., find the area of ΔABC.
(ii) If BC = 3·7 in., and if the area of ΔABC is 6·24 sq. in., find AD.
(iii) If BE = 5 in., AB = 6 in., CF = 4 in., find AC.
(iv) If BD = 18 in., AD = 24 in., DC = 7 in., find the lengths of AB, AC, the area of ΔABC, and the lengths of BE, CF.

(i)



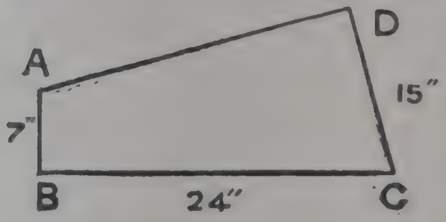
(ii)



5. Find the area of an equilateral triangle whose side is 6 in.

6. If $\angle ABC = \angle ADC = 90°$, calculate the length of AD and find the area of ABCD. Find also the lengths of the perpendiculars from B, D to AC. [Fig. (iii).]

7. In the parallelogram ABCD, AB = 8·3 in., BC = 5 in.; the perpendicular from A to CD is 2·7 in. Find the perpendicular from B to AD.



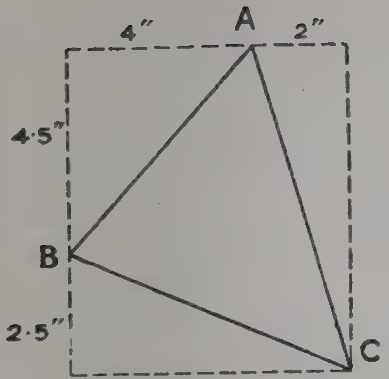
(iii)

8. Find the area of a rhombus whose diagonals are of lengths 4·6 in., 5·5 in.

9. The triangle ABC is enclosed in a rectangle as shown. Find its area. [Fig. (iv).]

10. Two parallel sides of a four-sided field are 160 yd. and 196 yd., and the distance between them is 134 yd. Find the area of the field in acres.

11. AP, BQ are perpendicular to DC. Find the area of ABCD. [Fig. (v).]



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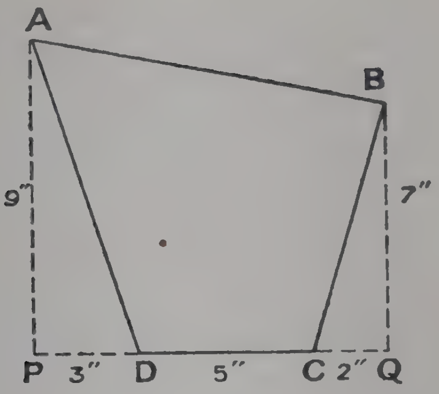
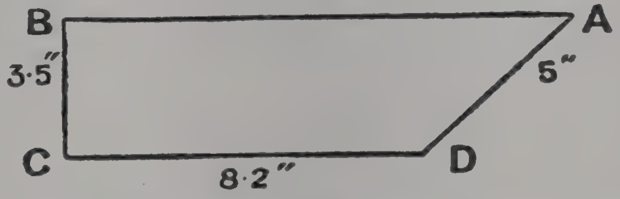


(v)

12. $\angle ABC = \angle BCD = 90°$. Calculate the length of the perpendicular from C to AD produced. [Fig. (vi).]

(vi)



13. ABCD is a rectangle; AB=12 in., AD=8 in. Find the length of the perpendicular from B to AC.

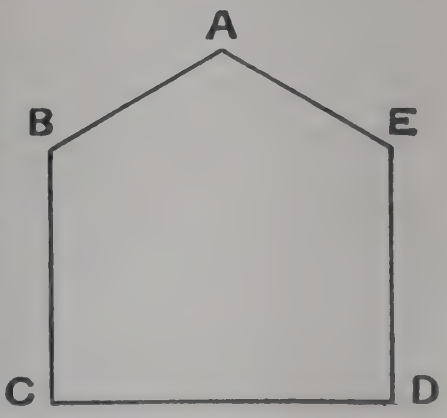
14. Use the *s*-formula for the area of a triangle to calculate the area of a triangle whose sides are 5, 6, 7 cm.; find also its greatest altitude.

15. Use the *s*-formula for the area of a triangle to calculate the area of the quadrilateral ABCD, given that AB=7·4 in., BC=6·7 in., CD=9·2 in., DA=10·8 in., AC=8·4 in.

16. The sides of a triangular field measure 585, 522, 453 links. Find the area of the field in acres.

17. A section of a railway cutting is a trapezium 15 ft. wide at the bottom and 25 ft. wide at the top: the depth is 18 ft. What is the area of the cross-section and the number of cubic feet of soil removed per yard run?

18. Find the area of the end ABCDE of a shed, given BC=ED=22 ft. 6 in., CD=18 ft. 4 in., and that A is 27 ft. above CD. [Fig. (vii).]



(vii)

19. If the shed in No. 18 is 36 ft. long, find its volume.

20. A right prism of length 12 cm. has a triangular base whose sides are 3, 4, 5 cm. Find its volume.

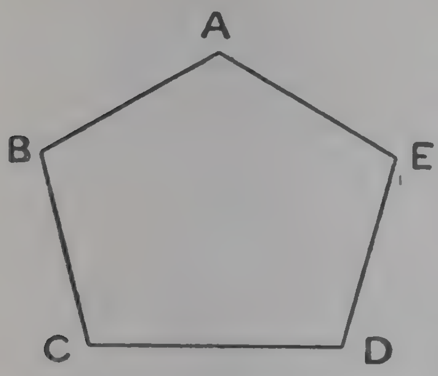
21. A trench is dug whose cross-section is a trapezium. The width of the bottom is 3 ft. 4 in. and at the top is 7 ft. 6 in.; the depth is 5 ft. 6 in. Find the number of cubic feet excavated in making 50 yd. of trench.

22. A greenhouse 20 ft. long is built against a wall, where its height is 16 ft.; in front the height is 8 ft. and the width is 12 ft. It is built of brick up to a height of 4 ft. both in front and at the sides, the remainder being glass and wood. Find the number of panes of glass required, each being 16 in. by 12 in., assuming that 5 per cent. of the surface is taken by the woodwork.

23. Find the volume of the greenhouse described in No. 22.

24. A coal-shed is built with a sloping roof; it is 9 ft. 6 in. high at the back, 4 ft. 6 in. high in front, and is 10 ft. wide from back to front, and is 13½ ft. long. How much coal will it hold if a ton of coal occupies 45 cu. ft.?

25. A swimming-bath, 80 ft. long, 20 ft. wide, 9 ft. deep at one end and 4 ft. deep at the other, is full of water. How many cubic feet of water does it contain? How many cubic feet of water must be run off in order to expose half of the floor area?



26. Water flows in a V-shaped gutter at 2 ft. per second, the sides of the gutter being at right angles. If the depth of water is 9 in., find the number of gallons discharged per hour, taking 1 cu. ft. = 6·23 gallons.

27. ABCDE represents the section of a haystack 24 ft. long. The heights of A, B, E above the ground are 18, 12, 12 ft. CD = 18 ft., BE = 24 ft. Find the volume of the stack.

28. The great pyramid has a square base whose perimeter is about half a mile. Its height is about 486 ft. Find its approximate volume in cubic feet.

29. A pyramid stands on a base 6 ft. square, and its volume is 48 cu. ft. Find its height.

30. The base of a hollow pyramid 1 ft. high is a square of side 8 in. If the base is level and uppermost, and if the pyramid contains water to a depth of 6 in., find the volume of the water.

31. A pyramid stands on a base 8 ft. square; each of the four slant edges is 9 ft. long. Find the volume of the pyramid.

32. Find by calculation or by drawing and measurement the volume of a regular tetrahedron, each of whose four faces is an equilateral triangle of side 4 in.

33. A mound is in the shape of a truncated prism: its base is a rectangle 12 ft. by 8 ft., and its top is a rectangle 6 ft. by 4 ft., and it is 5 ft. high. Find its volume.

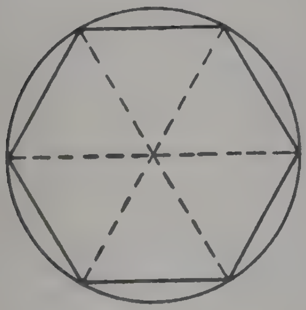
Plot on squared paper the following points, and find exactly the areas of the figures obtained by joining the points.

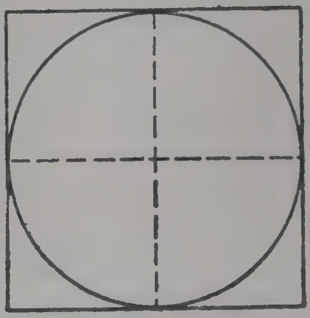
34. A (0, 3); B (4, 0); C (5, 4).
35. A (1, 1); B (2, 5); C (4, 7); D (6, 0).
36. A (−2, 1); B (1, 4); C (3, −2).
37. A (−2, 2); B (1, 5); C (4, 3); D (5, −1).
38. A (−1, 2); B (0, 3); C (1, 6); D (4, 1); E (3, −2).

The Circle

The Circumference of a Circle

Suppose the diameter of the circle is d in.





If we inscribe a regular hexagon in the circle, each side is equal to the radius and is therefore $\frac{1}{2}d$ in. long, so that the perimeter of the hexagon is $3d$ in. Consequently the length of the circumference of the circle must be *greater* than $3d$ in.

If we circumscribe a square about the circle, each of its sides is d in. long, so that its perimeter is $4d$ in. Consequently the length of the circumference of the circle must be *less* than $4d$ in.

$$\therefore \frac{\text{circumference}}{\text{diameter}} = \text{some number } \textit{between } 3 \text{ and } 4.$$

The approximate value of this number should now be found by measurement. Take a cylindrical tin and measure its diameter with a ruler; various methods may then be used to measure its circumference: (*a*) allow it to roll along a graduated ruler; (*b*) wind a cotton thread round it several times and then measure the thread; (*c*) wrap a sheet of paper round the cylinder so that it overlaps and prick through the overlapping portion, then unwrap the paper and measure the distance between the pin-pricks.

This number is denoted by π, and it can be proved that $\pi=3{\cdot}14159\ldots$; a convenient approximation for π is $\frac{22}{7}$. The Egyptians took π as $(\frac{4}{3})^4=3{\cdot}16\ldots$; Archimedes proved that π lies between $3\frac{1}{7}$ and $3\frac{10}{71}$; in India it was often taken to be $\sqrt{10}$; in 1573 a German writer, Valentinus Otto, suggested $\frac{355}{113}$, which is correct to six places. The value of π to 35 places of decimals was calculated by Ludolph van Ceulen (1615), a professor at Leyden, who asked that the result should be engraved on his tombstone. It was not until 1761 that Lambert proved that π is incommensurable, *i.e.* it cannot be represented exactly by any fraction.

We write then $\dfrac{\text{circumference}}{\text{diameter}}=\pi$

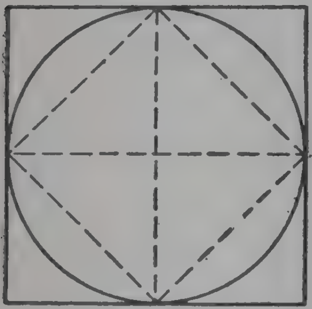
$$\text{or circumference}=\pi\times\text{diameter}.$$

For a circle of radius r in., circumference $=2\pi r$ in.

Alternatively, radius $=\dfrac{\text{circumference}}{2\pi}$.

The Area of a Circle

Suppose the radius of the circle is r in. If we circumscribe a square about the circle, we can divide it into four equal squares, each of area r^2 sq. in., so that the area of the circle must be *less* than $4r^2$ sq. in. But we can divide each small square in half as shown, and from this we see that the area of the circle must be *greater* than $4(\frac{1}{2}r^2)$, or $2r^2$ sq. in.



$\therefore\ \dfrac{\text{area of circle}}{\text{square on radius}}=$ some number between 2 and 4.

The approximate value of this ratio should now be found by measurement. Draw on inch-squared paper a circle of radius (say) 2 in., and find its area by counting squares. The value of this ratio is actually the same number π described above.

We write then $\dfrac{\text{area of circle}}{\text{square on radius}}=\pi$

$$\text{or area of circle (radius } r \text{ in.)}=\pi r^2 \text{ sq. in.}$$

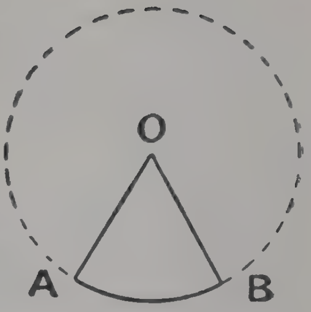
If the diameter of a circle is d in., its area $=\pi(\frac{1}{2}d)^2=\frac{1}{4}\pi d^2$ sq. in.

The Area of a Sector of a Circle

AB is an arc of a circle, centre O.

$$\text{Then}\qquad \frac{\text{area of sector OAB}}{\text{area of circle}}=\frac{\text{arc AB}}{\text{circumference of circle}}.$$

Let the radius be r in., and let the arc AB$=l$ in.



Then
$$\frac{\text{area of sector OAB}}{\pi r^2}=\frac{l}{2\pi r}.$$

$$\therefore \text{ area of sector OAB}=\frac{l}{2\pi r}\times \pi r^2=\tfrac{1}{2}rl \text{ sq. in.}$$

Example.—Find the area of the ring between two concentric circles whose radii are 3·24 *in. and* 4·36 *in.*

The required area is $\pi(4{\cdot}36)^2-\pi(3{\cdot}24)^2$ sq. in.

$$\begin{aligned}&=\pi[(4{\cdot}36)^2-(3{\cdot}24)^2]=\pi[(4{\cdot}36+3{\cdot}24)(4{\cdot}36-3{\cdot}24)]\\&=\pi(7{\cdot}6)(1{\cdot}12)\\&=26{\cdot}7 \text{ sq. in.}\end{aligned}$$

EXERCISE XVIII (b)

1. Find the circumference and area of a circle whose diameter is 16·2 in.

2. Find the diameter of a circle whose circumference is 21·3 in.

3. Find the area of the ring between two circles of radii 6·24 in. and 7·76 in.

4. The area of a circle is 17·4 sq. cm. Find its radius.

5. The circumference of a tin is 8·4 in. Find its diameter.

6. A wheel of diameter 4 ft. makes 1420 revolutions in 3 miles. What loss of distance arises from the slipping of the wheel ?

7. A gear-wheel has 48 teeth of pitch 1½ in. What is the diameter of the pitch circle ? [The pitch is the distance along the arc between the centres of two adjacent teeth.]

8. A circular running track has an internal radius of 70 yd. and is 12 ft. wide. Find the area of the track. What is the difference between the lengths of the inner and outer boundaries of the track ?

9. The minute-hand of "Big Ben" is 11 ft. long. Find the distance its tip moves in 5 minutes.

10. If the diameter of the driving-wheel of an engine is 7 ft., find the number of revolutions per minute at a speed of 60 miles an hour, if there is no slipping.

11. A flat piece of sheet iron 8 ft. wide is made into corrugated iron. If its new shape is a series of semicircles, find its new width.

12. A bucket is raised from the bottom of a well 26 ft. deep by a

cord which is wound on an axle of diameter 15 in. How many turns of the axle are required to raise the bucket ?

13. A steam piston is 2 ft. in diameter, and the piston rod is 5 in. in diameter. Find the pressure on each face for a steam pressure of 80 lb. per square inch.

14. The angle of a sector of a circle is 144° and its radius is 4·6 cm. Find the area of the sector.

15. A piece of flexible wire is in the form of an arc of a circle of radius 7·2 cm., and subtends an angle of 108° at the centre of the circle. It is bent into a complete circle. What is the diameter of this circle ?

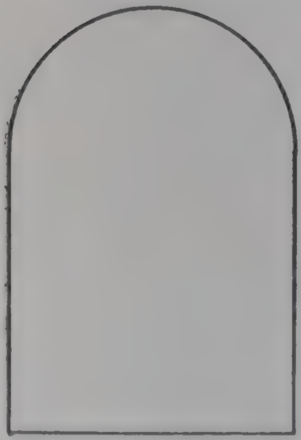
16. A horse is tethered by a rope 20 ft. long to a ring which can slide along a low straight rail 10 yd. long. What is the area over which the horse can graze ?

17. Taking the mean distance of the Earth from the Sun as 93,000,000 miles, and assuming the Earth moves in a circle, find its average speed in miles per second.

18. The statement that the gear of a bicycle is x means that a single revolution of the pedals causes the bicycle to advance a distance equal to the circumference of a circle of diameter x in. If the gear of a bicycle is 90, what is the number of revolutions of the pedals per mile ?

19. In a certain frame, the pull in one of the tie-rods is 12 tons. If the tie-rod is solid and circular, and if the material should not be made to support a tension of more than 3 tons per square inch, find the least diameter of the rod.

20. A circular metal disc 8 in. in diameter weighs 4 lb. What is the weight of a disc of the same material and thickness and of diameter 6 in. ?



21. A quadrant of radius 5 in. is cut away from each corner of a square of side 10 in. What is the area and perimeter of the remaining figure ?

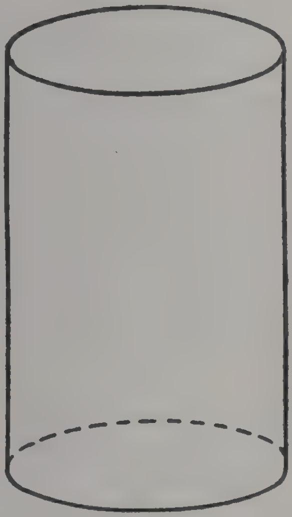
22. A piece of cardboard in the shape of a semicircle is of area 20 sq. in. What is its perimeter ?

23. A window consists of a rectangle surmounted by a semicircle : its width is 4 ft. and its greatest height is 8 ft. Find its area.

24. Three cylinders, each of diameter 4 in., are bound together, with their axes parallel, by an elastic band. What is the stretched length of the band ?

Circular Cylinder

It is easy to name objects in everyday use of cylindrical shape: a water-pipe, a pencil of circular section, a penny, a tumbler, a garden roller, etc.



To find the area of the *curved* surface of a circular cylinder, take a sheet of paper and wrap it round the cylinder and crease it so that it just covers the cylinder without overlapping. When we unwrap it, we obtain a rectangle whose height is the height of the cylinder, and whose breadth is equal to the perimeter of the circular base of the cylinder.

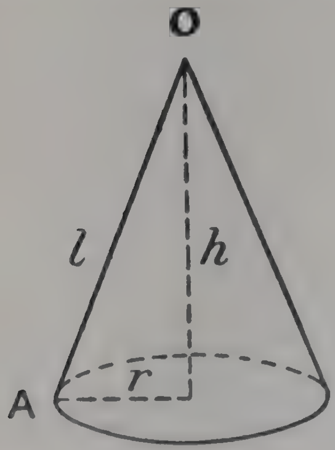
Suppose the cylinder is of radius r in. and height h in., then **the area of the curved surface** $= h \times 2\pi r = 2\pi rh$ sq. in.

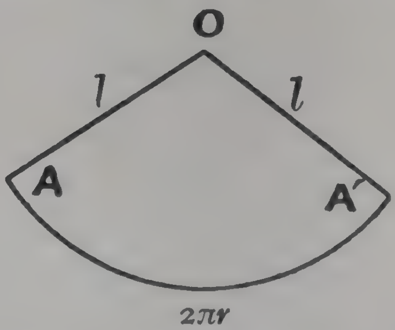
The **volume of the cylinder** = base-area × height

$$= \pi r^2 \times h = \pi r^2 h \text{ cu. in.}$$

Circular Cone

The shade of an electric light is very often made in the shape of a circular cone. If we cut down a slant side of the cone from the vertex O and unwrap the curved surface, we obtain a sector of a circle centre O, and the rim of the base of the cone becomes the circular arc AA′ bounding the sector.





Suppose the slant side of the cone is l in. long and the base-radius is r in., then $OA = l$ in., and arc $AA' = 2\pi r$ in.

$\therefore$ **area of curved surface of cone** $=$ area of sector OAA′

$$=\tfrac{1}{2}l\times 2\pi r=\pi rl \text{ sq. in.}$$

Suppose the height of the cone is h in., then, as for a pyramid,

volume of cone $=\frac{1}{3}$ base-area $\times$ height

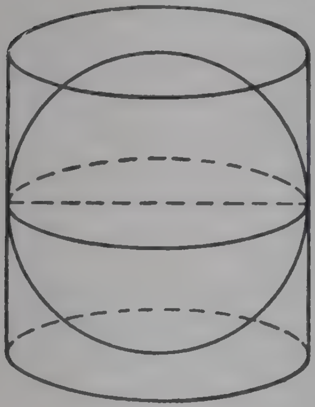
$$=\tfrac{1}{3}\pi r^2h \text{ cu. in.}$$

Note.—By Pythagoras, we also have $l^2=r^2+h^2$.

If the top part of a cone or pyramid is cut off by a plane parallel to the base, the remainder is called a **frustum.**

The Sphere

Suppose a sphere rests inside a cylindrical tube and just fits it, so that the radius of the sphere is equal to the internal radius of the tube. If any two planes are drawn each parallel to the plane of contact of the sphere and the tube, then the area of the surface of the sphere intercepted between these two planes (called a **zone** of the sphere) is equal to the area of the curved surface of the cylinder intercepted between these two planes. This result was discovered by Archimedes.



If the radius of the sphere or cylinder is r in., and if the distance between the parallel planes is d in., the area of the curved surface of the cylinder between the planes is $2\pi rd$ sq. in.

$\therefore$ **the area of the surface of a zone of a sphere** of height d in. is $2\pi rd$ sq. in.

Putting d equal to $2r$, we have the result

total area of surface of a sphere $=2\pi r\,.\,2r=4\pi r^2$ sq. in.

It can also be proved that the volume of the sphere is two-thirds of the volume of the circumscribing cylinder bounded by planes touching the sphere at its highest and lowest points.

Suppose you have a graduated cylinder and three equal spheres, each of which will just fit into it: let the diameter of each sphere be d cm. Introduce water into the cylinder so that the depth exceeds $3d$ cm. When the three spheres are inserted, it will be found that the rise of water-level is $2d$ cm., thus showing that the total volume of the three spheres is equal to twice the volume of a circumscribing cylinder of height d cm.

$$\therefore \text{ the volume of a sphere} = \tfrac{2}{3} \cdot \pi r^2 \cdot 2r = \tfrac{4}{3}\pi r^3.$$
$$= \tfrac{4}{3}\pi(\tfrac{1}{2}d)^3 = \frac{4\pi}{3} \cdot \frac{d^3}{8} = \frac{\pi d^3}{6}.$$

EXERCISE XVIII (c)

1. Find the volume of a circular cylinder, given that—(i) height = 10 in., radius = 6 in.; (ii) height = 5 ft., diameter = 3 ft.; (iii) diameter = 1·5 in., length = 12 ft.; (iv) length = 8 ft., girth = 4 in. Find also the volume of metal in a tube (*a*) length 6 ft., bore 9·5 in., thickness 1 in.; (*b*) length 3 ft., bore 1·5 in., thickness 0·2 in. [*See* Example, p. 233.]

2. Find the radius of a cylinder whose length is 3 ft. and volume 10 cu. ft.

3. Find the total surface of a closed cylinder, height 8 in. and radius 5 in.

4. A telegraph pole is 21 ft. high and 10 in. in diameter. Find its weight if the wood weighs 35 lb. per cubic foot.

5. A cylindrical tank is 9 ft. in diameter and is 12 ft. high. How many gallons of water will it hold?

6. A cylindrical tank of diameter 12 ft. holds 2000 gallons of water. What is its depth?

7. A 2-inch pipe (*i.e.* diameter 2 in.) delivers 15 gallons of water per minute. What is the speed of the water in feet per second?

8. What is the volume of metal in a pipe 20 ft. long, external diameter 3 in. and 0·2 in. thick?

9. One pound of tobacco is packed in a cylindrical tin of diameter 4 in. and height 8 in. What would be the height of a tin of diameter 3 in., which would hold $\frac{1}{4}$ lb. of tobacco, similarly packed?

10. How many cylindrical glasses 2 in. in diameter can be filled to a depth of 3 in. from a cylindrical jug of diameter 5 in. and height 12 in.?

11. The volume of a coil of wire is 5·3 cu. cm., and its length is 6·5 m. Find its diameter.

12. What area of sheet metal is required to make 1000 cylindrical tins, closed at one end only, if the diameter is 4·5 in. and the height is 6 in.?

13. Water flows into a bath through two 4-inch pipes at 5 feet per second. The bath is 50 ft. long, 24 ft. wide. How long will it take to fill it to a depth of 8 ft.?

14. A cylindrical measure which holds a gallon is 5 in. deep. What is its diameter?

15. A hall is warmed by pipes of total length 160 ft. and diameter 5 in. What is the area of the heating surface?

16. The length of a column of mercury in a glass tube is 12·4 cm., and the weight of the mercury is 267·3 gr. Given that 1 cu. cm. of mercury weighs 13·6 gr., find the mean diameter of the tube.

17. Find the volume of the greatest circular cylinder that can be cut from a rectangular block whose edges are 8, 10, 12 cm.

18. Two cylinders, diameters 15, 10 cm., are filled with water to depths 20, 12 cm. respectively; they stand side by side on a level table and are connected at the bottom by a tube with a tap. When the tap is turned on, what is the resulting depth in each cylinder?

19. A garden roller is 2 ft. 9 in. in diameter and is 4 ft. wide. What area does it roll over in 30 revolutions?

20. A wire fence is supported by 35 cylindrical posts each of diameter 6 in. and height 4 ft. The surface (side and top) of each is to be painted. What area of surface is this?

21. Find the volumes of the solids whose plans and elevations are shown on opposite page.

22. Find the volume of a circular cone, given that

(i) height = 10 in., base-radius = 6 in.
(ii) height = 8·5 cm., base-diameter = 4·6 cm.

23. The base of a conical tent is 12 ft. in diameter and its height is 8 ft. Find (i) the volume of the tent; (ii) the area of canvas required for making it.

24. A sector of a circle of radius 8 cm. and angle 120° is bent to form the surface of a cone. Find the radius of its base.

25. A conical wine-glass is 5 cm. in diameter and 6 cm. deep. How much wine will it hold?

26. Find (i) the volume, (ii) the area of the surface of a ball of diameter 12 cm.

27. A hollow spherical steel shell has an external diameter of 6 in. and is $\frac{1}{2}$ in. thick. What volume of steel is used in making it?

28. Taking the radius of the Earth as 3960 miles, find the area of its surface.

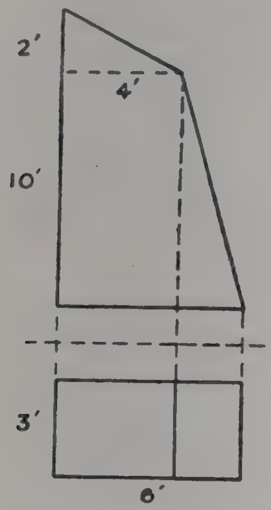
29. How many spherical shot, each of diameter $\frac{1}{4}$ in., can be cast from 1 cu. ft. of lead?

30. A cylindrical tank of diameter 3 ft. contains water: five spheres, each of diameter 6 in., are placed in the tank and wholly submerged. What is the increase of height of the water-level?

31. A hemispherical bowl holds 2 gallons of water. What is its diameter?

32. A solid is formed of a hemisphere surmounted by a cone whose height equals the diameter of the hemisphere. The diameter is 1 foot. Find the volume and the area of the total surface.

(i)



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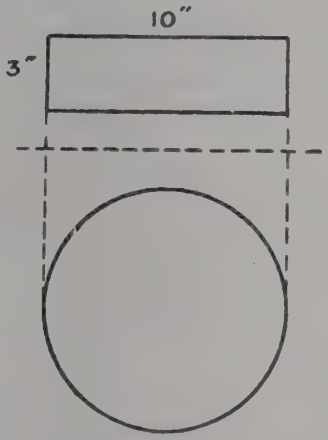

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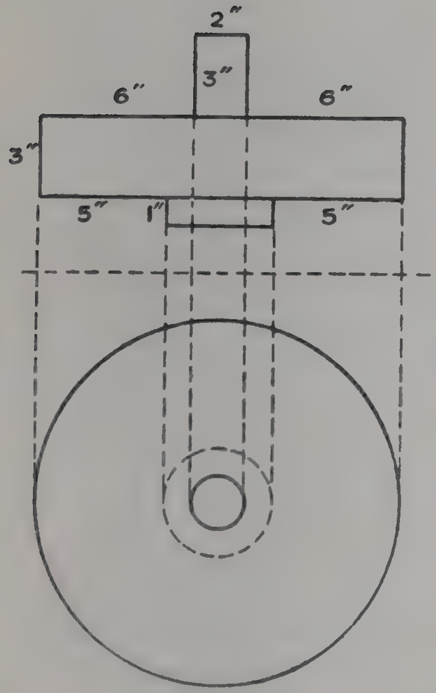

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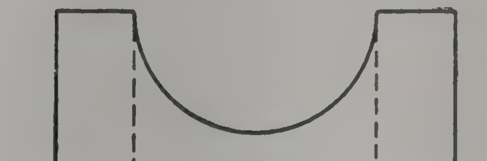


33. A cylindrical boiler has hemispherical ends; its internal

diameter is 7 ft. and its total internal length is 20 ft. Find its volume and its internal surface.

34. A funnel is in the shape of the frustum of a cone; the diameters of its upper and lower rims are 6 in., 2 in., and its slant edge is 3 in. Find the area of tin-plate needed for its construction.

35. A frustum of a cone has a top of radius r_1 and a base of radius r_2, the slant height being l. Prove that the slant height of the part cut off is $\frac{r_1 l}{r_2 - r_1}$. Hence show that the area of the curved surface of the frustum is $\pi l(r_1 + r_2)$.

CHAPTER XIX

DENSITY AND SPECIFIC GRAVITY

THE **Density** of a substance is the weight of a unit of volume of the substance. The density therefore depends on the units chosen.

Thus 1 cu. cm. of water weighs 1 gram.

$\therefore$ the density of water is **1 gram per cu. cm.,**

and 1 cu. ft. of water weighs 62·3 lb.,

$\therefore$ the density of water is also **62·3 lb. per cu. ft.**

The **Specific Gravity** of a substance is the ratio of the weight of any volume of that substance to the weight of an equal volume of water. Specific gravity, being a ratio, is a pure number and does not depend on the units chosen.

Thus, since 1 cu. cm. of water weighs 1 gram and 1 cu. cm. of mercury weighs 13·6 grams, the specific gravity of mercury = 13·6; and since 1 cu. ft. of water weighs 62·3 lb., 1 cu. ft. of mercury weighs 13·6 × 62·3 lb.

The following table gives the specific gravity of various substances:

Platinum	21·5	Wrought iron	7·8	Pitch pine	·6
Gold	19·3	Cast iron	7·2	Elm	·56
Mercury	13·6	Zinc	7·0	Sulphuric acid	1·8
Lead	11·4	Aluminium	2·7	Linseed oil	·94
Silver	10·5	Plate glass	2·8	Alcohol	·79
Copper	8·8	Ebony	1·2	Petrol	·68
Steel	7·8	Beechwood	·69	Cork	·24

The volume of a given weight of a given substance varies slightly with its temperature; consequently it is necessary to specify the temperature in the definition of specific gravity; but for ordinary changes of temperature this variation can be neglected in actual practice.

Example.—*The density of Spanish mahogany is 53 lb. per cubic foot. Find the weight of a panel 6 ft. long, 4 ft. wide, and $\frac{1}{8}$ in. thick.*

The volume of the panel is $6 \times 4 \times \frac{1}{8} \times \frac{1}{12}$ cu. ft. $= \frac{1}{4}$ cu. ft.

$\therefore$ its weight $= \frac{1}{4}$ of 53 lb. $= \underline{13\frac{1}{4}\text{ lb.}}$

Example.—*The density of Spanish mahogany is 53 lb. per cubic foot. Find its specific gravity.*

1 cu. ft. of mahogany weighs 53 lb.
1 cu. ft. of water weighs 62·3 lb.

$$\therefore \text{ specific gravity} = \frac{53}{62{\cdot}3} = \underline{0{\cdot}85.}$$

Example.—*What is the weight of a gallon of petrol, specific gravity 0·68?*

1 gallon of water weighs 10 lb.
$\therefore$ 1 gallon of petrol weighs $0{\cdot}68 \times 10 = \underline{6{\cdot}8\text{ lb.}}$ approx.

EXERCISE XIX (a)

[Where necessary, the data in the table on p. 240 should be utilised.]

1. The density of marble is 170 lb. per cubic foot. Find the weight of a mantel-shelf 6 ft. long, 9 in. wide, 1 in. thick.

2. The density of chalk is 120 lb. per cubic foot. Express this in ounces per cubic inch.

3. The average density of granite is 168 lb. per cubic foot. What is its specific gravity?

4. What is the weight in grams of 50 cu. cm. of sulphuric acid?

5. A flask when empty weighs 16 grams, and when full of alcohol (specific gravity 0·8) weighs 46 grams. What is the capacity of the flask?

6. What is the weight in ounces of 1 cu. in. of gold?

7. A sheet of plate glass $\frac{1}{4}$ in. thick weighs 8 lb. What is its surface area?

8. A flask when empty weighs 18 grams, and when full of water

weighs 54 grams, and when full of oil weighs 51 grams. Find the specific gravity of the oil.

9. A block of oak, 8 in. long, 6 in. wide, 2 in. thick, weighs 2·7 lb. What is the density of the oak in lb. per cubic foot ?

10. How many cubic inches of gold will balance 8 cu. in. of silver ?

11. The specific gravity of air is 0·0013. What is the weight of air in a hall 30 ft. long, 20 ft. wide, 12 ft. high ?

12. The cross-section of a cork is 1·6 sq. cm., and its length is 5 cm. What is its weight ?

13. A bottle when empty weighs 11 oz., when full of water weighs 36 oz., and when full of another liquid weighs 31 oz. Find the specific gravity of the liquid.

14. A vessel whose cross-section is 30 sq. in. contains 1 cwt. of mercury. What is the depth of the mercury ?

15. A vessel weighs 6 oz. when empty and 10 oz. when full of water. What will it weigh when full of alcohol ?

16. A mixture is made (i) of equal volumes, (ii) of equal weights of two liquids of specific gravity 0·6 and 1·2. What is the specific gravity of the mixture, there being no chemical reaction ?

17. If 50 grams of a liquid of specific gravity 1·4 are mixed with 200 grams of a liquid of specific gravity 0·8, find the specific gravity of the mixture.

18. If 20 cu. ft. of gravel, specific gravity 1·8, is mixed with 5 cu. ft. of sand, specific gravity 1·6, find the specific gravity of the mixture.

19. Which is the heavier, 10 litres of petrol or a cubic decimetre of copper, and by how much ?

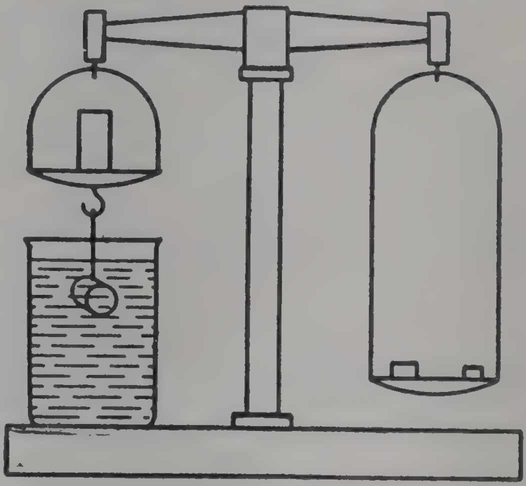
20. There are two equal glasses ; one is empty, the other contains a cubic centimetre of lead. Both are now filled to the brim with water. What will be the difference of their weights ?

Principle of Archimedes

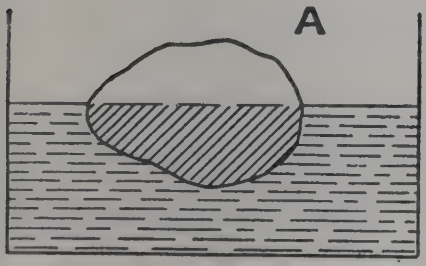
Experiment

Obtain a metal cylinder and make a water-tight case into which it exactly fits. This can be done by making a wooden base equal in diameter to the cylinder and using paper steeped in paraffin wax for the curved part.

Suspend the metal cylinder from the short pan of a Hydrostatic Balance, place the case in the pan, and counterpoise. Bring up a beaker of water so that the cylinder is completely immersed. The short pan will rise and, when water is poured into the case, equilibrium will be restored as soon as the case is full. This illustrates the Principle of Archimedes, which states that: **when a body is immersed wholly or partially in a fluid, the upthrust of the fluid on the body equals the weight of fluid displaced.**



If a body floats in a fluid, the upthrust of the fluid must equal the weight of the body—hence by the Principle of Archimedes the weight of the body equals the weight of fluid displaced. If we imagine the body A removed and the cavity (cross hatched) filled up with fluid, it is clear that this fluid would be supported by the upward thrust of the neighbouring fluid upon it. This upthrust therefore equals the weight of fluid occupying the cavity. The presence of A, instead of this fluid, will not affect the behaviour of the rest of the fluid in the vessel, which therefore exerts an upthrust on A equal to the weight of fluid which would occupy the cavity, *i.e.* the weight of fluid displaced by A.



In the first experiment above, it was noticeable that the introduction of the beaker of water caused the cylinder to rise so that it appeared to weigh less when weighed in water than when weighed in air. The fact that a body is supported of course does not reduce its weight, so it is better to refer to the **apparent** weight of a body in the fluid rather than to its weight in the fluid.

Example.—A block of wood of specific gravity 0·65 is floating in water; the volume of the block is 69·2 cu. cm. What is the volume of the portion submerged?

The weight of the block is 69·2 × 0·65 grams = 45 grams.

∴ the weight of the water displaced is 45 grams.

∴ the volume of the water displaced, *i.e.* the volume of the portion of the block submerged, is 45 cu. cm.

Example.—*A block of wood of specific gravity* 0·65 *is floating in alcohol; the volume of the block is* 69·2 *cu. cm. What is the volume of the portion submerged, if the specific gravity of alcohol is* 0·79?

The weight of the block is $69{\cdot}2 \times 0{\cdot}65$ grams.

∴ the weight of the alcohol displaced is $69{\cdot}2 \times 0{\cdot}65$ grams.

But 1 cu. cm. of alcohol weighs 0·79 gram.

∴ the volume of the alcohol displaced, *i.e.* the volume of the submerged portion of the block, is $\frac{69{\cdot}2 \times 0{\cdot}65}{0{\cdot}79}$ cu. cm.

$$= \underline{57 \text{ cu. cm.}}$$

Example.—*A lump of lead weighs* 94·62 *grams in air and* 86·34 *grams in water; it weighs* 88·70 *grams in another liquid. Find the specific gravity of* (*i*) *the lead,* (*ii*) *the other liquid.*

The support it receives from the water is $94{\cdot}62-86{\cdot}34=8{\cdot}28$ grams.

∴ the weight of the water displaced is 8·28 grams.

∴ the weight of a volume of water equal to that of the lump is 8·28 grams.

∴ the specific gravity of the lump is $\frac{94{\cdot}62}{8{\cdot}28}=\underline{11{\cdot}4}$.

Also the support it receives from the fluid is $94{\cdot}62-88{\cdot}70=5{\cdot}92$ grams.

∴ the weight of a volume of the fluid equal to that of the lump is 5·92 grams.

But the weight of the same volume of water is 8·28 grams.

∴ the specific gravity of the fluid is $\frac{5{\cdot}92}{8{\cdot}28}=\underline{0{\cdot}715}$.

Example.—*A body of volume* V *floats in water with a volume* V_1 *below the surface. Show that the specific gravity of the body is* $\frac{V_1}{V}$.

Let s be the specific gravity of the body, and w the weight of unit volume of water. Then a volume V of water weighs Vw and a volume V of the body weighs sVw. But the weight of the body equals the weight of water displaced, which is V_1w.

$$\therefore\ sVw=V_1w. \quad \therefore\ s=\frac{V_1}{V}.$$

EXERCISE XIX (b)

1. A body is floating so that the volume of the portion submerged is 52 cu. cm. What is the weight of the body (i) if in water, (ii) if in a fluid of specific gravity 0·75 ?

2. A body of weight 43·62 grams is floating in a liquid. What is the volume of the portion submerged (i) if in water, (ii) if in a fluid of specific gravity 0·84 ?

3. A wooden cuboid floats in water so that one quarter of it projects above the surface. What is the specific gravity of the wood ?

4. A wooden cuboid floats in water: the specific gravity of the wood is 0·6. What fraction of the cuboid is submerged ?

5. A wooden cuboid floats in water so that its depth below the surface is 6·4 cm., and its upper face, which is horizontal, is 3·6 cm. above the surface. When placed in another fluid, the same face as before being horizontal, the block projects only 3 cm. above the surface. Find the specific gravity (i) of the wood, (ii) of the other fluid.

6. A body of specific gravity 0·55 is floating in a fluid of specific gravity 0·90. What fraction of the body is submerged ?

7. A wooden cylinder of volume 6·4 cu. in. floats in water: the volume of the part submerged is 4·3 cu. in. What is the specific gravity of the wood ?

8. A body is floating in water so that the volume of the submerged portion is 5·7 cu. in.; when placed in another liquid it floats so that he volume submerged is 4·8 cu. in. Find the specific gravity of the iquid.

9. A body is attached by a string to the hook of a spring balance and rests partly submerged in a fluid; the reading on the spring balance is 27·4 grams, and the true weight of the body is 91·4 grams. What is the volume of the portion of the body submerged (i) if in water, (ii) if in a liquid of specific gravity 0·75 ?

10. A body is attached by a string to the hook of a spring balance nd rests wholly submerged in water; the reading on the spring balance is 42·2 grams, and the true weight of the body is 49·2 grams What is the specific gravity of the body ?

11. A body of volume 31·3 cu. cm. is attached by a string to the ook of a spring balance and rests wholly submerged in a fluid; ts true weight is 226·5 grams. Find the apparent weight (i) if in water, (ii) if in a fluid of specific gravity 0·86.

12. The true weight of a body is 173·4 grams; its apparent weight in water is 147·7 grams. What is its specific gravity? What will be its apparent weight when weighed in a liquid of specific gravity 0·82? What is the specific gravity of a liquid if the apparent weight of the body when weighed in it is 156·2 grams?

13. A lump of lead weighed in air is 104·88 grams, and weighed in water is 95·68 grams. A lump of cork is now attached to the lead, and the composite body weighs 109·06 grams in air and 87·56 grams in water. Find the specific gravity of the cork.

14. A wooden cube of edge 5 cm. and specific gravity 0·65 is floating in water. What is the weight of the body which, when placed on the upper face of the cube, will make it just level with the surface of the water?

15. Repeat Exercise No. 14 if the water is replaced by a fluid of specific gravity 0·85.

16. Explain why a balloon rises. When does it cease to rise? Why should the throwing out of ballast cause a further rise?

17. A piece of glass, specific gravity 2·5, apparently weighs 25·0 grams in air and 15·6 grams in oil. Find the specific gravity of the oil.

18. A block of wood 15 cm. long is supported lengthwise from a dynamometer. It is then slowly lowered into water and the following readings are taken:

Dynam. reading .	780	720	660	600	540	grams.
Length of block immersed . .	0	1	2	3	4	cm.

Represent these figures on a graph and from it deduce the density of the wood.

19. A pontoon in the shape of a rectangular box is to be used as the mid-support of a bridge. The pontoon is 20 ft. wide. Find its length, so that an additional load of 20 tons shall increase its immersion by not more than 6 in. When floating freely, the pontoon is immersed to a depth of 6 ft. Find its weight.

20. A balloon, which with its car and ropes weighs, when empty, 100 Kg., is inflated with 1250 cu. metres of coal-gas. If 1 cu. metre of air weighs 1·3 Kg. and 1 cu. metre of coal-gas weighs 0·75 Kg., find the additional weight which the balloon is capable of lifting.

21. A ship and its cargo weigh 3341 tons, and the section of the ship by the plane of the water is 9120 sq. ft., the sides being vertical in the neighbourhood of the water-line. Find the distance by which the water-line is lowered as the ship sails from fresh to salt water of specific gravity 1·03.

22. A rectangular stone trough is 3 ft. 8 in. long, 2 ft. 7 in. wide, nd 1 ft. 2½ in. deep externally, and weighs when empty 863 lb. It ill hold 39 gallons of water. Find the specific gravity of the stone.

23. An iron pipe has a bore of 8 in., the thickness of the metal eing 0·5 in. If the pipe weighs 41·6 lb. per foot length, find the ensity of the iron.

24. Two pieces of iron of specific gravity 7·7, suspended from the wo arms of a balance, the one in water and the other in alcohol pecific gravity 0·85), are found to balance one another. Find the tio between their true weights.

25. The specific gravity of gold is 19·3 and of silver 10·5. What is he composition of an alloy of gold and silver whose specific gravity s 17·6, no change in volume being supposed to accompany the dmixture of the metals?

26. A piece of lead (specific gravity 11·4) and a piece of sulphur specific gravity 2·0), suspended from the ends of a balance, just alance when both are immersed in water. Compare their volumes.

27. A flask which, when filled with water, weighs altogether 410 rams, has 80 grams of a solid introduced and, being then filled up ith water, weighs 470 grams. Find the specific gravity of the solid.

28. An empty biscuit tin weighs 1 lb.: the area of its base is 2 sq. in.; it floats upright in water. To what depth is it immersed?

29. A tank measuring 2 ft. each way contains water to a depth of 8 in. Find the edge of the largest cube of specific gravity 0·8 which an float in the tank without spilling any water, and find the depth o which it is immersed.

30. A body is totally immersed in water and appears to weigh grams. The body is then totally immersed in a liquid of specific ravity s, and appears to weigh Q grams. Find expressions in terms f P, Q, s for the weight of the body, its volume, and its specific ravity.

REVISION PAPERS

(R 41–50)

R 41

1. Write down to one significant figure the values of

(i) $(0{\cdot}8973)^3$. (ii) 60391×8172. (iii) $913 \div 0{\cdot}0785$.

2. Evaluate to three significant figures by logarithms

(i) $\dfrac{8{\cdot}672}{109 \times 3{\cdot}174}$. (ii) $\sqrt[3]{(0{\cdot}5374)}$.

3. Find the weight of a block of lead (specific gravity 11·4) 9·6 cm. long, 5·5 cm. wide, 2·5 cm. deep.

4. What is $3\frac{3}{4}$ per cent. of £5, 4s. 2d. ?

5. Iron weighs 450 lb. per cubic foot. If sheet iron weighs 15 lb. per square foot of surface, what is the thickness of the sheet in inches ?

6. Discs of diameter $\frac{7}{8}$ in. are stamped out of a rectangular sheet of rubber 8 in. square and $\frac{1}{4}$ in. thick. How many can be cut out, and what volume of rubber is wasted ?

R 42

1. What decimal of £7 is £5, 14s. 4d. ?

2. Evaluate $(\cdot 0264)^{-\frac{1}{3}}$.

3. A farmer floods for skating a field 500 yd. by 80 yd., the average depth of water being 5 in. How much per 1000 gallons was he charged for water if the company's bill was £10 ?

4. Find the diameter of a circle whose area is 236 sq. cm.

5. Find the volume of metal in a tube 48 ft. long, whose bore is 3·6 in., the thickness of the metal being 0·35 in.

6. A solution is made of acid diluted with water in the ratio 2 : 7. How much acid will there be in 500 c.c. of solution ?

R 43

1. The length and breadth of a rectangle are given as $6\cdot 3 \pm 0\cdot 02$, $4\cdot 8 \pm 0\cdot 02$ cm. Express its area in a similar form.

2. If the radii of the plane ends of the frustum of a cone are x, y inches, and if the height of the frustum is h inches, the volume of the frustum is $\frac{\pi h}{3} \times \frac{x^3 - y^3}{x - y}$ cu. in. Evaluate this expression if $x = 3\cdot 61$, $y = 2\cdot 17$, $h = 1\cdot 83$.

3. An insurance premium is 1s. 9d. per £100. What is the premium on a policy for £3200 ?

4. A train travels up an incline of 1 in 120 at 45 miles an hour. How many feet does it rise vertically in 3 minutes ?

5. How many cubic feet of earth must be removed to sink a well 5 ft. in diameter and 40 ft. deep ?

6. Telegraph wire weighs 500 lb. per mile, and the material from which it is made weighs 460 lb. per cubic foot. Find the area of the cross-section of the wire ?

R 44

1. Find the H.C.F. of 22,100 and 18,759.

2. Find the area of a trapezium whose parallel sides are 16 ft. 4 in. and 19 ft. 8 in., the altitude being 5 ft. 2 in.

3. In a town whose rateable value is £105,000 the School Committee requires £4156, 5s. Find the amount of school rate paid by a person whose house has a rateable value of £66, 13s. 4d.

4. Find the weight of $2\frac{1}{4}$ cu. ft. of lead whose specific gravity is 11·4.

5. Workmen demand an increase of 6·95 per cent. in their wages to bring them up to the level of the previous year before they were reduced. By how much per cent. had they been reduced ?

6. 2,615,423 tons of goods are valued at £786,514, 15s. Find, without logarithms, the average value per ton to the nearest penny.

R 45

1. Find the value of

(i) $\frac{3}{4} \div 1\frac{1}{2} - \frac{1}{4}$ of 3. (ii) $\frac{3}{4} \div [(1\frac{1}{2} - \frac{1}{4})$ of $3]$.

2. Evaluate to three significant figures by logarithms

(i) $\dfrac{1}{(17{\cdot}41)^2} - \dfrac{1}{(23{\cdot}64)^2}$. (ii) $412 \times 4\frac{1}{4} \times \frac{29}{365} \times \frac{1}{100}$.

3. The area of a square field is $4\frac{1}{2}$ acres. What is the length of each side correct to the nearest yard ?

4. If 35 per cent. of a sum of money is 40 guineas, what is 45 per cent. of the same sum ?

5. The internal diameter of an iron pipe is 8 in., and the metal is half an inch thick : the pipe is 10 ft. long. What is its weight if 4 cu. in. of the metal weigh 1 lb. ?

6. A lump of iron, specific gravity 7·2, weighs 3·62 Kg. in air. What is its apparent weight in water ?

R 46

1. Find to four decimal places the value of $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ . \ . \ .$

2. Find the area of the ring between two concentric circles whose radii are 11·4 cm. and 9·6 cm.

3. Find the value of $\dfrac{\sqrt{17{\cdot}6} \times (1{\cdot}23)^2}{\sqrt[3]{{\cdot}0684}}$.

4. When a cork whose specific gravity is 0·24 is placed in water, what fraction of its volume will be immersed? If the volume immersed is 62·4 c.c., find the weight of the cork.

5. Show, without using tables, that $\log_{10}3{\cdot}16 \simeq 0{\cdot}5$ and that $\log_{10}1{\cdot}778 \simeq 0{\cdot}25$.

6. Some tobacco, after losing 14 per cent. of its weight by evaporation of moisture, then weighs 13 cwt. 6 lb. What was its original weight?

R 47

1. Multiply 9s. 11d. by 1200.

2. Find the square root of (i) 21·4369, (ii) $27\frac{46}{49}$.

3. The number of cubic feet of gas which pass per hour through a main of diameter d inches and length l yards is $1100\sqrt{\left(\frac{2{\cdot}2d^5}{l}\right)}$. Find the number of cubic feet per minute for a main of diameter $1\frac{3}{4}$ in. and length $\frac{1}{4}$ mile.

4. A swimmer who takes 30 strokes per minute can swim a mile in 38 min. 20 sec. What is the average length in yards of each stroke?

5. In 1913 a firm paid £7000 in wages and £2100 for other expenses and made a profit of £1400. In 1924 it paid 75 per cent. more in wages and 45 per cent. more for other expenses, but its receipts had increased by 60 per cent. By what percentage had its profits risen?

6. The area of the curved surface of a solid circular cylinder is 8 sq. in. and the diameter is 3 in. Find its volume.

R 48

1. Arrange in ascending order the fractions $\frac{5}{11}$, $\frac{8}{17}$, $\frac{14}{31}$, $\frac{15}{32}$.

2. A metal cylinder is to be 35 cm. long and 3·8 cm. in diameter. If both length and radius are made 1 per cent. too large, what is the percentage error in the volume?

3. A bullet is to be made of an alloy of lead (specific gravity 11·4) and antimony (specific gravity 6·7). How many c.c. of lead are there in every 100 c.c. of the alloy, if the specific gravity of the bullet is 9·5?

4. Write the formula $\frac{4}{3}\pi r^3$ for the volume of a sphere in terms of d, the diameter. Find the diameter of a sphere whose volume is 216·4 c.c.

5. A grocer prices his tinned fruit so as to make 8 per cent. profit on each tin. If $\frac{1}{20}$th of the stock proves unfit for sale, what is his percentage loss or gain on the whole stock?

6. 6456 tons of tin are sold for £1,763,842. What is the average price per ton to the nearest shilling?

R 49

1. A exceeds B by 40 per cent., and exceeds C by 20 per cent. Find the ratio of B to C.

2. Find correct to three significant figures the diameter of a sphere of volume 7 cu. in. [$V=\frac{4}{3}\pi r^3$.]

3. Find with as little working as possible the value of (i) 9997×2731, (ii) $\sqrt{(2^3\times3^4\times5^2)}$, taking $\sqrt{2}=1{\cdot}4142$.

4. The driving-wheel of an express train is 6 ft. 5 in. in diameter. How many times does it turn round in a run of 12 miles?

5. A cube of pitch pine of edge 1 in. weighs 0·025 lb.; a cubic foot of water weighs 62·3 lb. If the cube is placed in water, what is the height of its upper surface above the water level?

6. The transport of the world's yearly production of petroleum would require a pipe of 6 ft. 5 in. internal diameter, in which the oil flows continuously at 2 m.p.h. Find correct to two significant figures in cubic feet the yearly production.

R 50

1. How many articles, each worth £3, 17s. 8d., can be bought for £578, 12s. 4d.?

2. Evaluate $$\frac{\sqrt[3]{0{\cdot}164\{(52{\cdot}3)^2-(17{\cdot}8)^2\}}}{21{\cdot}4+16{\cdot}73\times0{\cdot}8412}.$$

3. The external dimensions of a metal vessel of cylindrical shape closed at both ends are: height 4 ft. 6 in., diameter 2 ft. 9 in. The metal is $\frac{1}{4}$ in. thick, and its specific gravity is 7·0. Find the weight of the vessel.

4. An article is sold for £21, 9s. at a gain of 4 per cent. What was the cost price, and what selling price would have given a gain of 8 per cent.?

5. Draw a graph to show the cost at $3\frac{3}{4}$d. per yard of any number of yards up to 20. From it find how many yards can be bought for 4s.

6. If 12 tons are carried 34 miles for £6, 12s., find the cost of transporting 51 tons for 30 miles.

PROBLEM PAPERS

(P 41–50)

P 41

1. What is the percentage change in the area of a rectangle if each side is (i) increased by 10 per cent., (ii) decreased by 10 per cent. ?

2. A bucket full of water weighs 25 lb. : half full it weighs 15 lb. How many gallons of water does the bucket hold, assuming that a pint of water weighs a pound and a quarter ?

3. What is x if (i) $10^x=12$, (ii) $8^x=12$?

4. If a number is greater than 100 and less than 1000, and is divisible by 9 and 11, show that the sum of its first and last digits must be 9.

5. A wooden cylinder A, 1 in. in diameter, 6 in. high, floats with its axis vertical in water in a cylindrical tin B, 2 in. in diameter and 6 in. high. The top of A is 1 in. above the top of B, and the water is 5 in. deep. If A is pressed down to touch the bottom of B, its axis being still vertical, how much will the water-level rise ?

P 42

1. A man buys six houses for £5000 in all : the rent of each is £70 a year. Four of them are occupied for a whole year and the other two are empty for half a year each. Repairs amount to £3 per house per year. What annual percentage does the owner obtain on his investment ?

2. Express the price of coal in (Swiss) francs per tonne (1000 Kg.) when coal is at 49s. a ton and £1=25·2 (Swiss) francs, taking 1 Kg.=2·20 lb.

3. Mr Gladstone was forty in the year x^2 A.D. When was he born ?

4. A small stoppered bottle when empty weighs 23·51 grams, and when full of water weighs 74·77 grams. If it is filled with alcohol, it weighs 65·72 grams. Find the specific gravity of the alcohol.

5. The length of a circular running track, measured 12 in. from the inner edge, is 440 yd. What is the length of the inner edge ? If the track is 15 ft. wide, how many spectators will be able to stand along the outer edge of the track, if each spectator takes up 2 ft. ? How many rings of spectators would be formed if there are 2000 spectators, allowing 2 ft. breadth per ring ?

P 43

1. Sir Isaac Newton made a thermometer with the freezing-point of water marked 0° and the heat of the human body 12°. On the Fahrenheit scale these are 32° and 98°. What is the temperature of boiling water (212° F.) on Newton's scale?

2. An amateur photographer spoils x per cent. of the photographs he takes. How many does he take if he has 100 successes?

3. A piece of lead was found to weigh 105·3 grams in air and 96·02 grams in water. A piece of cork is fastened to the lead, and together they weigh 121·5 grams in air and 73·54 grams in water. Find the specific gravity of (i) the lead, (ii) the cork.

4. If the number of dry-bobs at Eton is 40 per cent. greater than the number of wet-bobs, and if they are respectively greater by 204 and 12 than the total numbers at Winchester, find the numbers at Eton and Winchester.

5. Shrapnel bullets are spherical and weigh 41 to the lb., and are made of metal weighing 682 lb. per cubic foot. Find the diameter of a bullet in inches, correct to $\frac{1}{100}$ in.

P 44

1. A fast train leaves Winchester at 1.0 p.m. and travels to London (66 miles) at 50 m.p.h. without a stop; a slow train leaves Winchester at 12.30 p.m., stops for 10 minutes at Basingstoke (18 miles), and otherwise travels at 30 m.p.h. Find how far from Winchester the fast train passes the slow train.

2. A lawn is 110 ft. long and 50 ft. wide: there is no slope lengthways, but one side is 3 ft. 6 in. higher than the other. How many cubic yards of soil must be *shifted* to make it level? [The 50 ft. may be taken as the width measured horizontally.]

3. If a motor of horse-power H drives a fan of diameter d feet, the fan delivers V cubic feet of air per second where $V=14d^2\sqrt{H}$. Find V if the diameter is 10 in. and $H=0·7$.

4. If the specific gravity of ice is 0·9, find the weight of debris that can be carried on an ice-floe 25 ft. thick with a level surface of 2 acres. Take 1 cu. yd. of water to weigh $\frac{3}{4}$ ton.

5. The lengths of two altitudes of a triangle are 3, 5 in. Prove that the length of the third altitude must be less than $7\frac{1}{2}$ in. What length must the third altitude exceed?

P 45

1. A housekeeper has her supplies sent from stores in London, where prices are 5 per cent. below the local tradesman's. Carriage and postage cost 17s. 8d., yet she saves £1, 4s. What would the supplies have cost her locally?

2. In a race the scratch man A passes a competitor B 200 yd. before the finish, which he reaches 25 sec. afterwards, 3 sec. before B. Assuming uniform speed, find how far B was behind A when A passed the post.

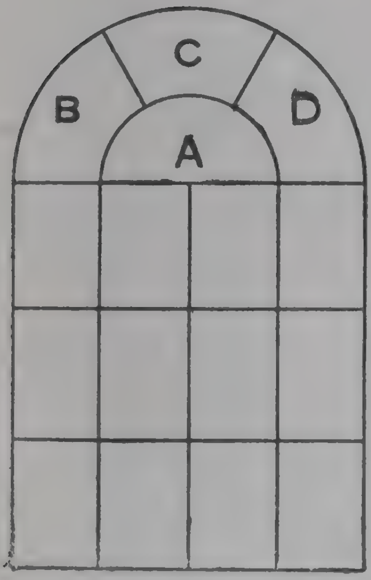
3. A boy who is told that his answer is 12 per cent. too small increases it by 12 per cent. and is then told that his error is 3·2. What was the correct answer?

4. A man owes £215, 5s. now. He agrees to pay two equal instalments, one now and one in a year's time. How much should each instalment be, reckoning 6 per cent. per annum?

5. A watch which gains 5 sec. in every 3 min. was set right at 8 a.m. What was the true time in the afternoon of the same day when the watch indicated 3 hr. 15 min.?

P 46

1. Two cog-wheels geared together have 36 and 54 teeth respectively. If the smaller makes 340 revolutions per minute, find the speed of the other.



2. The window has a semicircular top divided into a semicircular pane A, and three equal panes, B, C, and D, with 12 equal panes below. If the width of the window is $4a$, find the area of each pane and compare the areas of A and B. Find the height of each rectangular pane if it is to have the same area as A. When this is the case, find the height of the window and compare the perimeters of the various panes.

3. The ore from a lead mine contains at first 15·9 per cent. of lead. After washing, in which no lead is lost, it contains 87·45 per cent. of lead. How much material is washed away from 216 tons 5 cwt. of the original ore?

4. How many whole numbers are there between 1800 and 2400?

5. A man's income, which increased by the same percentage for four consecutive years, rose from £1620 to £2140. Find the percentage.

P 47

1. A machine costing £1050 can do the work of 5 men, each earning £2, 12s. 6d. a week. It needs a mechanic at a wage of £3, 3s. a week and a boy at £1, 5s. a week, and consumes 33 gallons of petrol a week at 2s. 4d. a gallon. After 5 years' use, it is sold for £185. Taking a year to consist of 50 working weeks, find how much the employer gains during 5 years by installing the machine, assuming that he could have got £250 interest by investing the money used to buy the machine, and neglecting the interest on the money saved in wages.

2. A tailor allows a customer 5 per cent. off his bill. How much per cent. must he add to the cost of his goods if he is to make a net profit of 10 per cent. ?

3. Two pendulums are set swinging together. One beats seconds accurately, but the other is slightly slower. It is found that they are swinging together at intervals of 36 min. Find the time of swing of the second pendulum.

4. A bar of iron, specific gravity 7·2, totally submerged, is suspended in water from a spring balance which reads 12·4 lb. What would the balance read if a liquid of specific gravity 0·8 were substituted for the water ?

5. The radii of the two ends of a truncated cone 2 ft. high are 2 ft. 6 in. and 1 ft. 8 in. Find the height of the complete cone and the volume of the truncated cone.

P 48

1. When the duty on food is increased by 17 per cent. it is said to increase the cost of living by 3 per cent. Express the duty paid before the increase as a percentage of the cost of living.

2. The capital of a company is £50,000. In 1923 the gross income was £325,000, which, after paying working expenses, left a balance equal to 25 per cent. of the capital. In 1924 the gross income increased by 10 per cent. and the working expenses increased by 12 per cent. Find the balance, and find what percentage it was of the capital.

3. To watch a road 60 miles long, a cyclist and a trooper start at the same time, one from each end, to patrol the road from end to end continuously. If the average speeds are 10 mls./hr. and 6 mls./hr., find where they meet for the second time.

4. A cylindrical can contains water to a depth of 2 ft. A long

cylindrical bar is placed vertically in the water, so that it reaches the bottom of the can and the water rises 1 in. Compare the radii of the bar and the can.

5. A mixture contains 68 per cent. water, 18 per cent. alcohol, and the remaining volume is solid matter. How much water must be added to every gallon of the mixture to reduce the percentage of alcohol in it to 15?

P 49

1. Evaluate $(0{\cdot}208)^{1{\cdot}4}$.

2. A country has eleven million men of military age. Of these 6 per cent. are unfit for any service. Of those who can be utilised, 700,000 are required for civilian work, and 64 per cent. of the remainder are chosen by ballot as conscripts. How many are left who can still serve as volunteers?

3. A fine thread is wound exactly 10 times round a cylinder: the total length of thread required is 39·27 in., correct to $\frac{1}{100}$ of an inch. What is the diameter of the cylinder?

4. The pressure of water at any point in a vertical pipe is proportional to the depth of that point below a certain level. At 30 ft. below this level the pressure is 14 lb. per square inch. The pressure in a tap on the ground floor is 50 lb. per square inch, and in a tap on the fourth floor is 28 lb. per square inch. Find the height of the fourth floor.

5. How many cubic feet of concrete will be required to make an open tank in the shape of a rectangle with semicircular ends? Inside dimensions: total length 15 ft., width 5 ft., depth 6 ft.; walls to be 6 in. thick, and floor 1 ft. thick.

P 50

1. A distribution is to be made of the following sums: £1, 16s.; £3, 12s.; £1, 4s.; 16s.; £3, 4s.; £1; £2, 8s.; 8s. It is then found that the average amount can be increased by 9s. Alter the amounts in the same proportion to allow for this.

2. Water is pumped at the rate of 1000 gallons per minute out of a reservoir with a surface of 5400 sq. ft. into one with a surface of 3600 sq. ft. How many minutes will it take to diminish the original difference between their levels by 1 inch?

3. Cloth is wound tightly round a wooden cylinder of diameter 1 ft. 6 in., and the total diameter of the roll is 1 ft. 10 in. Find what diameter the cylinder should have for the total diameter to be 1 ft. 6 in. when the cloth is rolled on it.

4. Two hundred and forty-two tents 13 ft. high are made of canvas. If the height is reduced to 12 ft., how many tents of a similar shape can be made with the same amount of canvas ?

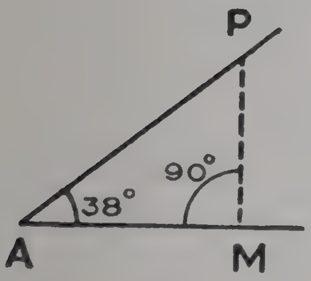
5. After a census it was found that in a certain district of a country 24 per cent. of the people were foreigners, while in the rest of the country only 16 per cent. were foreigners, and in the whole country 18 per cent. were foreigners. What percentage of the foreigners in the country lived in that district ?

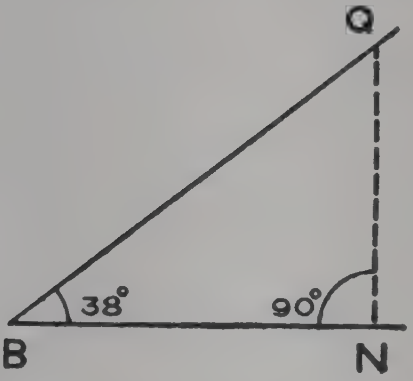
CHAPTER XX

TRIGONOMETRICAL RATIOS

The Tangent of an Angle

DRAW two angles A, B, each (say) 38°, and from points P, Q on one arm of each angle draw the **perpendiculars** PM, QN to the other arm. Then the triangles PAM, QBN are equiangular and are therefore the same shape ; we may in fact regard the triangle PAM as a scale drawing of the triangle QBN.





$$\therefore \text{ the ratio } \frac{PM}{AM} \text{ equals the ratio } \frac{QN}{BN}.$$

$\therefore$ the value of the ratio $\frac{PM}{AM}$ does not depend on the length of AP, but only on the size of the angle PAM.

The ratio $\frac{PM}{AM}$ is called the **tangent of the angle PAM**, and is written tan PAM, in this case tan 38°, since $\angle PAM = 38°$. In other words, if a perpendicular is let fall from any point on one arm of an angle to the other arm,

$$\textit{the tangent of the angle} = \frac{\textit{opposite side (PM)}}{\textit{adjacent side (AM)}}.$$

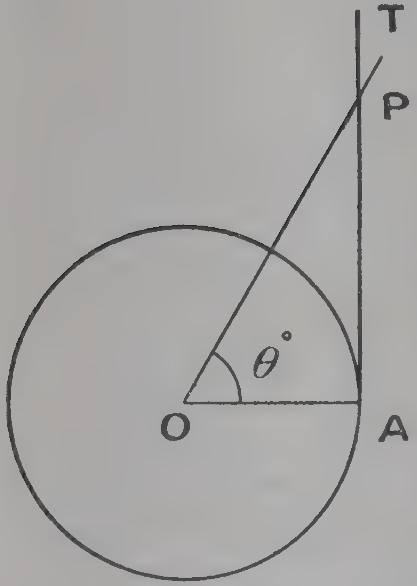
The ratio $\frac{PM}{AM}$ is also known as the *gradient* of the line AP.

We may find the approximate value of the tangent of any angle by measurement and computation.

Here $\tan 38 = \frac{QN}{BN} = \frac{2\cdot5 \text{ cm.}}{3\cdot2 \text{ cm.}}$ approx. $= 0\cdot78$ approx.

The tangents of angles are calculated, once for all, and entered in a table from which they can be obtained when required. It should be noted that in the figure $\angle NQB = 90° - 38° = 52°$.

$$\therefore \tan 52° = \frac{BN}{QN} = \frac{1}{\tan 38°}.$$



The tangent of any angle is in fact equal to the reciprocal of the tangent of its complement, $\tan \theta° = \frac{1}{\tan (90° - \theta°)}$.

Draw a circle of unit radius, say 1 dm. Let O be the centre and OA any fixed radius. Let AT be the tangent at A to the circle. Draw a line making any given angle $\theta°$ with OA, and let it cut AT at P. Suppose AP $= x$ dm., then $\tan \theta = \frac{AP}{OA} = \frac{x}{1} = x$.

∴ the tangent of the angle is equal to the measure of the portion cut off on the tangent, the radius being of unit length. The name "tangent of an angle" is derived from this property. The values of the tangents of angles of different sizes may be found by drawing and measurement.

Example.—Find by drawing the value of tan 31° *and the angle whose tangent is* 1·8.

(i) To find tan 31°. Draw on squared paper a line AB of unit length: BD is the perpendicular at B to AB. Construct $\angle BAC = 31°$

and let AC cut BD at P. By measurement BP=0·6 units.

$$\therefore \tan 31^\circ = \frac{BP}{BA} = \frac{0{\cdot}6}{1} = 0{\cdot}6 \text{ approx.}$$

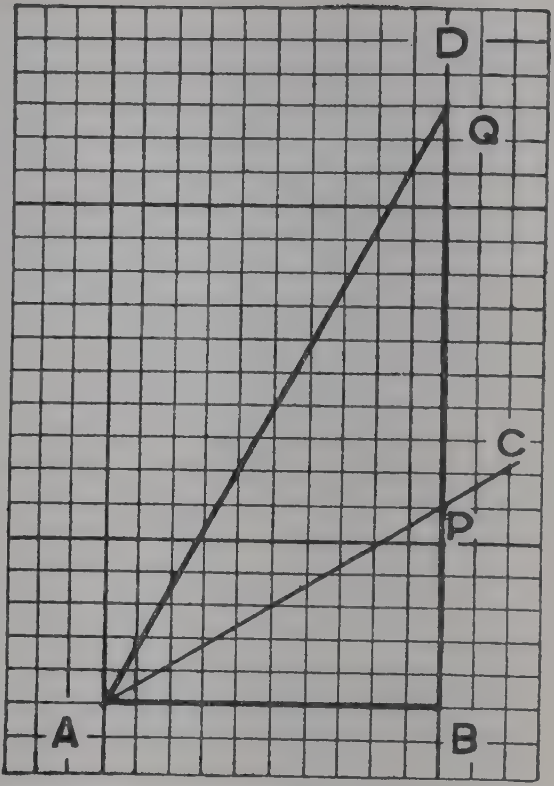
(ii) To find the angle whose tangent is 1·8. From BD cut off BQ equal to 1·8 units. By measurement ∠BAQ=61°.

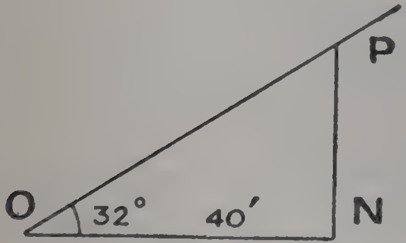
$$\therefore \tan 61^\circ = \frac{BQ}{BA} = \frac{1{\cdot}8}{1} = 1{\cdot}8.$$

∴ the angle whose tangent is 1·8 is 61° approx. This is often written in the form $\tan^{-1}(1{\cdot}8) = 61^\circ$.

Example.—A vertical pole casts a shadow 40 ft. long when the Sun's elevation is 32°. Find the height of the pole.

PN is the pole; ON is the shadow.





$$PN = ON \times \frac{PN}{ON}$$

$$= 40 \times \tan 32^\circ = 40 \times 0{\cdot}625 = 25 \text{ ft.}$$

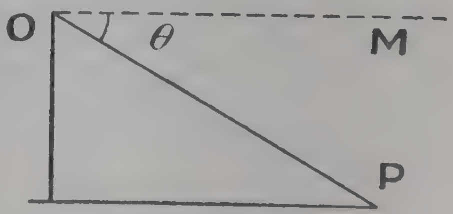
N.B.—Always write down the side required (PN) and express it in terms of the known side (ON) thus: $PN = ON \times \frac{PN}{ON}$.

Example.—The shadow of a stick 6 ft. long is 8 ft. Find the Sun's elevation.

If PN is the pole and ON is the shadow

$$\tan PON = \frac{PN}{ON} = \frac{6}{8} = 0{\cdot}75.$$

∴ from the tables, ∠PON=37° approx.



Note.—(i) If you use four-figure tables, you will find that a closer approximation for ∠PON is 36° 52′.

(ii) If the observer (O) is below an object (P), the angle between the horizontal ON and the line joining O to P is called the *angle of elevation* of P from O.

If the observer O is above P, then the angle θ between the *horizontal* OM and OP is called the *angle of depression* of P from O.

EXERCISE XX (a)

1. Find by drawing and measurements approximate values of (i) tan 25°, (ii) tan 45°, (iii) tan 65°.

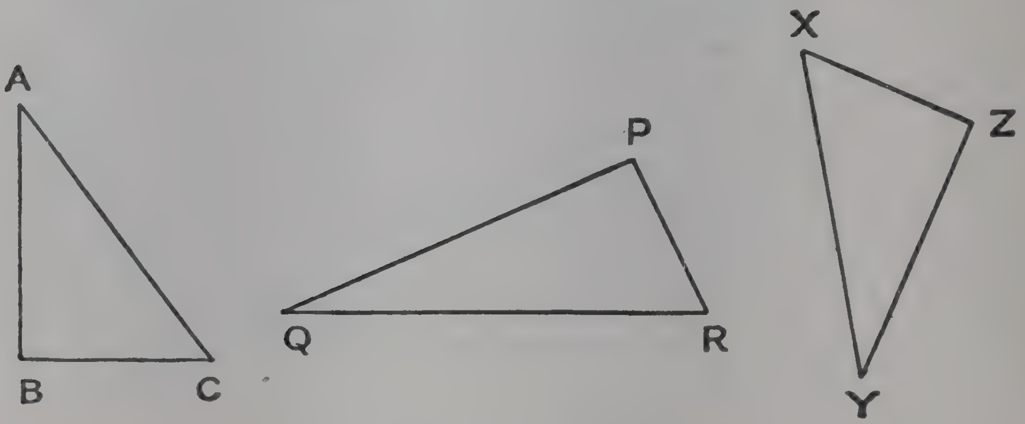
2. Find by drawing and measurements the angles whose tangents are (i) 0·4, (ii) 0·7, (iii) 1·6.

In the diagram, the angles at B, P, Z are each 90°.

Write down expressions for the following :

3. tan BCA. 4. tan PQR. 5. tan XYZ. 6. tan BAC.

7. $\frac{PQ}{PR}$. 8. $\frac{BC}{AB}$. 9. $\frac{YZ}{XZ}$. 10. $\frac{PR}{PQ}$.



Using the same diagrams, make the following calculations :

11. BC=10 in., ∠ACB=49° ; find AB.
12. QP=5 in., ∠PQR=38° ; find PR.
13. XZ=20 in., ∠YXZ=62° ; find YZ.
14. AB=5·8 cm., BC=2 cm. ; find ∠ACB.
15. PQ=4 cm., PR=3 cm. ; find ∠PQR.
16. XZ=2·7 cm., YZ=3·8 cm. ; find ∠YXZ.
17. AB=10 cm., ∠ACB=50° ; find BC. [What is ∠BAC ?]
18. PQ=6 cm., ∠PRQ=65° ; find PR. [What is ∠PQR ?]

19. A man, standing 100 ft. from the foot of a tower, observes that the angle of elevation of the top of the tower is 48°. What is the height of the tower ?

20. What is the angle of elevation of the top of a tower 135 ft. high from a point on the ground 90 ft. from the foot of the tower ?

21. A man on the top of a cliff 150 ft. high sees that the angle of depression of a boat is 31°. How far is the boat from the foot of the cliff ?

22. A man starts from O and walks 160 yd. east and then 84 yd. north. What is his final bearing from O ?

23. The pole of a bell tent is 8 ft. long, and the diameter of the base is 14 ft. What angle does the slant side of the tent make with the ground ?

24. A chord of a circle is 9 cm. long and subtends an angle of 80° at the centre of the circle. What is the distance of the chord from the centre ?

25. The side of an equilateral triangle is 6 in. What is its height ?

26. A ladder leaning against a house makes an angle of 63° with the ground and just reaches a window 20 ft. above the ground. How far is the foot of the ladder from the wall ?

27. A rectangular sheet of paper is 10 in. long, 7 in. wide. At what angle do the diagonals cut each other ?

28. The steps of a staircase are 14 in. deep and 6 in. high. What is the slope of the staircase ?

29. Two men stand on opposite sides of a tower 97 ft. high and measure the angles of elevation of the top of the tower as 27° and 36° respectively. How far apart are the men ?

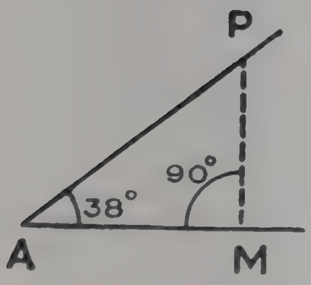
30. A man, standing on a cliff 140 ft. high, observes two boats in a vertical plane with him. The angles of depression are 37°, 53°. What is the distance between the boats ?

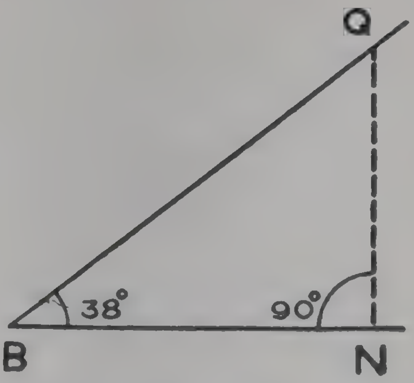
31. The shadow of a telegraph pole is 14 ft. long when the Sun's elevation is 58°. What is the length of the shadow when the elevation is 29° ?

32. Tangents are drawn from a point to a circle of radius 6 cm. ; the angle between the tangents is 84°. Find the length of each tangent.

Sine and Cosine of an Angle

Using the same figure as on p. 257, the triangle PAM is a scale





drawing of the triangle QBN, and therefore $\frac{PM}{AP}=\frac{QN}{BQ}$; and so the value of the ratio $\frac{\text{PM}}{\text{AP}}$ does not depend on the length of AP, but only on the size of the angle PAM.

The ratio $\frac{\text{PM}}{\text{AP}}$ is called the **sine of the angle PAM**, and is written sin PAM, in this case sin 38°, since $\angle\text{PAM}=38°$.

By the same argument, we see that the value of the ratio $\frac{\text{AM}}{\text{AP}}$ also does not depend on the length of AP, but only on the size of the angle PAM.

The ratio $\frac{\text{AM}}{\text{AP}}$ is called the **cosine of the angle PAM**, and is written cos PAM, in this case cos 38°.

In words, if a perpendicular is let fall from any point on one arm of an angle to the other arm,

$$\textit{the}\ \text{sine}\ \textit{of the angle}\ (\textit{sin}\ \text{PAM})=\frac{\textit{opposite side}\ (\text{PM})}{\textit{hypotenuse}\ (\text{AP})},$$

$$\textit{the}\ \text{cosine}\ \textit{of the angle}\ (\textit{cos}\ \text{PAM})=\frac{\textit{adjacent side}\ (\text{AM})}{\textit{hypotenuse}\ (\text{AP})}.$$

Since the hypotenuse is greater than either of the other sides, neither the sine nor the cosine of an angle can exceed 1.

It is also important to notice that the sine of any angle and the cosine of the complementary angle are equal. Suppose in the figure, $\angle\text{PAM}=\theta°$, then $\angle\text{APM}=90°-\theta°$; and we have

$$\sin\theta°=\frac{\text{PM}}{\text{AP}}=\cos\text{APM}=\cos(90°-\theta°);$$

thus $\sin 20°=\cos 70°$ or $\cos 10°=\sin 80°$. The cosine tables can therefore be deduced from the sine tables by writing them backwards.

It has been stated that the gradient of a *line* is measured by the tangent of the angle of slope, but the gradient of a road is measured by the sine of the angle. Thus a road whose gradient is 1 in 12 is one which rises 1 ft. for every 12 ft. along the road.

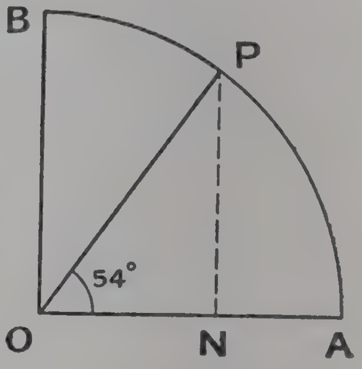
Example.—Find by drawing and measurement the values of sin 54° *and cos* 54°.

Draw a circle centre O, radius 10 cm. [The figure in the text is on a reduced scale.] Take a radius OA, and draw a line making 54° with

OA, and let it cut the circle at P. Draw PN perpendicular to OA. Measure PN, ON. We find PN=8·1 cm., ON=5·9 cm. Then

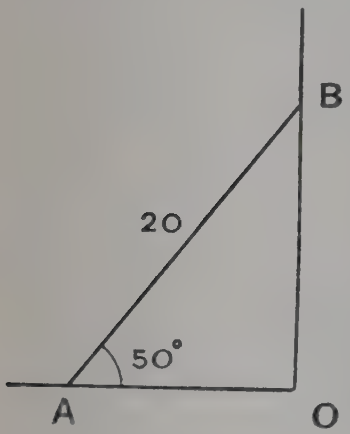
$$\sin 54^\circ = \frac{PN}{OP} = \frac{8{\cdot}1}{10} = 0{\cdot}81$$

$$\cos 54^\circ = \frac{ON}{OP} = \frac{5{\cdot}9}{10} = 0{\cdot}59.$$



Example.—A ladder 20 ft. long leans against a wall and is inclined at 50° to the ground. How far from the wall is the foot of the ladder? How high up the wall does the ladder reach?

AB is the ladder.



$$AO = AB \times \frac{AO}{AB}.$$

$$\therefore\ AO = 20 \times \cos 50^\circ = 20 \times 0{\cdot}643 = \underline{12{\cdot}9 \text{ ft.}}$$

Further $\quad BO = AB \times \dfrac{BO}{AB}$

$$= 20 \times \sin 50^\circ = 20 \times 0{\cdot}766$$

$$= \underline{15{\cdot}3 \text{ ft.}}$$

Note.—Since $\angle OAB = 50^\circ$, $\angle OBA = 40^\circ$, we could therefore say $AO = AB \times \dfrac{AO}{AB} = AB \times \sin OBA = 20 \times \sin 40^\circ$.

EXERCISE XX (b)

1. Find by drawing and measurement approximate values of (i) sin 37°, (ii) cos 37°, (iii) sin 63°, (iv) cos 63°.

2. Find by drawing and measurements the angle (i) whose sine is 0·39, (ii) whose cosine is 0·39, (iii) whose cosine is 0·92. [These could be written $\sin^{-1}(0{\cdot}39)$, $\cos^{-1}(0{\cdot}39)$, $\cos^{-1}(0{\cdot}92)$.]

In the diagram (p. 264) the angles at A, Q, Z are each 90°. Write down expressions for the following, giving all possible answers :

3. $\dfrac{AB}{BC}$. **4.** $\dfrac{QR}{PR}$. **5.** $\dfrac{XZ}{XY}$. **6.** $\dfrac{AC}{BC}$.

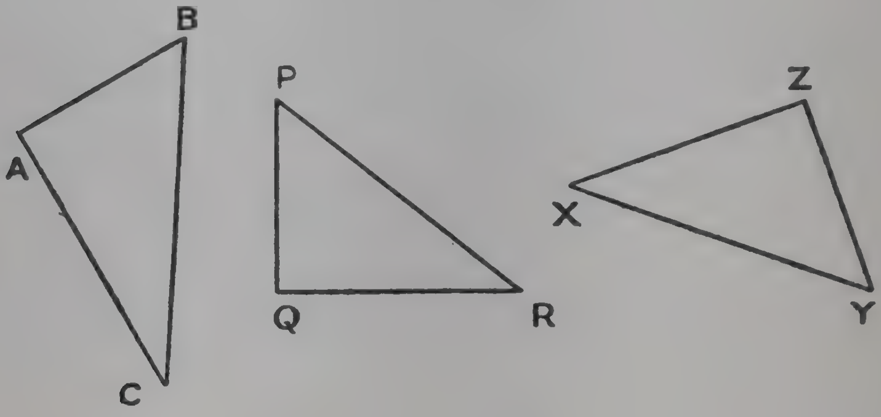
7. $\dfrac{XZ}{ZY}$. **8.** $\dfrac{PQ}{PR}$. **9.** $\dfrac{PQ}{QR}$. **10.** $\dfrac{AC}{AB}$.

11. sin ACB. **12.** cos PRQ. **13.** cos ZYX. **14.** cos ABC.

15. sin QPR. **16.** tan ZXY. **17.** cos QPR. **18.** tan ABC.

Using the same diagrams, make the following calculations :

19. $BC=20$ cm., $\angle ACB=25^\circ$; find AB.
20. $PR=5$ in., $\angle QPR=32^\circ$; find PQ.
21. $XY=7$ cm., $\angle ZYX=54^\circ$; find XZ, YZ.
22. $AB=4$ in., $BC=7$ in.; find $\angle ABC$.
23. $XZ=4$ in., $XY=5$ in.; find $\angle XYZ$.



24. $PR=3$ cm., $\angle QRP=51^\circ$; find QR.
25. $XY=5$ in., $\angle XYZ=55^\circ$; find XZ.
26. $QR=3$ in., $\angle QRP=59^\circ$; find the perpendicular from Q to PR.

27. A ladder 25 ft. long stands against a vertical wall, and its foot is $6\frac{1}{2}$ ft. from the wall. What angle does the ladder make with the ground ?

28. A hill slopes upwards at an angle of 27° with the horizontal. What height does a man rise when he walks 30 yd. up the slope ? What is the gradient of the path ?

29. A man starts at O and walks 200 yd. in a direction 27° east of north. How far east of O is he ? How far north of O is he ?

30. The string of a kite is 650 ft. long and makes an angle of 58° with the ground. How high is the kite ?

31. The pole of a bell tent is 8 ft. long and a slant side of the tent is 10 ft. long. What angle does the side of the tent make with the ground ?

32. A door is 4 ft. wide and can only be opened to an angle of 51°. What is the widest packing case that can be pushed in, without being lifted ?

33. At what angle does the ground slope, if a man rises a height of 114 ft. after walking 200 yd. up hill ? What is the gradient of the slope ?

34. A picture of height $2\frac{3}{4}$ ft. rests with its lower edge against a wall and makes an angle of 12° with it. How far is the top of the picture from the wall?

35. A regular pentagon is inscribed in a circle of radius 3 inches. What is the length of its side?

36. A pendulum 4 ft. long swings backwards and forwards through an angle of 14° each side of the vertical. How high does the tip of the pendulum rise?

37. A man walks 200 yd. up a slope of 14° and then 150 yd. up a slope of 9°. What is the total height he rises?

38. A chord of a circle of radius 8 cm. subtends an angle of 110° at the centre. What is the length of the chord?

39. The sides of a parallelogram are 6 in., 4 in., and one angle is 63°. What is the area of the figure?

40. The legs of a pair of dividers are each 5 in. long, and they are opened so that the points are 2·3 in. apart. What is the angle between the legs?

41. A man walks 4 miles in a direction 27° east of north, and then 6 miles in a direction 57° east of south. How far east and how far north is he of his starting-point?

42. A, B, C are three points on a circle of radius 5 cm.; $\angle ACB = 72^\circ$. Calculate the length of AB. [What angle does AB subtend at the centre of the circle?]

43. The floor of a room is a rectangle ABCD, and the ceiling is a rectangle PQRS with P vertically above A; AB=20 ft., BC=15 ft., AP=11 ft. Calculate $\angle BAC$, $\angle PCA$.

44. A regular pyramid of height 8 in. stands on a square base of side 5 in. Find the angles which (i) each slant edge, (ii) each slant face makes with the base.

45. Prove that $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$.

46. Prove that $(\sin\theta)^2 + (\cos\theta)^2 = 1$.

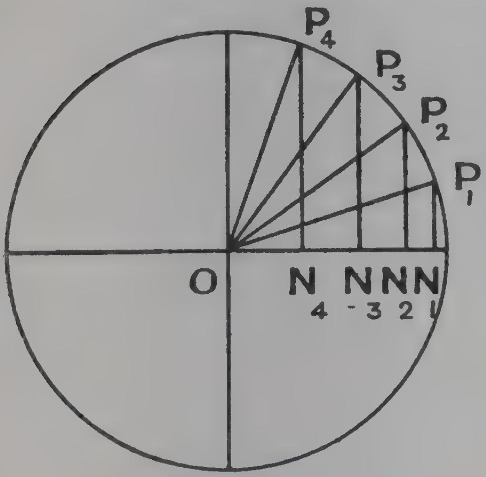
[It is usual to write $(\sin\theta)^2$ as $\sin^2\theta$, and $(\cos\theta)^2$ as $\cos^2\theta$.]

Other Trigonometrical Ratios

The reciprocals of the Sine, Cosine, and Tangent of an angle are called respectively the **Cosecant, Secant** and **Cotangent,** and are written more shortly as cosec, sec, and cot.

Thus $\quad \operatorname{cosec} \theta = \frac{1}{\sin \theta} \quad : \quad \sec \theta = \frac{1}{\cos \theta} \quad : \quad \cot \theta = \frac{1}{\tan \theta}$.

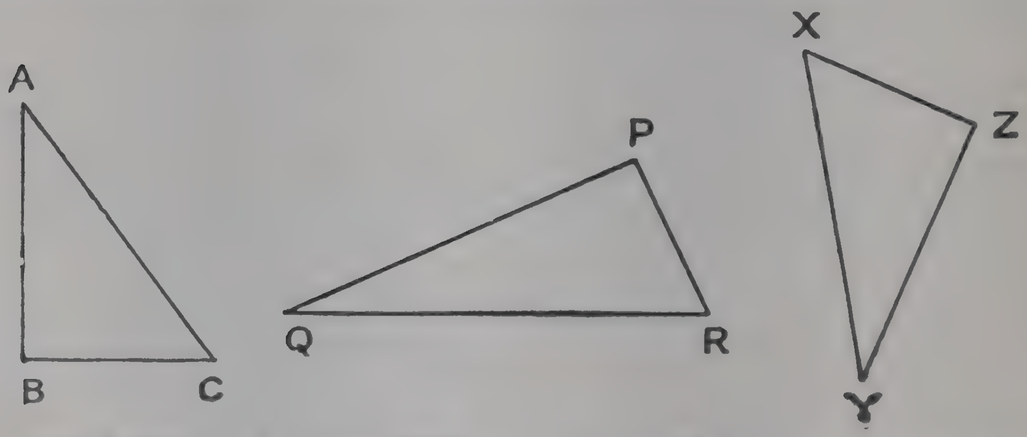
Since the sine and cosine of an angle cannot be greater than 1, the cosecant and secant cannot be less than 1.



The diagram shows an angle gradually increasing in size with the hypotenuse OP remaining unaltered. An inspection of the figure will show that as the angle NOP (θ) increases, $\sin \theta$, $\tan \theta$, and $\sec \theta$ **increase,** but cosine θ, cosec θ, and cotangent θ **decrease.** If four-figure tables are used, the difference columns must be **subtracted** for an increase of θ, in the case of $\cos \theta$, cosec θ, and $\cot \theta$.

EXERCISE XX (c)

In the diagram the angles at B, P, Z are each 90°. Write down expressions for the following, giving all possible answers:



1. sec ACB. 2. cosec PQR. 3. cot ZXY. 4. cosec BAC.

5. sec ZYX. 6. cot PRQ. 7. $\frac{XY}{XZ}$. 8. $\frac{AC}{BC}$.

9. $\frac{PR}{PQ}$. 10. $\frac{QR}{QP}$. 11. $\frac{BC}{AB}$. 12. $\frac{ZY}{ZX}$.

13. BC sec ACB. 14. PQ cot PRQ. 15. XZ cosec ZXY.

16. PR sec PRQ.

17. In the triangle ABC, AC=3 cm., CB=4 cm., $\angle ACB=90°$; CN is the perpendicular from C to AB. Write down the values of (i) sec ABC, (ii) cot CAB, (iii) cosec ABC, (iv) length of CN.

18. Draw a right-angled triangle ABC such that tan BAC$=\frac{5}{12}$, and use it to calculate sec BAC, cosec BAC, cot BAC.

19. Draw a right-angled triangle ABC such that cos BAC=0·4, and use it to calculate sin BAC, cot BAC, cosec BAC.

20. Show that

$$\text{(i)}\ \frac{\cos\theta}{\sin\theta}=\cot\theta, \qquad \text{(ii)}\ \frac{\operatorname{cosec}\theta}{\sec\theta}=\cot\theta,$$

and simplify $\tan\theta \operatorname{cosec}\theta$.

21. Prove that tan 20°=cot 70°, and generalise this result.

22. Prove that cosec 40°=sec 50°, and generalise this result.

23. Prove that $1+(\tan\theta)^2=(\sec\theta)^2$; this is written

$$1+\tan^2\theta=\sec^2\theta.$$

24. Prove that $1+(\cot\theta)^2=(\operatorname{cosec}\theta)^2$; this is written

$$1+\cot^2\theta=\operatorname{cosec}^2\theta.$$

25. A chord of length 10 cm. subtends an angle of 84° at the centre of a circle. What is the radius?

26. A man on the top of a cliff 120 ft. high measures the angle of depression of a boat as 33°. What is the distance of the boat from the man?

27. The diagonals of a rectangle contain an angle of 68°; the longer side is 7 cm. What is the length of a diagonal?

28. A ladder leans against a wall with its foot 7 ft. from the wall and at an angle of 68° with the horizontal. What is the length of the ladder?

29. AD is an altitude of the triangle ABC. If AD=6 in., $\angle ABC=42°$, $\angle ACB=52°$, find the lengths of AB, AC, BC.

30. ABCDE is a regular pentagon of side 8 cm. Find the distance of A from CD.

31. Use tables to find as shortly as possible the values of

(i) $\frac{1}{\sin 50}$. (ii) $\frac{10}{\sec 20}$. (iii) $\frac{\sin 40}{\cos 40}$.

(iv) tan 70 cos 70. (v) $\frac{\sin 80}{\cos 10}$. (vi) sec 25 cot 25.

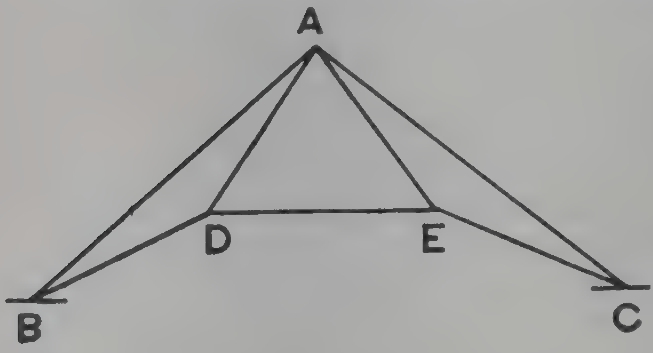
32. AB is a diameter of a circle; AP is a chord x cm. long and $\angle PAB = \theta$; PN is the perpendicular from P to AB. Find in terms of x, θ (i) the radius of the circle, (ii) the length of NB, (iii) the length of PB, (iv) the area of triangle APN.

33. A pendulum swings through an angle of 16° each side of the vertical, and the tip of the pendulum rises 4 in. above its lowest point. Find the length of the pendulum.

34. Two circles are of radii 5 cm., 3 cm., and their centres are 9 cm. apart. What is the angle between (i) their exterior common tangents, (ii) their interior common tangents.

35. A sphere of radius 5 cm. rests inside a hollow cone of vertical angle 70° and base radius 20 cm. What is the distance of the centre of the sphere from the base of the cone?

36. The diagram represents a roof-truss in which BD=AD=AE=EC, AB=AC, $\angle DAE = 70°$, $\angle BDA = 150°$, DE = 7 ft. Find the lengths of AB, BC.



37. A girder carrying a bridge is in the form of a circular arc; the length of the span is 40 ft. and the rise of the arch is 7 ft. Find the angle subtended by the arc at the centre of the circle and the radius of the circle.

38. A cylinder of radius 10 cm. is placed between two inclined planes sloping in opposite directions at angles of 50°, 72° with the horizontal, and placed so that their line of intersection is horizontal. Find the vertical depth of this line below the axis of the cylinder.

39. A desk slopes at 18° to the horizontal; a line on the desk makes an angle of 35° with the line of greatest slope. Find its inclination to the horizontal.

40. Find the angle between the diagonal AP of a cube, and (i) one of the faces of the cube, (ii) an edge AB of the cube.

MISCELLANEOUS EXAMPLES

M (VI)

1. Find the greatest number less than 10,000 which leaves a remainder of 17 when divided by 41.

2. Find the error per cent. if a value $3{\cdot}271 \times 10^{-4}$ is calculated to be $3{\cdot}127 \times 10^{-4}$.

3. The rate of postage for letters is 1½d. for the first 2 oz. and ½d. for each additional 2 oz. or part of 2 oz. What can you say about the weight of a letter which requires a 4d. stamp ?

4. A lamp is 9 ft. above the floor. What length of shadow will be cast by a horizontal stick 4 ft. long which is held 5 ft. from the floor ?

5. Show that 1st March and 1st November always fall on the same day of the week.

6. Use tables to find the length of the hypotenuse of a right-angled triangle whose sides are 3·28, 4·81 in.

7. The return railway ticket from a suburb to London costs 4s. 7d., and the annual ticket costs £27, 10s. How many days in the year must a man travel to save money by taking an annual ticket ?

8. A rectangular tray with a ledge round its rim measures internally 2 ft. by 1 ft. 6 in. How many halfpennies can be laid flat on the tray and in contact with the ledge ? The diameter of a halfpenny is 1 in.

9. A rectangular photograph which measures 8·5 cm. by 5·5 cm. is mounted on cardboard so as to leave a margin of 5 mm. all the way round. Find the area of the margin.

10. A man sold his car to a friend at 45 per cent. below cost price; the friend sold it to a dealer for 30 per cent. of what he gave for it ; the dealer sold the car for £181, 10s. and gained 10 per cent. What was the original cost ?

11. Find any integer whose cube root lies between 3·1 and 3·2.

12. How many jugs, each holding 2¼ pints, can be filled from a 5-gallon cask ? How much is left over ?

13. By selling cards at 5 for 4d. a man gained 20 per cent. How much a dozen did he give for them ?

14. A rectangular plot of building land with a frontage of 40 ft. and a depth of 45 yd. was let on a building lease at the rate of £80 per acre. What was the rent correct to the nearest shilling ?

15. A boy on the outside edge of a merry-go-round describes a circle of radius 6 yd. He pays a penny for a dozen revolutions. What is the charge per mile ride?

16. Write down the cost of 480 things at 1s. $3\frac{1}{2}$d. each.

17. A costermonger charges 20 per cent. more than the cost price of his goods. What must be his takings before he gets a profit of £3?

18. What approximate multiplying factor converts a pressure in Kg. per sq. cm. into lb. per square inch? [1 metre = 39·37 in., 1 Kg. = 2·205 lb.]

19. Water flows through a cutting in the shape of a trapezium whose sloping sides are equally inclined to the vertical. It is 20 ft. wide at the bottom, 30 ft. wide at the top, and 10 ft. deep. What is the width 6 ft. from the bottom?

20. What is the error per cent. in saying that the weight of water falling on an acre of ground per inch rainfall is 100 tons?

21. Through how many right angles does a cylindrical reel of diameter 3 in. turn when winding on 10 yd. of fine thread?

22. A pyramid is 6·5 in. high, and the edge of its square base is 4·2 in. Find the area of a section parallel to the base and 2 in. from it.

23. The floor of an attic measures 10 ft. by 8 ft. The ceiling consists of two parts, one of which is horizontal and measures 10 ft. by 5 ft., and the other slopes down from a height of 7 ft. to a height of 4 ft. What is the air-space in the attic?

24. Simplify $\dfrac{11-(\frac{4}{7}\text{ of }17)}{2-\frac{4}{5}} - \dfrac{2-\frac{4}{5}}{11-(\frac{4}{7}\text{ of }17)}$.

25. The marks on an examination paper were from 17 to 68. They were ranged from 30 to 100. What was the ranged mark for a boy who got 38 on the paper?

26. The area of a semicircular sheet of cardboard is 1 sq. ft. What is its perimeter correct to $\frac{1}{10}$th in.?

27. A triangular corner shelf whose equal sides are each 2 ft. is to be made of wood, of which a rectangular specimen 6·6 in. by 4·5 in. and ·8 in. thick weighs 124·6 grams. What will be the weight of the shelf?

28. Two cogged wheels with 6 and 10 teeth respectively work into each other: the first is making one revolution every four seconds. How many revolutions per minute is the other making?

29. A school of boys and girls contains 449 pupils. The number of girls is almost exactly 0·39 of the number of boys. Find the number of boys.

30. A culvert in the form of a trapezium with sides of equal slope is 4 ft. across at the bottom, 7 ft. at the top, and 4 ft. deep.

Calculate the width x ft. from the bottom, and find the distance across the surface when the culvert is $\frac{3}{4}$ full of water.

31. A man measures the length and breadth of a rectangular field by pacing it, and then deduces that the area of the field is 12 acres. He believes that his average pace is 1 yd., whereas it is really 33 in. What is the true area of the field correct to $\frac{1}{10}$th acre ?

32. A railway mounts from sea-level to a point 740 ft. above sea-level. For the first $9\frac{1}{3}$ miles it is on a gradient of 1 in 110, and for the remainder of the distance on a gradient of 1 in 60. What is the length of the railway ?

33. A small stoppered bottle when empty weighs 23·51 grams, and when full of water weighs 74·77 grams. If it is filled with methylated spirit it weighs 65·72 grams. Find the specific gravity of the methylated spirit.

34. A cylindrical glass holds half a pint ; its internal diameter is 2·5 in. What is its internal depth, correct to $\frac{1}{10}$th of an inch ?

35. A diver and his dress weigh 2 cwt. A leaden weight of 33 lb. is required to sink him in sea-water. Find the volume of the diver and his dress in cubic feet, given that the density of sea-water is 1024 oz. per cubic foot and the density of lead is 11,300 oz. per cubic foot.

Trigonometrical Questions

36. From a point 40 ft. away from the wall of a house, angles of elevation of the top and bottom of a window are 29° and 24°. Calculate the height of the window.

37. Tangents are drawn to a circle of radius 6 cm. from a point 9·3 cm. from the centre. Find the angle between them.

38. The legs of a pair of compasses are 12 cm. long. What is the distance between the tips, if the legs contain an angle of 154° ?

39. A bridge is thrown across a ravine through which a stream 17 ft. wide runs ; the sides of the ravine slope upwards right from the banks of the stream at angles of 80° and 70° respectively with the horizontal. The bridge is 85 ft. long. What is its height above the stream ?

40. Two circles of radius 2, 3 in. have their centres 4 in. apart. Find the angle between their common tangents.

41. What is the area of a regular pentagon inscribed in a centre of radius 10 cm. ?

42. What is the area of the minor segment of a circle of radius 5 in. cut off by a chord of length 4 in. ?

43. The elevation of a mountain-top as seen from a place A due wes of it is 27°, and is 18° at a place due north of A and 1 mile from A Find the height of the mountain in feet.

44. A ring 10 in. in diameter is suspended by 8 equal string attached to its circumference at equal intervals from a point 1 f above its centre. Find the angle between two consecutive strings.

45. A sector of a circle, angle 140°, is made into the curved surfac of a circular cone. Find the vertical angle of the cone.

46. Prove that $(1 - \sin^2\theta)(1 + \tan^2\theta) = 1$.

47. A small writing-desk is $3\frac{1}{2}$ in. high at the back, $2\frac{1}{4}$ in. high at th front; the base is $13\frac{1}{2}$ in. from back to front, and is 8 in. broad Find the slope of the desk and the slope of the line joining the to right-hand corner to the bottom left-hand corner of the desk.

48. A man walks 6 miles from A to B in a direction 20° north o east, then 4 miles from B to C in a direction 42° east of north, the 5 miles from C to D in a direction 22° west of north. How far i D (1) east of A, (2) north of A? Find the distance AD measured in straight line.

49. A shaft slopes downwards at an angle of 15° to the horizonta for 200 yd. and then at an angle of 12° for 340 yd. What vertica depth has then been reached?

50. A horizontal bar 2 ft. long is suspended by two vertical string at its ends each 4 ft. long. The bar is twisted round its centre, th strings being kept taut and the bar horizontal, until its centre ha risen 3 in. vertically. Through what angle has the bar been twisted

PART IV

CHAPTER XXI

SIMPLE AND COMPOUND INTEREST AND DISCOUNT

Simple Interest

JUST as you pay money for being allowed to use someone else's house, the payment being called rent, so you pay money for being allowed to use someone else's money, and this payment is called **Interest.** The money lent is called the **Principal.** The total obtained by adding to the Principal the Interest for any length of time is called the **Amount** after that time. The percentage payment per year is called the **rate per cent. per annum,** and is often called simply the rate per cent. Thus, interest at 4 per cent. means that £4 is the interest on £100 for one year.

If the borrower pays each year the interest due to the lender, the debt (*i.e.* the principal) remains unchanged. An arrangement of this kind is called **simple interest.** If, however, the borrower retains the interest, the debt (*i.e.* the principal) increases by that amount each year, and consequently the interest on the principal also increases each year. An arrangement of this kind is called **compound interest.**

Interest is often paid more frequently than once a year. Thus, interest at 8 per cent. paid quarterly means that on £100 the interest due is £2 each quarter of a year.

Example.—Find the simple interest on £450 for 3 years at 6 per cent.

The interest on £100 for 1 year is £6.

$$\therefore \text{ the interest on £100 for 3 years is £}6\times 3.$$

$$\therefore \text{ the interest on £450 for 3 years is £}\frac{450\times 6\times 3}{100}=\underline{\text{£}81.}$$

Example.—Find the simple interest on £P for T years at R per cent.

The interest on £100 for 1 year is £R.

$$\therefore \text{ the interest on £100 for T years is £}R\times T.$$

$$\therefore \text{ the interest on £P for T years is £}\frac{P\times R\times T}{100}.$$

Hence we have the following formula:
If the simple interest on £P for T years at R per cent. is £I,

$$I=\frac{P\times R\times T}{100}.$$

This formula can be put into various other forms:

$$\frac{P\times R\times T}{100}=I. \qquad \therefore\ P\times R\times T=100I$$

$$\therefore\ P=\frac{100I}{R\times T};\quad R=\frac{100I}{P\times T};\quad T=\frac{100I}{P\times R}.$$

Example.—Find the interest on a loan of £3500 at 6 per cent. from March 20 to April 8 in 1925, correct to the nearest penny.

March 20 to April 8 equals 19 days [*N.B.*—Only one of the given days must be counted, *e.g.* March 20 to March 21 equals 1 day, *not* 2 days], and there are 365 days in the year.

$$\therefore\ \text{interest}=£\frac{3500\times 6\times \frac{19}{365}}{100}=£\tfrac{798}{73}$$

$$=£10{\cdot}932=\underline{£10,\ 18\text{s. }8\text{d.}}$$

Note.—Always consider whether it is best to work in fractions or decimals. If you are finding the interest on an odd sum of money, *e.g.* £163, 15s. 8d., it is usually best to express it in £ and a decimal of a £; the answer is probably required correct to the nearest penny.

EXERCISE XXI (a)

[Take a year as 365 days, unless the contrary is stated.]

Find the simple interest on

1. £300 for 2 years at 5%.
2. £250 for 3 years at 4%.
3. £50 for 4 years at 6%.
4. £350 for 1 year at 8%.
5. £800 for 1 year at 3½%.
6. £60 for 2½ years at 4%.
7. £900 for 4 months at 5¾%.
8. £640 for 3 months at 7½%.
9. £2000 for 15 months at 4½%.
10. £112, 10s. for 1½ years at 4%.
11. £61, 5s. for 4½ years at 3⅓%.
12. £43, 15s. for 16 months at 4½%.

Find correct to the nearest penny the simple interest on

13. £68 for 2 years at 4%.
14. £224 for 3 years at 3½%.
15. £162 for 6 months at 7%.
16. £1000 for 20 days at 8%.
17. £81, 15s. for 2 years at 7%.
18. £768 for 1½ years at 4½%.
19. £38, 14s. for 3 years at 5½%.
20. £3260 for 90 days at 6%.

21. £164, 17s. 4d. for 4 years at $4\frac{1}{2}$%.

22. £337, 8s. 7d. for $2\frac{1}{2}$ years at $3\frac{1}{2}$%.

23. £2800 from June 3 to August 11 at 4%.

24. £7360 from September 9 to December 31 at $5\frac{1}{2}$%.

25. The directors of an industrial concern wish to borrow £175,000 and state that they expect to pay quarterly a dividend at the rate of 7 per cent. per annum. How much will it be necessary for them to distribute each quarter?

26. Which receives more interest: £120 invested at 4 per cent., or £190 at $2\frac{1}{2}$ per cent.? What is the yearly difference?

27. What is the amount of £620 after 4 years, reckoning 5 per cent. simple interest?

28. The Post Office Savings Bank pays $2\frac{1}{2}$ per cent. per annum interest. What is the interest per £ per month?

29. A moneylender advances £80 at 120 per cent. per annum. If the whole debt is discharged after 9 months, how much is paid?

30. A man owes his tailor £65. Simple interest at the rate of 6 per cent. is charged as long as the account stands. How much must be paid to settle it when it is 18 months overdue?

31. A man borrows £120 from a moneylender and agrees to pay him 5 per cent. per month simple interest. He pays back £100 after 6 months. How much does he then still owe?

32. Just before the War the National Debt was 712 million pounds, and the rate of interest $3\frac{1}{2}$ per cent. How much was paid each year in interest?

33. A man is paid a salary of £2500 a year for 5 years. At the end of the first year he invests 18 per cent. of his salary in War Bonds, on which he receives 5 per cent. interest, and he does the same at the end of each year afterwards. What is the total amount of interest he has received at the end of the fifth year?

34. A man has £60 in the Post Office Savings Bank, where he receives $2\frac{1}{2}$ per cent. interest and is not taxed. He withdraws his money and lends it at $4\frac{1}{2}$ per cent. interest, and the borrower deducts income tax of 2s. 6d. in the £ from the interest before he pays it. How much more interest per annum does the man get by taking his money out of the bank?

35. The capital of a company is £50,000 Debentures and £125,000 Preference. It pays 6 per cent. interest on the Debentures and

8 per cent. interest on the Preference. Its annual profits are £15,000. How much is left after the interest on the Debentures and Preference has been paid ?

36. A man is due to pay £3627, 8s. 9d. on April 5, but is unable to do so until April 9. How much more will he have to pay, reckoning interest at 6 per cent. ?

37. What is the interest on £$2x$ for $5y$ years at $4z$ per cent. ?

38. What is the simple interest on x shillings for y months at z per cent. per month ?

Example.—In what time does the interest on £750 at 8 per cent. amount to £150 ?

$$P=750,\ R=8,\ I=150,\ T=\ ?$$

$$\frac{P\times R\times T}{100}=I\,;\quad\therefore\ \frac{750\times 8\times T}{100}=150.$$

$$\therefore\ T=\frac{150\times 100}{750\times 8}=\frac{5}{2}=\underline{\mathbf{2\tfrac{1}{2}\ years.}}$$

Or, the interest on £750 at 8 per cent. for 1 year $=\dfrac{750\times 8}{100}$
$=$£60.

$\therefore$ £60 is the interest for 1 year ;

£150 is the interest for $1\times\dfrac{150}{60}$ years $=\underline{\mathbf{2\tfrac{1}{2}\ years.}}$

Note the use of the word " amount." In 5 years at 4 per cent. the interest on £100 amounts to £20 ; but the amount of £100 is £120. The interest at a given rate is proportional to the number of years, but the amount is not proportional to this number, *e.g.* if £100 amounts to £120 in 5 years, it will not amount to £240 in 10 years, but to £140.

Example.—What principal will amount to £259, 12s. in 3 years at 6 per cent. ?

The interest on £100 for 3 years at 6 per cent. = £18.

$\therefore$ £118 is the amount of £100 ;

£259·6 is the amount of £100 $\times\dfrac{259{\cdot}6}{118}=$ £220.

EXERCISE XXI ((b)

1. Copy the following table and fill in the gaps :

	Principal.	Rate per cent.	Number of Years.	Simple Interest.
i	£100	—	4	£12
ii	£300	6	—	£54
iii	£250	8	$\frac{3}{4}$	—
iv	—	$4\frac{1}{2}$	3	£108
v	£450	—	5	£135
vi	£1200	$5\frac{1}{2}$	—	£99
vii	—	$3\frac{1}{2}$	$1\frac{1}{2}$	£336
viii	£x	y	—	£z
ix	£a	—	n	£z
x	—	c	k	£q

2. Copy the following table and fill in the gaps :

	Principal.	Rate per cent.	Number of Years.	Simple Interest	Amount.
i	£240, 16s.	4	3	—	—
ii	£360, 15s.	4	—	£72, 3s.	—
iii	—	4	5	£52	—
iv	—	3	6	—	£295
v	—	2	$5\frac{1}{4}$	—	£939, 5s.
vi	£860, 10s.	3	—	£90, 7s. 1d.	—
vii	£2700	$2\frac{3}{4}$	—	—	£3219, 15s.
viii	—	5	$7\frac{1}{2}$	—	£1246, 13s. 4d.
ix	—	$2\frac{1}{2}$	3	—	£7697
x	—	$4\frac{1}{2}$	$3\frac{1}{2}$	£273	—

3. A man borrows £300 and discharges the debt after 6 months by paying £312. What is the rate of interest ?

4. After what time will £1 amount to 25 shillings at 10 per cent.?

5. A man receives four guineas as one year's interest from the Post Office Savings Bank, which pays $2\frac{1}{2}$ per cent. How much had he in the bank?

6. A man borrows £45 on the condition that he repays £54 at the end of 8 months. To what rate of interest is this equivalent?

7. In what time will £4500 amount to £5141, 5s. at 3 per cent.? To what will £4500 amount at 6 per cent. in the same time?

8. The interest on a debt for 2 months at 5 per cent. is one guinea. What is the debt?

9. To what will £100 amount in $2\frac{1}{2}$ years at 4 per cent.? What sum of money will amount to £99 in $2\frac{1}{2}$ years at 4 per cent.?

10. A charge of one penny a month simple interest is made on a loan of half a crown. What is the rate of interest?

11. What sum of money will amount to £70 in 18 months at 8 per cent.?

12. A sum of money amounts at simple interest to £60 after 1 year and to £65 after 18 months. What was the original principal, and what is the rate of interest?

13. The rate of interest on a loan is increased from 6 per cent. to $6\frac{1}{2}$ per cent., and the debtor accordingly has to pay £10 a year more. How much is the loan?

14. If £142 amounts to £148 in 9 months, find to what it will amount in 1 year.

15. At what rate per cent. must £180 be invested in order to produce the same income as £250 invested at 6 per cent.?

16. In how many years will money double itself at 15 per cent. simple interest?

17. A man borrowed £100 from his bank at $7\frac{1}{2}$ per cent. on January 1, 1923. When he repaid it, the interest, correct to the nearest penny, was £2, 9s. 4d. For how many days did the loan run?

18. Cecil Rhodes founded at Oxford 60 Colonial scholarships of £300 a year, 104 American scholarships of £300 a year, and 15 German scholarships of £250 a year. The cost of management is £1500 a year. What capital sum was needed for the bequest, reckoning interest at 3 per cent.?

19. A man invests half his money at 5 per cent., one-third of it

at 8 per cent., and the remainder at 12 per cent. What rate of interest does he get on the average for the whole ?

20. The year's profits of a company are £35,250. Out of this sum, interest must first be paid at the rate of $5\frac{1}{2}$ per cent. on £150,000 Debentures ; then £10,000 is put aside as a reserve. The remainder is divided among £200,000 ordinary shares. How much per cent. do these receive ?

21. Two partners invest £2000 and £1200 respectively in a business and agree that 20 per cent. of the annual profit is to be divided equally between them and the remainder is to be treated as interest on capital invested. The profit for the first year is £260. How will this be shared between them ?

22. If £x amounts to £y in one year at simple interest, after how many more years will it amount to £$(y+z)$?

Compound Interest

To find the compound interest for a given time, it is necessary to calculate the principal at the end of each period for which the interest is paid, using a practice method.

Example.—Find the compound interest on £234 *for* 3 *years at* 4 *per cent., correct to the nearest penny.*

The interest for 1 year is £$\frac{4}{100} \times 234$; we therefore multiply 234 by 4 and start two places to the right, so as to divide by 100.

It is unnecessary to keep more than four places of decimals in the work, to get a result correct to the nearest penny.

	234·	
	9·36	
	243·36	Amount at end of 1st year.
	9·7344	
	253·0944	Amount at end of 2nd year.
	10·1238	
	263·2182	Amount at end of 3rd year.
Deduct	234	
	29·218	Compound interest.

∴ compound interest = £29·218 = £29, 4s. 4d.

Example.—Find the compound interest on £162, 13s. 6d. for 3 years at $4\frac{3}{4}$ per cent., correct to nearest penny.

13s. 6d. = 13·5s.
= £·675.

To obtain $\frac{1}{2}$ per cent., divide the top line by 2 and start two places to the right.

To obtain $\frac{1}{4}$ per cent., divide the $\frac{1}{2}$ per cent. line by 2.

	162·675	
	6·5070|0	4 per cent. line.
	·8134	$\frac{1}{2}$ per cent. line.
	·4067	$\frac{1}{4}$ per cent. line.
	170·4021	Amount, 1st year.
	6·8161	
	·8520	
	·4260	
	178·4962	Amount, 2nd year.
	7·1398	
	·8925	
	·4462	
	186·9747	Amount, 3rd year.
Deduct	162·675	
	24·300	Compound interest.

∴ compound interest = £24·300 = £24, 6s. 0d.

Example.—Find the compound interest on £127 for $1\frac{1}{2}$ years at $4\frac{1}{2}$ per cent., payable half-yearly, correct to the nearest penny.

We require the amount at the end of three half-yearly periods, the rate of interest being $\frac{1}{2}$ of $4\frac{1}{2} = 2\frac{1}{4}$ per cent. per period.

	127·	
	2·54	
	·3175	
	129·8575	Amount, 1st period.
	2·59715	
	·32464	
	132·77929	Amount, 2nd period.
	2·65559	
	·33195	
	135·76683	Amount, 3rd period.
Deduct	127	
	8·767	

∴ compound interest = £8·767 = £8, 15s. 4d.

EXERCISE XXI (c)

Find correct to the nearest penny the compound interest on the following loans:

1. £180 for 3 years at 6%.
2. £325 for 4 years at 3%.

3. £85 for 2 years at 4%.

4. £620 for 3 years at 5%.

5. £240 for 3 years at 4½%.

6. £570 for 2 years at 3¾%.

7. £430 for 3 years at 5¼%.

8. £96 for 2 years at 5¾ per cent.

9. £155, 10s. for 3 years at 3½%.

10. £83, 17s. 6d. for 3 years at 5½%.

11. £61, 8s. 10d. for 2 years at 2¾%.

12. £109, 14s. 7d. for 3 years at 4·6%.

13. £75 for 2 years at 5%, payable half-yearly.

14. £162, 10s. for 1 year at 6%, payable quarterly.

15. £225 for 1½ years at 3½%, payable half-yearly.

16. £400 for 2 years at 5½%, payable half-yearly.

17. Find the difference between the simple and compound interest on £800 for 3 years at 4½ per cent.

18. A sum of money lent at compound interest amounts to £125, 8s. after 4 years and to £131, 0s. 10d. after 5 years. What is the rate of interest?

19. £1 at 3½ per cent. compound interest amounts to £1, 8s. 2½d. after 10 years. Show that money nearly doubles itself at this rate of compound interest in 20 years.

20. If £800 is lent at 5 per cent. compound interest, what is the actual interest for the third year?

21. A man borrows £400 at 6 per cent. compound interest: he pays £100 at the end of each year. What is the amount of his debt after his fourth payment?

22. Plant to the value of £1200 is installed in a factory, and at the end of each year 10 per cent. of the book-value is written off for depreciation. What is the book-value of the plant after 5 years?

23. A man invested £100 on January 1, 1920, and an equal amount on each succeeding first of January: the money accumulated at 5 per cent. compound interest. What was the total amount of the fund on December 31, 1924?

The method given on p. 279 becomes unwieldy if the number of years or periods is large. For business purposes compound interest tables are used, but the results may be calculated by the use of logarithm tables in the following way:

Example.—Find the compound interest on £127 for 20 years at 4 per cent., paid annually.

For one year the interest on each £1 is £0·04, and £1 therefore amounts to £1·04 in the course of each year.

The ratio of the amount at the end of any year to the amount at the beginning of that year is therefore $\frac{1\cdot04}{1}$.

During the 1st year £127 amounts to £$127\times\frac{1\cdot04}{1}$.

During the 2nd year £$127\times1\cdot04$ amounts to (£$127\times1\cdot04)\times\frac{1\cdot04}{1}$

$$=£127\times(1\cdot04)^2.$$

At the end of 20 years the amount will be £$127\times(1\cdot04)^{20}$.

If this is evaluated by logarithms, 7-figure tables must be used, as the error resulting from multiplying by 20 the 4-figure logarithm of 1·04 will affect the number of pounds in the answer.

Example.—Find (1) *by logs,* (2) *by tables, the compound interest on £127 for 12 years at 4 per cent.*

$$\begin{aligned}\text{Amount}&=£127\times(1\cdot04)^{12}\\&=10^{2\cdot1038}\times(10^{\cdot0170333})^{12}\\&=10^{2\cdot1038}\times10^{\cdot2043996}\\&=10^{2\cdot3082}\\&=203\cdot3\text{ (the decimal is approximate)}\\&=\underline{£203,\ 6\text{s.}}\end{aligned}$$

$$\therefore\ \text{compound interest}\ =\underline{£76,\ 6\text{s.}}$$

From the tables, amount of £1 = 1·60103.

$$\begin{aligned}\therefore\ \text{amount of }£127&=127\times1\cdot60103\\&=£203\cdot33\\&=£203,\ 6\text{s. }7\text{d.}\end{aligned}$$

$$\therefore\ \text{compound interest}=£76,\ 6\text{s. }7\text{d.}$$

$$\begin{array}{r}160103\\127\\\hline160103\\32021\\11207\\\hline\underline{203331}\end{array}$$

COMPOUND INTEREST TABLE

Amount of £1 at Compound Interest

Year.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1·02000	1·02500	1·03000	1·03500	1·04000	1·04500	1·05000
2	1·04040	1·05063	1·06090	1·07123	1·08160	1·09203	1·10250
3	1·06121	1·07689	1·09273	1·10872	1·12486	1·14117	1·15763
4	1·08243	1·10381	1·12551	1·14752	1·16986	1·19252	1·21551
5	1·10408	1·13141	1·15927	1·18769	1·21665	1·24618	1·27628
6	1·12616	1·15969	1·19405	1·22926	1·26532	1·30226	1·34010
7	1·14869	1·18869	1·22987	1·27228	1·31593	1·36086	1·40710
8	1·17166	1·21840	1·26677	1·31681	1·36857	1·42210	1·47746
9	1·19509	1·24886	1·30477	1·36290	1·42331	1·48610	1·55133
10	1·21899	1·28009	1·34392	1·41060	1·48024	1·55297	1·62889
11	1·24337	1·31209	1·38423	1·45997	1·53945	1·62285	1·71034
12	1·26824	1·34489	1·42576	1·51107	1·60103	1·69588	1·79586
13	1·29361	1·37851	1·46853	1·56396	1·66507	1·77220	1·88565
14	1·31948	1·41297	1·51259	1·61870	1·73168	1·85194	1·97993
15	1·34587	1·44830	1·55797	1·67535	1·80094	1·93528	2·07893

7-Figure Logarithms

No.	Log.	No.	Log.
1·02	·0086002	1·04	·0170333
1·025	·0107239	1·045	·0191163
1·03	·0128372	1·05	·0211893
1·035	·0149403	1·06	·0253059

EXERCISE XXI (d)

Find by logarithms and by tables the compound interest on

1. £1500 for 10 years at 3%. **2.** £216, 14s. for 8 years at 4%.

3. £167, 18s. for 14 years at 4½%.

4. £628, 14s. for 12 years at 3½%.

5. £2164 for 15 years at 5%.

6. £186, 17s. 6d. for 9 years at 2½%.

Find what sum of money amounts to

7. £2500 in 10 years at 2½%. 8. £525 in 12 years at 3½%.

9. £700 in 8 years at 4%. 10. £5200 in 7 years at 3%.

11. At what rate per cent. will £246 amount to £320 in 8 years ?

12. At what rate per cent. will £5240 amount to £6430 in 5 years ?

13. Find as accurately as four-figure tables permit the compound interest on £50 for 20 years at 4 per cent.

14. Use logarithm tables to ascertain after how many years a sum of money doubles itself at 5 per cent. compound interest.

15. A dozen dozen bottles of port when first bottled are valued at £36, and their owner believes that the value increases at 6 per cent. compound interest for the next 20 years. What is his estimate of their value after this time ?

16. What sum of money will accumulate to £1000 after 3 years at 6 per cent. compound interest ?

Compound Interest Law

Any quantities which increase or decrease at constant intervals by an amount which is a constant multiple of the quantity at the beginning of the interval are said to obey the Compound Interest Law.

Example.—The population of a town, after increasing by 5 per cent. each year for 4 years, amounts to 463,000. What was it at the beginning of the period ?

At the end of 1 year, the original population (P) becomes P(1·05), and at the end of 4 years it is $P(1{\cdot}05)^4$.

$$\therefore\ P(1{\cdot}05)^4=463{,}000.$$

$$\therefore\ P(10^{\cdot0212})^4=10^{5\cdot6656}.$$

$$\therefore\ P=\frac{10^{5\cdot6656}}{10^{\cdot0848}}=10^{5\cdot5808}$$

$$=\underline{\underline{381{,}000.}}$$

The factor 1·05 is called the " growth " factor. If there is a decrease of 5 per cent., then 0·95 is called the " decay " factor.

EXERCISE XXI (e)

1. A population increased by 2 per cent. every year. In January 1920 it was 863,670. What was it in January 1924 ?

2. A motor car cost £620 and depreciated 25 per cent. of this value

in the first year, and afterwards depreciated 10 per cent. of the value it had at the beginning of each year. Find its value after 5 years from purchase.

3. The contents of a reservoir diminish by 2·4 per cent. each day. How much water is used in 7 days if it starts with 5,600,000 gallons ?

4. A ball after each fall bounces $\frac{9}{16}$ of the height from which it fell. If it is dropped from a height of 16 ft., to what height does it rise after it has hit the ground for the sixth time ?

5. The population of a town is 22,491. Two years ago it was 20,400. Assuming the same annual rate of increase, find its population 3 years hence.

6. Find in what time a sum of money will double itself at 6 per cent. per annum compound interest.

7. A bullet, fired through a board, loses 12 per cent. of its speed in passing through the board. If it passes through five such boards and emerges with a speed of 800 ft./sec., with what speed did it reach the first ?

8. A business increased its profits by 4 per cent. every year. If its profits in 1921 were £862,000, what were they in 1925 ?

9. When light was passed through a sheet of glass it lost 10 per cent. of its illuminating power. What percentage was lost after passing through 12 such sheets ? How many sheets would be required to absorb 75 per cent. of the light ?

10. The plant of a factory cost £5600. Every year 12 per cent. was written off for depreciation. For how much was it valued after 10 years ?

11. A motor which cost £450 was valued at £150 after 4 years. Assuming a constant percentage depreciation each year, find this percentage.

12. A pump was made to remove $\frac{1}{9}$th of the air in the receiver after each stroke. How many strokes would be required to reduce the amount of air to $\frac{1}{1000}$th of its original amount ?

13. Draw a graph showing for 6 years the value of an article which depreciates 10 per cent. each year, given that its original cost was £100.

14. Find what constant percentage increase each year would cause a population of 3,615,282 to grow to 4,217,926 in 4 years.

15. The value of the output of a mine fell off by 6 per cent. for 5 consecutive years. If it was valued at £264,000 at the beginning of the period, find its value at the end, and find what percentage increase for the next 5 years would be required to bring it back to its original value.

Discount

Cash Discount.—Goods are often purchased "on credit." This means that the money is not paid when the goods are delivered, but at some future date, which may be fixed at the time of the sale or may be left undetermined. A shopkeeper will often offer special terms for "cash down" on delivery; this usually takes the form of a "discount" calculated as a percentage of the ticketed price of the goods. If a discount of 5 per cent. for cash is offered on an article marked at £40, the discount is £$\frac{5}{100}\times 40=$£2, and the cash price is therefore £40−£2=£38.

Trade Discount.—When a retail dealer buys goods from a wholesale dealer, the listed prices in the wholesale catalogue are always subject to a "trade discount." This is a rebate which is allowed to a retail dealer as opposed to a private purchaser. When the costs of manufacture alter, the wholesale dealer does not compile a new price list, but is able to allow for fluctuations in prices of labour, raw material, etc., by modifying the percentage allowed as trade discount. In this way he avoids the heavy expense of preparing and issuing a new sales-catalogue.

Example.—A car is listed at £175, but there is a trade discount of 40 *per cent. What price is paid by the retail dealer?*

The rebate to the retailer is £$\frac{40}{100}\times 175=$£70.

∴ the price paid by the retailer is £175−£70=$\underline{\text{£}105}$.

Banker's Discount.—When a merchant purchases goods from a wholesale dealer, the custom of the trade usually determines the length of time for which credit is allowed; it may vary from a few days up to three months or more. If the period is a long one, it is usual for a "Bill of Exchange" to be prepared. This is drawn up by the seller and "accepted" by the purchaser. The bill, which must be stamped, is a legal promise to pay a definite sum of money on a definitely named date to the "Drawer" of the bill or his agent, by the "Acceptor" of the bill, who acknowledges his liability by signing it. If the drawer of the bill requires the money before the date at which the bill matures, he takes the bill to a bill-broker, who will pay cash for it, if he is satisfied that the credit of the acceptor is good. But naturally the sale-value of the bill is less than its face-value. The bill-broker is said to "discount the bill," and the difference between the cash payment he makes and the face-value of the bill is called the "discount."

The *rate* at which a bill is discounted depends on various circumstances, such as the bank rate, the state of the money market, the credit of the acceptor. Suppose the rate is fixed at 5 per cent., the amount of the "banker's discount" is then equal to the interest at 5 per cent. per annum on the *face-value of the bill* for the number of days for which the bill has to run. The "discounted value" or the "present worth" of the bill is found by subtracting the banker's discount from the face-value of the bill. Discount is always calculated correct to the nearest penny.

In the United Kingdom, 3 *days' grace* are allowed by law; if, for example, a bill falls due on June 10, it need not be met till June 13. These three days must therefore be taken into account when the bill-broker calculates the discount on the bill. It must also be remembered that only one of the two extreme days must be counted: for example, if a bill is accepted on June 2 and matures on June 29, the number of days for which it runs is 27, *not* 28.

Example.—What discount is allowed on a bill of £3000, dated May 4, at 4 months, and discounted on June 20 at 5 per cent.?

Allowing for 3 days' grace, the bill must be met on September 7. The period June 20 to September 7 is 79 days.

$$\therefore \text{ the discount} = £3000 \times \tfrac{5}{100} \times \tfrac{79}{365}$$
$$= £32,\ 9\text{s. } 4\text{d., correct to nearest penny.}$$

True Discount.—Suppose a bill of £100 is discounted at 5 per cent. 1 year before it matures. The banker's discount is £5, and the discounted value is therefore £95. Consequently the drawer of the bill receives £95 one year before the date when £100 is due to him. Now this is rather less than the *true* present worth of the bill, because if this £95 is invested at 5 per cent. for the year, the interest will be $£95 \times \frac{5}{100} = £\frac{95}{20}$ = £4, 15s.; therefore at the end of the year the amount is only £95 + £4, 15s. = £99, 15s., instead of £100. It is therefore clear that the broker who discounts the bill gains slightly by reckoning the discount on the face-value of the bill. But it is the invariable commercial practice to calculate discount in this way, because it makes the arithmetical process so much easier and quicker to perform. *True Discount* is a fiction confined to text-books and examinations. An example will show how the calculation is performed.

Example.—Find the true discount and the true present worth if a bill of £500, due in 6 months, is discounted at 5 per cent.

The interest on £100 for 6 months is £2½.

∴ £100 **now** amounts to £102½ in 6 months' time.

∴ the true discount on £102½ due in 6 months is £2½

,, ,, ,, £500 ,, ,, ,, $£\frac{500}{102\frac{1}{2}} \times 2\frac{1}{2}$

$$= £500 \times \tfrac{2}{205} \times \tfrac{5}{2} = £\tfrac{500}{41}$$
$$= \underline{£12,\ 3\text{s. } 11\text{d.,}} \text{ correct to nearest penny.}$$

$$\therefore \text{ the true present worth is } £500 - £12,\ 3\text{s. } 11\text{d.}$$
$$= \underline{£487,\ 16\text{s. } 1\text{d.}}$$

Note.—The banker's discount is £5 × 2½ = £12, 10s., and the banker's present worth, or discounted value, is £487, 10s.

EXERCISE XXI (f)

1. An antique chair marked at £7, 10s. is sold for cash at a discount of 15 per cent. What is the cash price?

2. A tailor charges ten guineas for a suit of clothes, but allows a discount of 10 per cent. for cash. What is the cash price ?

3. A coal merchant charges 45 shillings per ton, but allows 5 per cent. discount for cash. What is the net cash price ?

4. A tailor who allows 10 per cent. discount for cash says that the cash price of an overcoat is £4, 10s. What is the credit price ?

5. My grocery bill, after a deduction of 2½ per cent. discount, amounts to £5, 11s. 2d. What was the gross amount ?

6. A retailer pays £30 for a piano listed wholesale at £50. What is the trade discount ?

7. If the trade discount is 25 per cent., what is the catalogue price of a carpet for which the retailer pays £36 ?

8. An armchair is listed wholesale at £12 and is bought by the retailer at £9. What is the trade discount ? For what price does the retailer sell it if he gains 25 per cent. ?

Find the discounted value of the following bills, allowing 3 days' grace (Nos. 9–13):

9. £250, drawn on March 10 for 2 months, and discounted on April 2 at 4 per cent.

10. £800, drawn on May 12 for 30 days, and discounted on May 15 at 3 per cent.

11. £1000, drawn on March 3 and due on June 10, if discounted on May 1 at 3½ per cent.

12. £362, 15s., drawn on July 2 for 90 days, and discounted on August 5 at 4½ per cent.

13. £720, 13s. 6d., drawn and discounted on April 10 for 1 month at 4¾ per cent.

Find the **true** discount and **true** present worth in the following cases (Nos. 14–16):

14. £250, due in 9 months at 5 per cent.

15. £420, due in 2 years at 4½ per cent.

16. £85, due in 6 months at 5½ per cent.

17. What sum of money, invested at 5 per cent. simple interest, will amount to £1000 after 3 years ?

18. What sum of money, invested at 4 per cent. compound interest, will amount to £1000 after 3 years ?

19. What is the rate per cent. if the true present worth of £450, due in 2 years' time, is £400 ? What is the answer if banker's discount is applied ?

20. What is the difference between true discount and banker's discount on a bill of £250, due in 9 months at 5 per cent. ?

CHAPTER XXII

HARDER PERCENTAGES, PROFIT AND LOSS

Miscellaneous Problems on Percentage

Example.—In an examination the top boy scored 72 per cent. and the bottom boy 16 per cent.; there were 196 marks between them. What was the least mark for passing if 40 per cent. was required?

The top boy scored $\frac{72}{100}$ of the total and the bottom boy scored $\frac{16}{100}$ of the total.

$\therefore$ the difference between them was $\frac{72-16}{100}$ or $\frac{56}{100}$ of the total.

$$\therefore \text{ the total was } \frac{100}{56}\times 196.$$

$$\therefore \text{ the least pass mark was } \frac{40}{100}\times\frac{100}{56}\times 196=\underline{\textbf{140 marks.}}$$

Note.—It is often easier to use algebraic methods:
Let the total be x marks.

The top boy scored $\frac{72x}{100}$ marks and the bottom boy scored $\frac{16x}{100}$ marks.

$$\therefore \frac{72x}{100}-\frac{16x}{100}=196.$$

$$\therefore 72x-16x=19600.$$

$$\therefore 56x=19600 \quad \therefore x=\frac{19600}{56}=350.$$

$$\therefore \text{ pass mark}=\frac{40}{100}\times 350=\underline{\textbf{140 marks.}}$$

EXERCISE XXII (a)

1. The population of a town was 26,720 in 1920 and increased by 5 per cent. in the next year. What was it in 1921?

2. What is (i) 50 per cent. of £28, (ii) 5 per cent. of £28? Hence write down 55 per cent. of £28.

3. The customs duty on a pound weight of tobacco of value 16 shillings is 10 shillings. What percentage is this?

4. The full marks for two papers are 80, 120. A boy obtains 56, 78 marks respectively. Express each as a percentage. Which is his better paper ?

5. Write down without any working approximate values of (i) 52 per cent. of 1 shilling, (ii) 66 per cent. of £6, (iii) 150 per cent. of £14, (iv) $12\frac{1}{2}$ per cent. of £1, (v) 5 per cent. of £17.

6. Five per cent. of a certain sum of money is £24. What is 10 per cent. of it, $2\frac{1}{2}$ per cent. of it, 50 per cent. of it ?

7. Two articles in a shop priced at £5 and £4 are each reduced by £1 in a sale. What is the percentage reduction of each ?

8. In 1902 the income tax was 1s. 3d. in the £ ; in 1922 it was 5s. in the £. What is the percentage increase ?

9. A man invests £250 in German marks and loses 95 per cent. of his money. How much is left ?

10. The duty on an article is £1, 7s., and this is 18 per cent. of its value. What is its value ?

11. A man's rent is £44, and this is 8 per cent. of his income. What is his income ?

12. A man's housekeeping bills are £450 a year, and this is 36 per cent. of his income. He spends 7 per cent. of his income on holidays. What do these cost ?

13. If 8 per cent. of a number is 28, what is 10 per cent. of it ?

14. A housekeeper puts down $5\frac{1}{2}$ thousand eggs, but 0·2 per cent. go bad. How many can be eaten ?

15. A man sells a house for £2800, but has to pay the agent $2\frac{1}{2}$ per cent. and the solicitor 2 per cent. on the sale price. What is his net receipt ?

16. A man who owes £1250 is unable to pay more than £275. What percentage of his debts remains unpaid ?

17. A steel bar is expected to be able to stand a strain of 45 tons per square inch, but for safety only 40 per cent. of this strain is put on it. To what strain is it subjected ?

18. If b per cent. of a number is c, what is the number ?

19. If x per cent. of a number is y, what is z per cent. of it ?

20. The true length of a line is 1·25 cm. What is the error per cent. in taking its length as 1·3 cm. ?

21. A problem on heights is to be solved by drawing to scale ; an error of 2 per cent. is allowed. The true height is 180 ft. Between what limits should an answer lie to be counted as correct ?

22. In September 1920, Government butter was £15, 3s. 4d. per cwt. wholesale and 3s. per lb. retail. By how much per cent. did the retail price exceed the wholesale price?

23. A reel of cotton before the War cost $1\frac{1}{2}$d., and during the War the price rose to 10d. What was the increase per cent.?

24. To pass an examination it is necessary to obtain at least 35 per cent. Full marks are 253. What is the least mark for passing?

25. In a musketry examination a boy estimates the range of an object to be 1500 yd. The real range is 2000 yd. Find his error per cent.

26. From what sum must 5 per cent. be deducted to leave £6, 13s.?

27. What is the number which, after being increased by 5 per cent., becomes x?

28. What is the number which, after being diminished by 12 per cent., becomes y?

29. If x is diminished by y per cent. it becomes z. Express z in terms of x and y.

30. When a number has been increased p per cent. it becomes 64. What was it originally?

31. In a sample of chocolate, 4 per cent. was fine grit. How many $\frac{1}{4}$-lb. slabs of chocolate contain a total of 1 lb. of grit?

32. The area of British Dominions in Africa is 3,450,000 sq. miles, and the area of Africa is 11,500,000 sq. miles. What percentage of Africa is British?

33. A bar of soap had a pre-War price of 3d. and a War price of 1s. 2d. Find the percentage increase to the nearest whole number.

34. An electric-light company which formerly charged 6d. per unit has raised its charges by 10 per cent. What bill is payable by a man whose house is supplied through 2 meters which read as follows:

	1st meter.	2nd meter.
Beginning of quarter .	3179	2575
End of quarter . .	3241	2610

35. A man measures the length of a straight road as 1934 yd. It is really 1917 yd. Calculate his error per cent. correct to one significant figure.

36. The Civil Service War bonus in 1920 was calculated as follows: to the first 35 shillings a week a percentage of 134 per cent. is added;

to the next portion up to £200 a year, 60 per cent. is added; to the remainder, 45 per cent. is added. What is the total War bonus for a salary of £400 a year? [Take a year as 52 weeks.]

37. In the Great War, 4·1 per cent. of the population of the United States were mobilised and 0·13 per cent. of the population were killed. The number mobilised was 3·8 millions. Find to two significant figures (i) the population, (ii) the number killed.

38. From what number must x per cent. be deducted in order to leave y?

39. The expenses of a household before the War were as follows: food £150, clothing £70, rent £35, sundries £95. If the cost of food has increased 50 per cent. and the cost of clothing 40 per cent., while the rent has remained the same, and sundries have increased by £24, find the new total expenditure and the increase per cent.

40. A man measures the floor of a room as 10 ft. by 8 ft., but his foot-rule is half an inch too short. What is the real area, and what is the percentage error in the area obtained?

41. A carpet, 15 ft. 9 in. by 12 ft., is placed in a room 18 ft. by 15 ft. What percentage of the floor remains uncovered?

42. In 1841 the total number of houses in Ireland was 1,328,800, of which 491,300 were made of mud. In 1901 the corresponding figures were 851,200 and 9900. Find what percentage of houses were made of mud at each date, correct to nearest whole number.

43. A man who weighs 18 stone undergoes a treatment which professes to reduce any person's weight by 10 per cent. in 3 months. What may he hope to weigh at the end of 9 months of the treatment, to the nearest lb.?

44. On a certain journey a motorist finds that he averages 20 m.p.h. for 70 per cent. of the distance, 30 m.p.h. for 10 per cent., and 8 m.p.h. for the rest. What is his average speed for the whole journey?

45. In 1902 the total length of submarine cable was 213,000 miles. In 1907 it was 259,000 miles. Find to two significant figures the percentage increase.

46. A square piece of thin metal of side x cm. is heated until its sides increase each by p per cent. Find the percentage increase in the area.

47. If butter was 2s. per lb. yesterday, rises 50 per cent. in price to-day, and falls 50 per cent. to-morrow, what will its price then be?

48. v and d are connected by the formula $v=\frac{0{\cdot}21d}{1+0{\cdot}25\sqrt{d}}$. What is the error per cent. in taking v as 1·6 when $d=16$?

49. In a certain constituency there were 10,000 electors, of whom 51 per cent. were Unionists and 49 per cent. Labour. There were two candidates. If 94 per cent. of the Unionists voted and 98 per cent. of the Labour electors voted, which candidate was elected, and what was his majority ?

50. A man spends 35 per cent. of his income on living expenses, 10 per cent. on taxes, and 7s. 6d. on insurance. The remainder is £241, 12s. 6d. What is his income ?

51. If railway fares are increased 50 per cent., by what percentage must they be reduced to return to the original rates ?

52. If the cost of living rises 100 per cent. in one year, and in the next year again increases by 50 per cent., by what percentage must it be reduced in the next year if it is then to be 50 per cent. above its original figure ?

53. It is required to measure two large quantities and to find the difference between them. If they measure 900 and 910 within ½ per cent., prove that the error in their difference may be as much as 90 per cent.

54. A boy brings up an answer and is told that it is 10 per cent. too large. He reduces his answer by 10 per cent. and is told that his error is now 2·5. What was the correct answer ?

55. A man bought an estate at 18 per cent. below its real value and sold it at 10 per cent. above its real value. If his profit was £1358, how much did he pay for the estate ?

56. A builder who has built a house for £1800 calculates that after paying £18 a year for ground rent and deducting 10 per cent. of the annual rent for repairs, etc., his outlay will bring him in 8 per cent. per annum. What is the rent of the house ?

57. By selling a house for £2576 a man gains 12 per cent. on his original outlay. How much per cent. would he have gained had the house cost him £200 less ?

58. A tradesman marks his goods to make a profit of 18 per cent. on the cost price. During a sale he allows 1d. in the shilling discount off the marked prices. What profit per cent. does this leave him ?

59. In 1923 the value of New Zealand exports was £45,967,000 and the value of her imports £43,487,000. Great Britain took 82 per

cent. of the exports and supplied 51 per cent. of the imports. What was the cash value of the balance due from Great Britain to New Zealand ?

60. A closed rectangular tank, 4 ft. long, $2\frac{3}{4}$ ft. wide, contains $1\frac{1}{4}$ tons of water. When the water freezes, the tank just bursts. If water increases 10 per cent. in volume when it freezes, find the depth of the tank. [Take 1 cu. ft. of water to weigh 1000 oz.]

Profit and Loss

In order to compare the gains on different transactions it is convenient to calculate what the profit would have been in each case if the **cost** of the articles sold had been 100 units of money. If the gain were £20 on £100, it would have been 20s. on 100s., or 20d. on 100d. In each case the gain would have been 20 per cent.

The gain is not calculated on the **number** of articles sold, for, if one has a capital of £100, it may possibly be more profitable to deal in pianos than in oranges. To buy 4 pianos for £100 and sell them for £150 represents a gain of £50 per cent., whereas a sale of 24,000 oranges for £120 which cost £100 represents a gain of only 20 per cent. Whatever the transaction, a fair comparison can only be made if the capital invested is always the same, and this is conveniently taken as being 100.

Tradesmen who calculate their profit on the selling price either do it in ignorance or for the purpose of concealing their real gain. The sale for £25 of an article costing £20 represents really a gain of 5 on 20, *i.e.* 25 per cent., whereas the gain, if calculated on the selling price, would be £5 on £25, *i.e.* 20 per cent.

A tradesman who calculates his profit on " what is in the till " would estimate that, on an article which cost him £1 and which he sold for £100, his gain was 99 on 100 or 99 per cent., whereas his gain was 99 on 1, *i.e.* 9900 per cent. !

We have therefore the following general rule :

Gain or loss per cent. is always calculated as a percentage of the Cost Price.

Example.— *A sells a car to B and gains 20 per cent., B sells it to C and loses 20 per cent., C sells it to D and loses 50 per cent. If D gave £480 for the car, what did it cost A ?*

C sold it for £480 and lost 50 per cent. ∴ his buying price and B's selling price was $\frac{100}{50}$ of £480.

B lost 20 per cent. ∴ his buying price and A's selling price was $\frac{100}{80}\times\frac{100}{50}$ of £480.

A gained 20 per cent. ∴ his buying price was $\frac{100}{120}\times\frac{100}{80}\times\frac{100}{50}$ of £480 $=$ £1000.

Example.— *A horse dealer makes a profit of 25 per cent. by selling a horse for £30. What would the sale price have been if he had made a profit of 40 per cent. ?*

If the cost price was £100, the first selling price would have been £125 and the second £140.

∴ ratio of the second selling price to first selling price $=$ 140 to 125.

$$\therefore \text{ second selling price} = \tfrac{140}{125} \text{ of first selling price.}$$
$$= £\tfrac{140}{125}\times 30 = £\tfrac{28}{5}\times 6$$
$$= £33{\cdot}6 = £33,\ 12\text{s.}$$

Example.— *An article is sold for £40 at a gain of 33⅓ per cent. What would have been the gain per cent. if it were sold for £5 less ?*

If the cost price were £100, the first selling price would be £133⅓.

∴ since the ratio of the selling prices is $\frac{35}{40}$, the second selling price would be $£133\frac{1}{3}\times\frac{35}{40} = £\frac{400}{3}\times\frac{7}{8} = £116\frac{2}{3}$.

∴ the profit would have been 16⅔ per cent.

Example.— *A shopkeeper buys eggs at the rate of £1 per 100 eggs and sells them at 3d. each. What is his gain per cent. ?*

One hundred eggs cost 240d. and are sold for 300d.

If the cost price is 240d., the selling price is 300d., and the profit is 60d.

Cost price 240d. Profit 60d.
,, 100d. ,, $\frac{100}{240}\times 60\text{d.} = 25\text{d.}$
∴ the profit is 25 per cent.

Note.—A profit of 25 per cent. means—if the cost is £100, the profit is £25; but it is *not* the profit on 100 articles. Where articles are bought at one rate and sold at another, choose *any convenient* number of them and find the cost price of that number and the sale price of the same number, and then compare these two prices with each other.

EXERCISE XXII (b)

1. Fill in the blanks in the following table, indicating gains by a + and losses by a −.

	Cost Price.	Sale Price.	Gain.	Gain per cent.
(i)	£200	—	—	+3
(ii)	£40	—	—	−5
(iii)	25s.	—	+8s.	—
(iv)	8d.	—	−1d.	—
(v)	8s.	10s.	—	—
(vi)	10s.	8s.	—	—
(vii)	£3, 15s.	—	−5s.	—
(viii)	£11	—	—	−7½
(ix)	£36	£40, 10s.	—	—
(x)	—	£84	—	+12
(xi)	—	12s. 9d.	—	−15
(xii)	—	£27	−£3	—

2. A house is sold for £1860 at a gain of 50 per cent. What did it cost ?

3. A piano is sold for £32 at a loss of 50 per cent. What did it cost ?

4. A man lost £50 by selling his car for £190. What was his loss per cent. ?

5. A coal merchant buys coal at 35s. a ton and sells it at 2s. 6d. a cwt. What is his gain per cent. ?

6. A shopkeeper buys cotton sheets at £9 per dozen pairs and sells them at a guinea a pair. What is his gain per cent. ?

7. A greengrocer who buys English potatoes by the ton gains 60 per cent. by selling them at 12s. per cwt. How much did he pay per ton ?

8. A man buys francs when they stand at 100 to the £ and sells them when they stand at 90 to the £. What is his profit per cent. ?

9. A man buys some mining shares at 25 shillings each and sells them at 28 shillings each. What is his profit per cent. ?

10. A horse costs £23 and is sold at a profit of 15 per cent. What is its sale price ?

11. A draper bought some shirts at 13s. 6d. each and marked them for sale at 18s. 9d.; but when selling off he gave 10 per cent. discount on the marked price. What was his gain per cent.?

12. A coal merchant buys coal at 32s. per ton and, in addition, pays 13s. a ton for carriage and cartage. He sells it at 3s. 6d. per cwt. What is his gain per cent.?

13. A man buys 100 eggs for 15 shillings; 6 are broken, and the rest are sold at 4 for a shilling. What is his gain per cent.?

14. A speculative builder buys a plot of land for £120 and erects on it a row of six cottages at a cost of £1000: he sells each end cottage for £325 and the others at £275. Find his gain per cent.

15. By selling eggs at 4 for a shilling, a shopkeeper gains 50 per cent. What did he pay per score?

16. A shopkeeper marks his goods at prices which give him a profit of 60 per cent., but he gives 5 per cent. discount for cash. What is his profit per cent. on a cash purchase?

17. A bookseller sold 5000 copies of a book at 3s. 6d. each and thereby made a profit of £187, 10s. What was his gain per cent.?

18. Cheese is bought at 5 guineas per cwt. and sold at 1s. 4d. per lb. What is the gain per cent.?

19. An article passes through the hands of 3 dealers, each of whom makes a profit of 50 per cent. (i) If it cost the first dealer £4, for what did the third dealer sell it? (ii) If the third dealer sold it for £54, what did the first dealer pay for it?

20. A man gains 20 per cent. if he sells an article for 30 shillings. What did it cost? What selling price would give him a profit of 30 per cent.?

21. A draper increases his profits by 12½ per cent. by raising the price of sheets by 2s. 3d. per pair. What did they cost? What is his profit per cent. if he sells them at 24s. per pair?

22. A motor car is sold for £336 at a profit of 40 per cent. What is the profit per cent. if the price is cut down to £300?

23. Oranges costing 15 a shilling are mixed in equal quantities with oranges costing 20 a shilling and are sold at a shilling per dozen. What is the gain per cent.?

24. A dealer makes 25 per cent. profit by selling an article for £x. What would he gain per cent. by selling it for £$2x$?

25. A reduction of 10 shillings in the price of an article changes a profit of 12 per cent. into a loss of 8 per cent. What did it cost?

26. An article is sold at a profit of 40 per cent. If the cost of production rises by 20 per cent. and the sale price is raised by 5 per cent., what is the new gain per cent.?

27. Christmas cards are bought at 5 for 4d. and are sold at 4 for 5d. What is the gain per cent.? What is the gain if 10 per cent. of the cards bought remain unsold and become valueless?

28. A bookseller buys second-hand school text-books at a flat rate of 4d. each; he finds that he sells 20 per cent. of them at an average of 1s. 6d. each, 40 per cent. of them at an average of 1s. each, and the rest remain on his hands. What is his gain per cent.?

29. The cost of a certain article is made up as follows: raw material 1s. 6d., labour 1s., other expenses 6d. The manufacturer sells it to the retailer at 6s. When the cost of labour increases 100 per cent., the price to the retailer is advanced 50 per cent. What is the change in the manufacturer's profit per cent.?

30. A certain fountain pen costs the manufacturer 4s. The manufacturer makes a profit of 25 per cent., the middleman makes a profit of 20 per cent., and the retailer makes a profit of 1s. 6d. What is the retail price?

Harder Percentages

The following examples illustrate special methods of dealing with more difficult problems:

Example 1.—The inhabitants of a town speak either Welsh or English: 74·8 per cent. speak Welsh and 82·4 per cent. speak English. What percentage speak both Welsh and English?

74·8 per cent. speak Welsh. ∴ 25·2 per cent. do not speak Welsh.
82·4 per cent. speak English. ∴ 17·6 per cent. do not speak English.

∴ 42·8 per cent. do not speak both languages.

∴ 57·2 per cent. speak both Welsh and English.

Example 2.—A tradesman, after selling his goods at a profit of 20 per cent., decides to reduce his profit to 15 per cent., and thereby increases his weekly sales by 60 per cent. What change per cent. takes place in his weekly receipts?

The **percentage** change will not depend on the number of articles he originally sold nor upon their actual cost, so we may suppose that originally he sold 1 article per week which cost £100, his takings therefore being £120.

He then sells $1\frac{6}{10}$ articles for £115 each, bringing in £184.

∴ his takings increase by £64.

∴ percentage increase in receipts $= \frac{64}{120} \times 100 = \underline{53\frac{1}{3}\text{ per cent.}}$

The argument may be clearer if we suppose that originally he sold x articles each costing £y. His receipts for x articles are therefore $\pounds\frac{120}{100}yx$. Afterwards he sells $\frac{160}{100}x$ articles for $\pounds\frac{115}{100}y$ each, bringing in $\pounds(\frac{160}{100}x \times \frac{115}{100}y) = \pounds\frac{184}{100}xy$.

$$\therefore \text{ increase on } \pounds\frac{120xy}{100} \text{ is } \pounds\frac{64xy}{100}.$$

$$\therefore \text{ increase on } \pounds 100 \text{ is } \pounds\tfrac{64}{120} \times 100 = \pounds 53\tfrac{1}{3}.$$

$$\therefore \text{ increase is } 53\tfrac{1}{3} \text{ per cent.}$$

EXERCISE XXII (c)

1. The normal work of an office was done by 3 men, A, B, C, working six days a week. On an average, A did 42 per cent. of the work and B did 40 per cent. One week the work was suddenly doubled. C did no more than usual, A increased his work by 150 per cent. for four days and then collapsed and stayed away. By how much per cent. did B increase his week's work to get everything done ?

2. The number of heavy draught horses, light draught horses, and mules with an army are in the ratio 4 : 7 : 5. Their daily corn rations in lb. per animal are in the ratio 15 : 12 : 10. If these are reduced, heavy draught by 20 per cent., light draught by 25 per cent., mules by 10 per cent., find the percentage of corn saved.

3. A man's total income increases by 20 per cent., and at the same time his income tax is raised from 2s. 6d. to 3s. 4d. in the £. What is the increase per cent. in his net income ?

4. A housekeeper finds that her expenditure is divided among three groups of commodities in the proportion 5 : 3 : 2. The prices of these three groups rise 45 per cent., 60 per cent., 30 per cent. What is the percentage increase in her total expenditure ?

5. When the price of coal goes up 20 per cent., by how much per cent. must a householder decrease his consumption of coal to avoid an increase in his bill ?

6. When the import tax on an article was reduced by 8 per cent., the revenue from the tax went up by 12 per cent. What was the percentage increase in the number of articles imported ?

7. Comparing August 1917 with August 1916, it was found that the export of cotton goods increased by 10·5 per cent. in quantity and by 43·6 per cent. in price. What was the average percentage increase in the price of cotton goods ?

8. A recruit is accepted by the Army if he is passed by the doctor, the dentist, and the oculist. Each man in a batch is examined by all three. The oculist passes 94 per cent., the dentist passes 88 per cent., and the doctor passes 85 per cent. What is the least percentage of the batch to be accepted ?

9. A cask contains 20 gallons, of which 56 per cent. is wine and the rest water. How much wine must be added to bring the percentage up to 84 ?

10. When the income tax was reduced from 5s. to 4s. 6d. in the £ the revenue derived from the tax was reduced 8 per cent. Find the percentage change in the total income which was available for taxation.

11. In 10 years, during which the population of the British Isles increased 10 per cent., the expenditure increased 39 per cent. By how much per cent. had the average expenditure per unit of the population increased ?

12. Between one census and the next the native population of a town increased by 8 per cent. and the foreigners decreased from 200 to 150. The increase in the whole population was 7 per cent. Find the number of the native population at the first census.

13. A manufacturer finds that by selling pins at 1s. 2d. per oz. he gains $34\frac{2}{5}$ per cent., whereas by selling them at 1d. per gross his gain is only $33\frac{1}{3}$ per cent. How many pins go to the ounce ?

14: The railway regulations assign 50 per cent. of the number of compartments of a train to be labelled " Smoking." Assuming that 80 per cent. of the male passengers desire to travel in smoking compartments, what is the company's assumption as to the relative numbers of female and male passengers which are to be expected ?

15. A cloth merchant marks his goods 16 per cent. above cost price, expecting to gain thereby 4d. a yard. Owing to his stock being damaged he is obliged to lower his prices and to sell at a loss of $3\frac{1}{4}$d. a yard. By how much per cent. did he reduce his marked prices ?

16. A tradesman sells goods at a profit of 20 per cent. He decides to reduce the price so that his profits will only be 15 per cent., and finds that the number of articles he sells per week increases by 75 per cent. By what percentage are his weekly profits increased or decreased by the change ?

17. In an examination, 60 per cent. of the men and 70 per cent. of the women pass. Of those who fail 64 per cent. are men. What percentage of the competitors were men ?

18. The weight of a cylindrical bar of diameter 7 cm. is to be reduced by 20 per cent. If this is done by turning it down in a lathe, find its new diameter.

19. The population of a town increased by 5 per cent. between 1901–1911 and again between 1911–1921. In 1921 it was 423,654. What was it in 1901?

20. A furniture dealer marks his goods at x per cent. above cost price and gives a discount of y shillings in the £ for cash. What is his net gain per cent.?

21. A tradesman marks goods at 25 per cent. above cost price. He allows cash customers a discount of 10 per cent. off the marked price and credit customers a discount of 4 per cent. Two-thirds of the sales are for cash. What percentage profit does he make on the cost price?

22. In a certain examination candidates may take Chemistry or Mechanics, or both. 70·3 per cent. of the candidates took Chemistry and 84·6 per cent. took Mechanics. What percentage took both?

23. When iron is completely converted into rust, its weight increases by 43 per cent. A certain lump of iron was observed to increase by 2·4 per cent. of its weight by the rusting of its surface. What percentage of the original iron had been rusted?

24. A tradesman marks his goods at 20 per cent. above cost price. If the cost price increases by 5 per cent., by how much per cent. must he increase his selling price if he is to make the same amount of profit as before, per article?

25. A company earns a profit of 25 per cent. on its capital and out of this pays a dividend of 8 per cent. on the capital. £1400 is placed to a reserve fund and £2000 is carried forward to the next year's account. What was the capital of the company?

26. 88·6 per cent. of the boys in a boarding-house play football and 79·3 per cent. play cricket. What percentage play both football and cricket, if all play at least one game?

27. The coal bill of a railway for the first half of a year was 11·25 per cent. of the total working expenses, and for the corresponding period of the following year it was 14·8 per cent. If the other working expenses remained unaltered, what increase per cent. does this represent in the price of coal, if the same amount was used in each period?

28. A manufacturer sells goods wholesale to a dealer at a price of 15 per cent. above the cost price. The dealer sells to a retailer at a profit of 20 per cent., and the retailer makes a profit of 10 per cent. by selling the goods for £12, 13s. Find the cost price to the manufacturer.

29. By treaty two countries, A and B, are made into one kingdom. In A 60 per cent. of the population were Poles, 15 per cent. Russians, and the rest Germans. In B there were only Poles and Germans. When united, 6 per cent. would be Russians and 30 per cent. Germans. Find the percentage of Poles and Germans in B.

30. A certain volume of a mixture is to be made of 3 substances, A, B, and C, the percentage composition by weight to be 33, 45, and 22 respectively. Find the error in the percentage by weight of each substance, if the mixture is made by taking the above numbers to represent the percentage composition of volume. 22 c.c. of A weigh as much as 35 c.c. of B or 40 c.c. of C.

CHAPTER XXIII

SHARES AND STOCKS

Shares

A MAN invents and patents some article for which he believes there will be a good demand, *e.g.* a hair-brush with a detachable back, the removal of which enables the hair-brush to be washed more quickly and efficiently than the common hair-brush. He calculates that it will require £30,000 to put his hair-brush on the market, taking account of costs of manufacture, advertising expenses, office expenses, etc. In order to get the necessary money he starts a company which he calls the **Sanitary Hair-brush Company, Limited,** and registers it at Somerset House; but although only £30,000 is required for the early working, in order to allow for future expansion, it is registered with a Nominal Capital of £50,000, divided into (say) 50,000 shares of nominal value £1 each. Other people who have become interested in the invention now buy the shares at £1 per share, and men called Directors are appointed to manage the business. They decide that, to begin with, they will not issue more than 30,000 shares, and when these have been sold the **issued capital** of the company is £30,000 and the **unissued capital** is £20,000. But they also decide that, to begin with, the company does not require as much as £30,000 cash, although perhaps six months later this sum may be required, and so the £1 shares are issued on the following terms: 1s. per share on application, 14s. per share on allotment, and the balance (5s.) at some future date at not less than a month's notice. The shares are then described as £1 shares (15s. paid), and are said to be liable to **a call of** 5s., and the **paid-up capital** of the company is £30,000 $\times \frac{3}{4}$. Anyone who possesses these shares is legally liable for this further call of 5s. per share, but their liability ends there; since the company is **limited**, whatever debts it may incur, there can be no further claim for payment against the shareholders. At the end of one year's working, it is found that the profit is £6728; the directors then decide to pay a dividend of 3s. per share; this absorbs $3 \times 30{,}000$s. $=$ £4500, and leaves a balance of £6728 $-$ £4500 $=$ £2228 to form a reserve fund. Consequently each shareholder who paid 15s. per share receives interest at the rate of $\frac{3}{15} \times 100 = 20$ per cent. **on his money.** As the result of this, other people are anxious to buy some of the shares, and those who hold the shares have no special wish to sell them, and so the value of the shares goes up; and a man who bought a partly paid share for 15s. can now sell it for 25s. or £1¼: the share is then said to stand at 1¼. The directors think there is now scope for employing more capital, and so they call up the remaining 5s. per share that is due, and the shares are then said to be **fully paid.**

The next year's working shows a profit of £10,316, and the directors decide to pay a dividend of 5s. per share; this absorbs $5 \times 30{,}000$s. $=$ £7500 and leaves a balance of £10,316 − £7500 = £2816 to be added to the reserve fund. An original shareholder, who has by now paid 20s. per share, therefore receives interest at the rate of $\frac{5}{20} \times 100 = 25$ per cent. **on his money.** Consequently the demand for the shares increases and their price now rises to (say) 35s., and the shares are then said to stand at 1¾ (35s. = £1¾). Further capital can now be usefully employed, and so the directors offer some of the hitherto unissued shares, of which there are altogether 20,000. Since the shares already issued are worth £1¾, they decide to offer 10,000 new shares at the price of £1, 5s. each, and this offer is in the first instance restricted to existing shareholders, each shareholder being allowed to subscribe for 1 new share for every 3 shares he at present holds. The new shares are said to be issued at a **premium** of 5 shillings, because the price exceeds the nominal value (£1) by 5 shillings, the new shares, just like the old shares, being still called £1 shares. But naturally the existing shareholders will exercise the right offered to them, because they are getting each share at a price 10s. below its cost in the open market. The issued capital of the company now becomes £40,000 in £1 shares, and the paid-up capital becomes £30,000 × 1 + £10,000 × 1¼ = £42,500.

In the company described above, there is only one kind of share, called **ordinary shares.** But in most companies there are several other different kinds.

(i) **Debentures.**—These are not really shares at all, but represent money the company has borrowed. Debenture holders are entitled to a fixed rate of interest, which is paid before any other dividends at all, and, if the interest is not forthcoming, the debenture holders can sell up the company and repay themselves from the assets of the company. A debenture is therefore a kind of mortgage on the actual property of the company.

(ii) **Preference Shares.**—The holder of a preference share is entitled to a fixed rate of interest (subject to the prior claim of debenture holders) before any dividend is paid on any ordinary share.

(iii) **Cumulative Preference Shares.**—If the fixed rate of interest is not paid in any one year, the deficit must be made up in future years before any further dividend is paid on the ordinary shares.

(iv) **Participating Preference Shares.**—The holder is entitled, firstly, to a fixed rate of interest, but in addition, if the dividend on the ordinary shares exceeds a named rate, the holder is also entitled to a share of these excess profits.

(v) **Deferred Shares.**—No dividend is paid on these shares until the preference and ordinary shares have received a named minimum rate of interest.

Shares are of various nominal values, but usually some multiple or submultiple of £1, such as 1s., 2s., 2s. 6d., 5s., 10s., £5, £50, £100. But shares of a high nominal value are uncommon, because they are more difficult to sell: the most frequent nominal values are £1 and 2s.

If the price of a share is stated to be $3\frac{11}{32}$, this always means £$3\frac{11}{32}$; if the price of a share is expressed in shillings, the fact is so stated: thus Cicely rubber 2s. ordinary shares are quoted at 10s. 6d., while Kelani Valley £1 ordinary shares are quoted at $4\frac{7}{8}$, *i.e.* £4, 17s. 6d.

Sale or Purchase of Shares

A man decides to buy 200 Imperial Tobacco (£1) ordinary shares. He applies to a stockbroker who acts as a commission agent. The stockbroker goes to a dealer called a *jobber*, who quotes a price for the shares, 97s. 6d. to 98s. 6d.: this means that the jobber will buy them for 97s. 6d. or sell them for 98s. 6d. per share. The difference of 1s. per share is called the *jobber's turn* and forms his profit on the transaction. The broker then contracts to buy 200 Imperial Tobacco shares at 98s. 6d. per share; the cost is therefore

$$200 \times 98\text{s. } 6\text{d.} = £10 \times 98\tfrac{1}{2} = £985.$$

But the broker also charges a commission; in this case it would be 9d. per share, *i.e.* 9×200d.=£7, 10s. In addition to this, there is a Government transfer stamp duty which varies with the cost of the shares, and in the present instance is £10 (*i.e.* nearly 1 per cent.), There is also a Government contract stamp duty, in this case 2s., and a company registration fee of 5s.

The purchaser of the shares therefore receives a bill, called a *contract note*, made out as follows:

BOUGHT.				£	s.	d.
200 Imperial Tobacco ordinary shares at 98s. 6d.				985	0	0
	£	s.	d.			
Commission at 9d. per share . .	7	10	0			
Transfer duty	10	0	0			
Contract stamp . . .	0	2	0			
Registration fee . . .	0	5	0			
				17	17	0
				£1002	17	0

The total cost, therefore, to him of the shares is £1002, 17s.

If he sells the shares, the total amount he receives is, of course, the price of the shares *less* the commission and stamp duties, etc.

After the transaction has been completed, the purchaser receives a certificate from the company that he is the registered holder of the shares, and when he wishes to sell these shares he sends this certificate to the broker.

If a share of nominal value £1 actually stands at 1, it is said to be **at par**; if it stands at $1\frac{1}{4}$ it is said to be at a **premium** of $\frac{1}{4}$, and if it stands at $\frac{7}{8}$ it is said to be at a **discount** of $\frac{1}{8}$, since $1-\frac{7}{8}=\frac{1}{8}$.

The price of a share is quoted in the papers, sometimes in the form $2\frac{1}{2}$–$2\frac{3}{4}$, and sometimes simply as $2\frac{5}{8}$. In the first case $2\frac{1}{2}$ is the selling price and $2\frac{3}{4}$ is the buying price; in the second case $2\frac{5}{8}$ is the mean between the selling and buying prices. If the words "ex div." are added to the price, it means that the new purchaser of the shares is not entitled to the last dividend that has been declared, even if the dividend is actually paid after the new purchaser has received the shares. In the contrary case the price is said to be "cum div."

Dividends

The phrase "8 per cent. preference shares" means that for shares of the **nominal** value £100, *e.g.* one hundred (£1) preference shares or twenty (£5) preference shares, the holder is entitled to £8 interest, *i.e.* he is entitled, if profits permit, to £8 interest on a **nominal** £100 preference share: the shares themselves may be worth much more or much less than £100 cash. The phrase "8 per cent. debenture" is used in the same sense. On ordinary shares the dividend is usually declared at so many pence or shillings per share; but if the phrase "8 per cent. dividend" is used, it always means a dividend of £8 on shares (or stock) of the **nominal** value of £100, which is usually quite different from their cash value.

EXERCISE XXIII (a)

1. A patent-razor company has a registered capital of £40,000, divided into 40,000 (£1) shares, of which only 25,000 shares have been allotted. The terms of issue are 5s. per share on application and a further 7s. per share on allotment, and the balance when called for.

(i) What is the issued capital? (ii) What is the paid-up capital? (iii) How are the shares described? (iv) What sum is required to pay a dividend of 2s. per share. (v) What will be the receipts from a final call of 8s. per share?

2. A wholesale trading company has issued 300,000 7 per cent. preference £1 shares and 800,000 ordinary £1 shares, all fully paid. What profit is distributed if the ordinary shares receive a dividend of 2s. per share?

3. The capital of a certain hotel combine is as follows :

5 per cent. debentures . .	£50,000.
6 per cent. first preference .	60,000 (£1) shares.
8 per cent. second preference	40,000 (£1) shares.
	80,000 (£1) ordinary shares.
	100,000 (1s.) deferred shares.

What profit is distributed (i) if the second preference receive 5 per cent. ; (ii) if the ordinary shares receive 6 per cent. and the deferred shares nothing at all ; (iii) if the ordinary shares receive 10 per cent. and the deferred shares receive 6d. per share ?

✓ 4. The capital of a company for manufacturing roofing material consists of 40,000 6 per cent. preference (£1) shares and 75,000 (£1) ordinary shares. A year's trading showed a profit of £6128, of which £728 was added to the reserve fund. What dividend did the ordinary shareholders receive ?

5. The capital of a rubber-manufacturing company consists of 100,000 (£1) A shares and 40,000 (1s.) B shares. No dividend is due to the B shares until the A shares have received 10 per cent. ; the remainder of the profits is then divided into two equal portions, one of which is divided between the A shareholders and the other between the B shareholders. If the total distribution is £12,000, what percentage does an A shareholder receive ? How much per share does a B shareholder receive, and what percentage is this of his nominal capital ?

6. The capital of the Home and Colonial Stores Limited is 180,000 6 per cent. first preference (£5) shares and 250,000 15 per cent. second preference (£1) shares, and 250,000 cumulative ordinary (£1) shares and 650,000 ordinary (£1) shares. How much was distributed in 1924, when the cumulative ordinary shares received 15 per cent. and the ordinary shares received 25 per cent. ?

7. What is meant by the following extracts from a newspaper (August 14, 1925) ?

Forestal Lands 6% Pref. 19/6–20/-
,, ,, ordinary 22/3–22/9
Spillers Milling ordinary 34/9–35/3
,, ,, 6% Pref. 21/3–21/9
,, ,, Def. 13/6–14/-
Aramayo Tin 3¾ ex. div.
New issue (2/-) Layang Rubber -/4½ prem.
New Zealand 4½% issued at 94½, present price $\frac{7}{16}$ disc.

8. A man sells 1000 Sepang Rubber (£1) shares at 1¼ ; the broker's

commission is 3d. per share, and the only other charge is the contract stamp of 3s. What are the proceeds of the sale ?

9. A man sells 525 Kuala Muda Rubber (£1) shares at 31s. ; the broker's commission is 4½d. per share, and the contract stamp is 2s. What are the proceeds of the sale ?

10. A man buys 300 Shell (£1) shares at 4¼ ; the broker's commission is 9d. per share, and the contract stamp is 3s. ; being the purchaser, he also pays £13 transfer duty and 5s. for registration. What is the total cost of the shares ?

11. A man buys 1000 Ropp Tin (4s.) shares at 14s. 6d. ; the broker's commission is 2d. per share, and the contract stamp is 2s. ; being the purchaser, he also pays £7, 10s. transfer duty and 5s. for registration. What is the total cost of the shares ?

12. A man can buy 7 per cent. Preference (£1) Power Securities for 25s., inclusive of commission and stamp duties. What is the yield, *i.e.* what is the percentage reckoned on the purchase money ?

13. In 1924 the dividend on Hovis ordinary (£1) shares was 10 per cent. What is the yield, if the shares can be bought for 35s., inclusive of commission and stamp duties ?

14. A company formed to place a patented article on the market is registered with a nominal capital of £100,000, divided into 100,000 (£1) shares. Initially, 40,000 shares are issued at par, and at the end of the first year a dividend of 30 per cent. is paid. The directors then offer 20,000 more shares at a premium of 10s., each existing shareholder being allowed to take up at this price 1 new share for every 2 shares he already holds ; at the end of the second year another dividend of 30 per cent. is paid on old and new shares alike. The shares then stand at 2½ ex. div., inclusive of commission and stamp duties. What is the position of a man who bought 1000 of the original shares and took up his quota of the new shares and sells his entire holding at the end of the second year ?

Examples on shares and stocks cannot be done intelligently unless the pupil has a clear idea of the real nature of the operations involved. The difficulty is due solely to the natural unfamiliarity with the machinery ; the actual arithmetic is always of a very simple character. It is suggested that examples should be taken from the financial columns of a daily newspaper, and, if copies are available for the use of the pupil, (i) of the current issue, (ii) of an issue (say) a fortnight old, there is abundant material for two or three interesting lessons which will cover all the types of questions that arise. Further information may be obtained from a "Monthly Investment List," which can be procured by application to any stockbroker. It may be of use to add that the Stock Exchange is closed on Saturdays, and so there are no quotations in a Monday issue, unless Friday's quotations happen to be reprinted.

In the next set of examples, the prices quoted may be assumed to include commission and stamp duties, unless otherwise stated. These vary, of course, with the price of the share and the total amount of money involved.

EXERCISE XXIII (b)

[The data in the following examples held good in October 1925.]

1. Hope Brothers ordinary (£1) shares stand at 24s. How many shares can I buy for £60 ? How much of the nominal capital of the company shall I then own ? What interest do I receive if the dividend is (i) 2s. per share, (ii) 10 per cent. ? What is the yield on the money I invest ?

2. Gramophone Limited ordinary (£1) shares stand at 41s. How much must I pay for 200 shares ? What interest do I receive if the dividend is 15 per cent. ? What is this dividend in shillings per share ?

3. Improved Wood Pavement Limited (£1) ordinary shares stand at $2\frac{7}{8}$. What do I pay for 400 shares ? What is the income from a dividend of 25 per cent. ? What is this dividend in shillings per share ?

4. Maypole Dairy Limited Cum. Pfd. (5s.) shares stand at 14s. 3d. What dividend do I receive if I hold 420 shares when a 20 per cent. dividend is declared ? How much of the nominal capital of the company do I possess ? How much cash shall I receive if I sell the shares ?

5. Cyprus Asbestos 10 per cent. participating (£1) preference shares stand at $1\frac{1}{4}$ and the deferred (1s.) shares stand at 8s. 3d. What must I pay for 250 preference shares and 500 deferred shares ? What dividend shall I receive if the dividend on the preference shares is 15 per cent. and on the deferred shares is 75 per cent. ?

6. Java Produce (£1) shares stand at $4\frac{3}{4}$ and pay a dividend of 25 per cent. What must I pay for 150 shares ? What is my dividend ? What is the yield per cent. on the money I invest ?

7. A man bought 4000 Sungei Matang (2s.) shares in 1924, when they stood at 3s. $2\frac{1}{2}$d., and sold them in 1925, when they stood at 5s. 7d. What profit did he make ?

8. A man bought 300 Tanjong Malim (£1) shares in 1920 at $1\frac{7}{8}$; he sold 200 of them in 1921 at $\frac{3}{4}$, and the remainder in 1922 at $\frac{9}{16}$. What did he lose ? If he had kept them till November 1925, he could have sold them at $2\frac{1}{16}$. What profit would he have made ?

9. National Provincial Bank £20 shares (£4 paid) are quoted $11\frac{11}{16}$–$11\frac{15}{16}$, and pay a dividend of 16 per cent. on the paid-up capital. How much will 300 shares cost ? How much is obtained by the sale of 300 shares? What is the dividend on 300 shares? A man wishes to invest £200; how many shares can he buy and what money remains over? What is the legal liability of a man who holds 75 shares. What is the yield per cent. on money invested in these shares?

10. United Dairies Limited (£1) ordinary shares rose from 31s. 9d. to 39s. 1d. in 1924. What profit could a man who bought 600 shares at the lower price, and sold at the upper price, make? If the dividend was 2s. 9½d. per share, find the dividend received by a man who bought 600 shares at 35s., and find the yield per cent. on the money he invested.

11. A man bought 900 Associated Cement (£1) shares at 15s. 3d. and sold them at 18s. 1½d. What profit did he make?

12. A man wished in 1924 to invest £500 in Calico Printers' ordinary (£1) shares when standing at $1\frac{15}{16}$. How many could he buy? He sold them in 1925, when they had fallen to 32s. 6d. What did he lose, taking into account the fact that he received in the interval a dividend of 3s. per share?

13. A man bought 640 Harrods (B.A.) (1s.) deferred shares at 47s. 6d. in 1924 and sold them at 33s. 9d. in 1925; allowing for a dividend of 5s. per share, what did he lose?

14. Vauxhall Motors (10s.) shares are quoted $\frac{5}{8}$–$\frac{3}{4}$. What is (i) the cost price, (ii) the sale price of 450 shares? If a 5 per cent. dividend is declared, how much will a holder of these shares receive?

15. 320 Clyde Valley Electric (£1) ordinary shares are bought at 27s. 6d. What is the purchase price? What is the yield per cent. on the money invested if a dividend of 1s. 7½d. per share is paid?

16. Allsopp's (£1) ordinary shares rose from 7s. 6d. in 1924 to 25s. 3d. in 1925. A man bought 800 at the lower price and sold at the higher price, and in the interval received a dividend of 5½ per cent. What was his total profit?

Stocks

Governments and other public bodies often borrow money by issuing stock on which a fixed rate of interest is promised. At the present time, for example, 3½ per cent. India Stock stands at 67½–68½. This means that a voucher for £100 stock face-value issued by the Indian Government can be bought for £68, 10s. or sold for £67, 10s., and that the holder is paid a fixed dividend of £3, 10s. a year. *A man who invests money in the shares of a company can only buy a whole number of shares, but stock can be bought in any quantity.*

Example.—A man invests £200 in $3\frac{1}{2}$ per cent. India Stock at $68\frac{1}{2}$. How much stock does he buy? What annual dividend does he receive?

$$£68\tfrac{1}{2} \text{ buys } £100 \text{ stock.}$$

$$\therefore\ £200 \text{ buys } £\frac{200}{68\frac{1}{2}} \times 100 \text{ stock}$$

$$= £291{\cdot}971 \text{ stock} = \underline{£291,\ 19\text{s. } 5\text{d. stock.}}$$

$$\text{The dividend on } £100 \text{ stock is } £3\tfrac{1}{2}.$$

$$\therefore \text{ the dividend on } £291{\cdot}971 \text{ stock is } £2{\cdot}91971 \times 3{\cdot}5$$
$$= £10{\cdot}219$$
$$= £10,\ 4\text{s. } 5\text{d.}$$

As in the case of shares, a charge is made by the broker who buys or sells the stock. The broker's commission is called the **brokerage**, and must be allowed for in calculating the price of the stock. The statement that the brokerage is $\frac{1}{8}$ per cent. means that the broker's commission for buying £100 stock is £$\frac{1}{8}$. Suppose the jobber quotes a stock at $67\frac{1}{2}$–$68\frac{1}{2}$, and suppose brokerage is $\frac{1}{4}$ per cent., a private purchaser must pay $£68\frac{1}{2}+\frac{1}{4}=£68\frac{3}{4}$ for £100 stock, and a private seller will receive $£67\frac{1}{2}-\frac{1}{4}=£67\frac{1}{4}$ for £100 stock. In other words, to obtain the net value of a stock, **add** the brokerage to the quoted price when buying, and **subtract** it when selling stock. In addition to this, there are the usual charges for transfer and stamp duty.

When a stock is issued by a government or corporation it is usual to attach some condition governing its redemption at face-value. For example, the Bristol Corporation has issued £1,500,000 stock at 6 per cent., with redemption dates 1940–1950. This means that in 1940, or at any later time, the Bristol Corporation can redeem the stock at face-value, *i.e.* by paying £100 cash for £100 stock, if it wishes to do so, and further, that any private individual who holds £100 stock of this issue in 1950 can require the Corporation to pay him £100 cash for it. At the present time (1925), corporations can borrow money at about $4\frac{3}{4}$ per cent.; if market conditions do not change, it is obvious that the Bristol Corporation will exercise their right to redeem in 1940, because they can borrow in its place new money at a much lower rate of interest.

When money is put into Trust, the Trustees are limited in their choice of purchase to certain stocks, enumerated in an Act of Parliament and known as "*Trustee Securities.*" The yield from a Trustee stock is naturally low, because it is selected for its safety. If a stock in this list rises to a price above its redemption value, Trustees are (with certain exceptions) no longer allowed to purchase it.

Example.—A man sells £900 Brighton Corporation 3 per cent. stock at 70 and invests the proceeds in $4\frac{1}{2}$ per cent. Brazil stock (1883) *at* 60. *Find the change of income.* [The prices quoted include brokerage, etc.]

The income on £100 Brighton stock is £3.

$\therefore$ the income on £900 Brighton stock is $£3 \times 9 = £27$.

He sells £100 Brighton stock for £70 cash.

∴ he sells £900 Brighton stock for £70 × 9 = £630 cash.

By investing £60 cash, he obtains £100 Brazil stock, and the income on this stock is £$4\frac{1}{2}$.

By investing £60 cash he obtains an income of £$4\frac{1}{2}$.

By investing £630 cash, he obtains an income of £$\frac{630}{60} \times 4\frac{1}{2}$

$$= £\frac{630}{60} \times \frac{9}{2} = £\frac{189}{4} = £47\tfrac{1}{4}.$$

∴ his income increases by £$47\frac{1}{4}$ − £27 = **£20, 5s.**

Note.—(i) Brazil stock (1883) means that the loan was issued in the year 1883 A.D.

(ii) Always distinguish clearly between money and stock by using phrases such as £900 **stock** or £630 **cash.**

Example.—What percentage do you get on your money by investing in 3 *per cent. stock at* 65? *Does this give as good a yield as a* 5 *per cent. stock at* 102?

In a 3 per cent. stock at 65 you get £3 interest per annum by investing £65 money.

∴ on £100 money you obtain £$3 \times \frac{100}{65}$ interest.

= **£4·6.**

In a 5 per cent. stock at 102 you get £5 interest by investing £102 in cash.

∴ on £100 money you get £$\frac{5 \times 100}{102}$ interest.

= **£4·9.**

∴ 5 per cent. at 102 gives the better yield.

Note.—In estimating the value of a stock, account must be taken not only of the yield, but also of (i) any conditions governing its redemption, (ii) the ability of the company or corporation to pay its fixed dividends.

EXERCISE XXIII (c)

[Unless otherwise stated, the prices quoted include brokerage, etc.]

What must be paid for the following stock, and what is the annual dividend?

1. £700 Brazil 5 per cent. (1908) stock at 78.

2. £1250 Buenos Aires 5 per cent. stock at 72.

3. £475 Greek 5 per cent. (1881) stock at 54.

4. £2400 Japan 4 per cent. (1905) stock at 87.

5. £1320 Siamese $4\frac{1}{2}$ per cent. (1907) stock at 90.

How much stock can be bought in the following cases, and what is the annual dividend ?

6. £300 cash in Swedish 3 per cent. 1894 stock at 75.

7. £875 cash in Bank of England stock at 250, paying 12 per cent.

8. £1300 cash in 3 per cent. Local Loans at 65.

9. £1000 cash in India 3 per cent. at 58.

10. £850 cash in L.N.E.R. 4 per cent. 1st preference at 71.

Find how much per cent. on money invested is obtained from the following stocks :

11. 4 per cent. at 75.

12. 6 per cent. at 106.

13. 5 per cent. at 101¼.

14. 6½ per cent. at 108⅜.

15. 3½ per cent. at $58\frac{3}{16}$.

16. 5 per cent. at $98\frac{5}{16}$.

Which of the following investments gives the better rate of interest ? State the difference in the percentage return on money invested.

17. 2½ per cent. at 51 or 5½ per cent. at 102.

18. 6 per cent. at 104½ or 5 per cent. at 99.

19. 7 per cent. at 112 or 6 per cent. at 106½.

20. 3½ per cent. at 62 or 7½ per cent. at 126¼.

21. Vancouver 4 per cent. consolidated stock stands at 80. What yield on his money is obtained by an investor ?

22. A man obtains 5 per cent. yield on his money by buying 4½ per cent. Pretoria stock. What was the purchase price of the stock ?

23. A man who holds £1600 4 per cent. Cape Town stock sells at 84 and invests the proceeds in 4 per cent. Lloyd Brasileiro at 70. How much of the latter did he buy ? What was his change of income ?

24. By investing £1344 in 4½ per cent. Chilean 1885 stock, I obtain an income of £63. What was the price of the stock ?

25. The German 7 per cent. 1924 loan stands at 102 and the Finland 6 per cent. 1923 loan stands at 90. Which gives the investor the higher yield on his money ?

26. What are the net capital proceeds of selling £840 2½ per cent. Consols at 55, £1250 4 per cent. Funding Loan at 87, and buying £1500 5 per cent. Tokyo Loan at 90 ? What is also the change of income ?

27. A man invested £900 in 4½ Brazil stock when standing at 45 in 1924, and sold when they reached 58 in 1925. What profit did he make, if he received one year's dividend in the interval ?

28. When 2½ per cent. stock Consols stand at 56, how much stock is obtained by investing £160, and what income is derived from it ?

29. What is the price of the Sudan $4\frac{1}{2}$ stock if it gives a yield of £4, 13s. 9d. per cent. on money invested ?

30. A man invests £1000, dividing it equally between $2\frac{1}{2}$ per cent. Consols at 55 and 5 per cent. Rio de Janeiro at 75. What is his income ?

31. What is obtained by selling £600 4 per cent. Japan Loan at $64\frac{1}{2}$, brokerage $\frac{1}{8}$ per cent. ?

32. How much India $2\frac{1}{2}$ per cent. stock at $48\frac{3}{4}$ is obtained by investing £400, brokerage $\frac{1}{8}$ per cent. ?

33. The price of $5\frac{1}{2}$ Sudan stock is quoted as 106–108, and the brokerage is $\frac{1}{4}$ per cent. What is the cost of £720 stock ? What is obtained by selling £720 stock ?

34. A man's holding of Plymouth 6 per cent. stock gives him an income of £450. He sells the stock when quoted 107–109, brokerage $\frac{1}{4}$ per cent. What does he receive ?

35. A man buys sufficient Penzance 3 per cent. stock to give him an income of £87. Its quotation is 72–74. Allowing brokerage $\frac{1}{8}$ per cent., and £12, 15s. for transfer charges, find the total price paid.

MISCELLANEOUS EXAMPLES

M (VII)

1. Which is the greater, $\frac{336}{106}$ or $\frac{355}{113}$? By how much per cent. ?

2. The charge for 7 tons 18 cwt. of coal is £10, 4s. 1d. What is the price per ton ?

3. Twenty complete turns of a screw-driver are required to drive a screw through a 2-in. plank. Through what angle must the screw-driver be turned to make the screw-head advance $\frac{3}{20}$ in. ?

4. A man walking at 4 mls./hr. takes 2 steps a second. What is the length of his step in inches ?

5. How many pieces of matting, each 5 ft. long and 1 ft. 9 in. wide, are required for the floor of a room 30 ft. long, 14 ft. wide ?

6. The daily ration of a man is $\frac{3}{4}$ lb. of meat and 2 lb. of potatoes. If meat costs £9, 6s. 8d. per cwt. and potatoes 7s. per cwt., find the cost of rationing a company of 32 men for 9 weeks.

7. A man sells goods, which cost him £900, through an agent. The agent keeps 10 per cent. of the selling price himself, but this still leaves the owner with a profit of 10 per cent. At what price did the agent sell the goods ?

8. How many days can a tourist stay at a Swiss hotel where he is paying 16 francs a day without his bill exceeding £15, when the rate of exchange is 25·20 francs to the £ ?

9. Simplify $\dfrac{\frac{9}{11}-\frac{2}{3} \text{ of } \left(\frac{5}{6}-\frac{1}{11}\right)}{\frac{2}{9}+\frac{7}{9}\times\frac{4}{3}}$.

10. An agent retains $2\frac{1}{2}$ per cent. of the rents he collects. If this commission amounts to £34, what does he pay to the owners ?

11. **Write down** the value of $78^2-79\times 77$.

12. If $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, use tables to find f correct to three significant figures, given $u=2{\cdot}36$, $v=8{\cdot}64$.

13. A path 8 ft. wide, which runs round a grass plot 50 ft. long, 30 ft. wide, is to be covered with gravel to a depth of 2 in. Find the cost if gravel is 12s. per cubic yard.

14. Eggs bought at the rate of £1 for 100 are sold at 4 for a shilling. Find the gain per cent.

15. A moneylender charges 3d. a week simple interest on a loan of £1. Express this as a rate per cent. per annum.

16. If £260 amounts to £305 at simple interest in 3 years, what will be the amount after another 2 years ?

17. 531441 is a perfect cube. Of what is it the cube. (Logs. not to be used.)

18. A sells an article to B at a profit of 20 per cent. B sells it to C at a profit of 15 per cent. C pays 15s. 10d. more than it cost A. What did it cost B ?

19. A man invests £1500, partly at 5 per cent. and the rest at 10 per cent. He obtains 8 per cent. on his money altogether. How much was invested at 10 per cent. ?

20. A tradesman, after allowing $\frac{1}{2}$d. in the shilling discount off the marked prices, makes 15 per cent. profit on his outlay. On the sale of a certain table his profit is £1, 13s. At what price was it marked ?

21. Evaluate to three significant figures $\frac{5Wl^3}{384EI}$, given $W=1500$, $l=240$, $E=600\times 2240$, $I=54$.

22. Find a rough approximation to 3^{50}.

23. What is the least integral value of x for which $\left(\frac{2}{3}\right)^x$ is less than $0{\cdot}001$?

24. A tradesman used to make a profit of 20 per cent. by selling an article for 6d. When the cost price increases by $1\frac{1}{4}$d. he alters the selling price to $7\frac{1}{2}$d. What is his new profit per cent. ?

25. A rectangular block of limestone measured 23 ft. by 14 ft. by 10 ft., and weighed 210 tons. Calculate its specific gravity.

26. A tailor, when forwarding clothes which have been ordered, encloses a bill for £22, 10s., but says that it may be discharged by an immediate payment of £21, 7s. 6d. What rate per cent. discount does the tailor allow for cash ?

27. The water from a flat rectangular roof 30 ft. by 12 ft. is collected in a cylindrical tub of diameter 4 ft. What rise of water-level in the tub is caused by a rainfall of $\frac{1}{4}$ in. ?

28. The price of admission to a show was lowered by 10 per cent., and as a result the number of visitors each day increased by 10 per cent. Find the percentage change in the daily receipts.

29. What sum of money will amount to £530 in 9 months at 8 per cent. simple interest ?

30. A bottle which holds a pint is 9 in. high. What would be the height of a bottle of the same shape which holds a gallon ?

31. From London to Rugby is 80 miles. How far from London will a train which leaves London for Rugby at 2.45 p.m. and travels at 45 mls./hr. meet a train which leaves Rugby for London at 1.45 p.m. and travels 25 mls./hr. ?

32. A person finds he can obtain £5 more per annum by investing in $3\frac{1}{2}$ per cent. stock at 96 than in 3 per cent. stock at 88. How much has he to invest ?

33. Prove that the weight in pounds of a foot of iron piping is given by $\frac{\pi t w}{144}(b+t)$ when the thickness is t in. and the bore is b in., w being the weight in pounds of a cubic foot of iron. Hence calculate the weight when $b=2\frac{1}{2}$, $t=\frac{1}{4}$, $w=460$.

34. Find the value of $6{\cdot}32\sqrt{{\cdot}019^2+2{\cdot}6({\cdot}0073)}$.

35. Find the compound interest on £645, 18s. for 2 years at $4\frac{1}{2}$ per cent. per annum, paid half-yearly.

36. A person buys a cask of wine at 24s. per gallon, but 12 per cent. of it is lost. At what price per gallon must he sell the rest to gain 10 per cent. on his outlay ?

37. A body weighs 21·2 grams in air. When fastened to a sinker and immersed in water the apparent weight is 74·5 grams. The sinker alone immersed in water apparently weighs 103·6 grams. Find the specific gravity of the body.

38. The duty on tea, coffee, etc., is increased by 35 per cent., and a workman says that this increases his cost of living by 4 per cent. If that is so, find what percentage of his cost of living was due to the duty on these articles before the change.

39. The average yearly birth-rate of a district is 42 per 1000, and the death-rate 16 per 1000 of the population at the beginning of each year. At this rate, find how long it would take to double the population.

40. The number of germs in a solution increased at the rate of 10 per cent. per hour. At 4 p.m. there were 125,400. How many were there at (i) 8 p.m., (ii) 2 p.m.?

41. A dealer obtains a commodity from U.S.A. at 15 dollars per short ton (2000 lb. avoirdupois). Taking £1 = 4·8 dollars, find the price per ton avoirdupois at which he must sell to make a profit of 10 per cent.

42. Evaluate to three significant figures $(0{\cdot}728)^{-1{\cdot}4}$.

43. Find the thickness of a spherical iron shell which weighs 851·8 Kg. when empty and 1363 Kg. when full of water, taking the specific gravity of iron to be 7·8.

44. A reservoir has a horizontal base 50 yd. square, and its sloping sides are all inclined at 45°. How many gallons of water will fill it to a depth of 10 ft.?

45. The total rates to be paid on a certain house from October 1, 1925, to March 31, 1926 (both dates inclusive), amounted to £10, 5s. 10d. The house was sold and the purchaser had to pay his share of the rates from and including January 20. How much did he pay, to the nearest penny?

46. Find the length of crossed belt required to connect two pulleys, a driving-wheel 2 ft. 6 in. diameter, and a driven wheel 5 in. diameter, the distance between their centres being 3 ft.

47. A conical vessel, whose rim is 6 in. in diameter and height 1 ft., is filled with water. A nugget of gold is then wholly immersed in the water and afterwards withdrawn. It is then found that the surface of the water has fallen $3\frac{1}{4}$ in. Find the volume of the nugget, to the nearest cubic inch.

48. A body weighing 8 oz. is placed in water and floats with $\frac{8}{9}$ths of its volume immersed. If it is then placed in a liquid of specific gravity 1·8, find the least weight which must be placed upon it so that it may be completely immersed.

49. The volume of Mars is calculated to be 0·168 that of the Earth and its mass to be 0·108 that of the Earth. If the specific gravity of the Earth is taken to be 5·5, what is the specific gravity of Mars?

50. An inkpot is formed by hollowing out a cylindrical hole $1\frac{1}{2}$ in. deep and $\frac{7}{8}$ in. diameter in a cube of glass of edge $2\frac{1}{8}$ in. Find the weight of the inkpot if 1 cu. in. of glass weighs 2·3 oz. and the lid weighs $1\frac{1}{4}$ oz.

CHAPTER XXIV

FURTHER VARIATION

Inverse Variation

THE length of a journey is 12 miles. At 6 m.p.h. it takes 2 hours; at 3 m.p.h. it takes 4 hours. Thus, if the speed is halved, the time taken is doubled. At v m.p.h. it takes t hours, where $vt=12$ or $t=\frac{12}{v}=12\times\frac{1}{v}$ or $\frac{t}{\frac{1}{v}}=12$. We then say that t **varies inversely** as v, or that t varies directly as $\frac{1}{v}$, or that $\frac{t}{\frac{1}{v}}$ is constant.

We may tabulate corresponding values of t, v as follows:

t	1	2	3	4	6	12	hours.
v	12	6	4	3	2	1	m.p.h.

The reader should draw the corresponding graph: he will find that it is a curve.

Now let us tabulate corresponding values of t, $\frac{1}{v}$.

t	1	2	3	4	6	12
$\frac{1}{v}$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{6}{12}$	$\frac{12}{12}$

The table shows that t varies directly as $\frac{1}{v}$, and when the reader plots the values of t against $\frac{1}{v}$, he will therefore obtain a straight line through the origin.

If t_1 and v_1 are corresponding values of t and v, we have

$$\frac{t_1}{t}=\frac{\frac{1}{v_1}}{\frac{1}{v}}.$$

If we are given a table of corresponding values of x and y, the existence of the relation $y \propto \frac{1}{x}$ may be tested in either of the following ways:

(i) Graphically: Plot y against $\frac{1}{x}$. Is the graph a straight line through the origin?

(ii) By Calculation: Evaluate $y \div \frac{1}{x}$ or xy. Is the value constant?

EXERCISE XXIV (a)

1. The area of a rectangular sheep-fold is 3600 sq. yd. Make a table showing possible corresponding lengths and breadths. What is the effect (i) on the breadth if you double the length, (ii) on the breadth if you halve the length, (iii) on the length if you treble the breadth? What is the variation relation between the length and breadth?

2. Given that the law is either $y \propto x$ or $y \propto \frac{1}{x}$, state without calculation which it is, for the following table:

x	8	10	12	20
y	1·5	1·2	1	0·6

What is x if $y=2$? What is y if $x=5$? What is the value of y in terms of x?

3. When the temperature of a gas is constant, the volume v cu. cm. of a given mass varies inversely as the pressure, p grams weight per sq. cm.; if $v=160$ when $p=1000$, find v when $p=480$, and find v when $p=500$.

4. x men take y days to level a large field which it would take 12 men 40 days to level. (i) Does $y \propto x$? (ii) What is the effect on x of doubling y? (iii) Express y in terms of x and draw a graph.

5. Corresponding values of x and y are given by the table:

x	0·4	0·8	2	6	10
y	20	10	4	1·33	0·8

What graph is obtained by plotting x against $\frac{1}{y}$, and why?

6. What is the relation between the number of cigarettes a man smokes per day and the number of days a certain consignment lasts him?

7. If y varies as $\frac{1}{x}$, complete the table:

$x=$	1	2	3	5	8	10
$y=$					7·5	

Plot the values of y against the corresponding values of $\frac{1}{x}$.

8. If y varies inversely as x, what is the effect (i) on y of trebling x, (ii) on x of trebling y, (iii) on y of multiplying x by 10, (iv) on y of dividing x by 5?

9. If A varies inversely as B, find the percentage change in A when B is increased by 25 per cent.

Other Types of Variation

(i) $y \propto x^2$. The area of a circle of radius r cm. is A sq. cm. where $A=\pi r^2$. If $r=3$, $A=9\pi$, and if $r=30$, $A=900\pi$. Hence we see that if r is increased in the ratio $10:1$, A is increased in the ratio $10^2:1$. We say that A varies directly as r^2.

A variable y varies directly as the square of x, provided that when x changes in any ratio, y changes in the square of that ratio. If y_1 and x_1 are corresponding values of y and x, we have $\frac{y}{x^2}=\frac{y_1}{{x_1}^2}$, *i.e.* the ratio of y to x^2 is a constant, or $y=kx^2$, if k is the constant value of $\frac{y}{x^2}$.

Also
$$\frac{y_1}{y}=\frac{{x_1}^2}{x^2}=\left(\frac{x_1}{x}\right)^2.$$

If we plot y against the corresponding values of x^2, the graph will be a straight line through the origin.

It is important to remember that the areas of similar figures and the areas of the surfaces of similar solids vary directly as the squares of corresponding linear dimensions.

If $y \propto x^2$, and y is doubled, then, since x^2 changes in the same ratio as y, it follows that x^2 is changed in the ratio $2:1$, *i.e.* x is increased in the ratio $\sqrt{2}:1$. If $y \propto x^2$ and x is doubled, x^2 changes in the ratio $2^2:1$. $\therefore$ y also increases in the ratio $2^2:1$.

(ii) $y \propto \frac{1}{x^2}$. If I is the illumination received from a source of light at a distance d, we know that if d is doubled, the new value of I is only $\frac{1}{4}=\frac{1}{2^2}$ of its old value, and if d is trebled, the new value of I is only $\frac{1}{9}=\frac{1}{3^2}$ of its old value. We say that I varies inversely as d^2, or varies directly as $\frac{1}{d^2}$.

If y varies inversely as the square of x, and if y_1 and x_1 are a corresponding pair of values of y and x,

then
$$\frac{y}{\frac{1}{x^2}}=\frac{y_1}{\frac{1}{x_1^2}}=\text{const.}$$

Also
$$\frac{y_1}{y}=\left(\frac{\frac{1}{x_1}}{\frac{1}{x}}\right)^2.$$

Many physical illustrations of this law will be found in the examples.

If $y \propto \frac{1}{x^2}$ and y is doubled, then $\frac{1}{x^2}$ also changes in the ratio 2 : 1, *i.e.* $\frac{1}{x}$ changes in the ratio $\sqrt{2} : 1$, and x changes so that its new value to the old is $1 : \sqrt{2}$.

(iii) $y \propto x^3$. An example of this relation is the geometrical theorem that the volumes of similar solids vary directly as the cubes of corresponding linear dimensions. If y changes to y_1 as the result of x changing to x_1, we have

$$\frac{y}{x^3}=\frac{y_1}{x_1^3} \quad \text{and} \quad \frac{y_1}{y}=\left(\frac{x_1}{x}\right)^3.$$

(iv) If $y \propto x^2$, then $\frac{x^2}{y}$ remains constant in value, and therefore $\frac{x}{\sqrt{y}}$ also remains constant in value, and therefore x varies directly as the square root of y. In symbols, if $y \propto x^2$, then $x \propto \sqrt{y}$. For example, if y is increased in the ratio 9 : 1, then x is increased in the ratio $\sqrt{9} : 1 = 3 : 1$.

Similarly if $y \propto \frac{1}{x^2}$, then $x^2 \propto \frac{1}{y}$ and $x \propto \frac{1}{\sqrt{y}}$.

Example 1.—*If the velocity of a stream is measured by a pressure gauge, the velocity is found to vary as the square root of the height in the gauge. If a velocity of 8 ft. per sec. causes a height of 12 in., find the velocity when the height is 4 in.; state also the relation between the velocity v ft. per sec. and the height h in.*

Since
$$v \propto \sqrt{h}, \text{ then } \frac{v_1}{v}=\frac{\sqrt{h_1}}{\sqrt{h}}.$$

Now $v=8$ when $h=12$; it is required to find v_1 when $h_1=4$.

$$\therefore \frac{v_1}{8}=\frac{\sqrt{4}}{\sqrt{12}}. \quad \therefore v_1=\frac{8\sqrt{4}}{\sqrt{12}}=\frac{16}{\sqrt{12}}=4\cdot 6.$$

∴ when the height is 4 in., the velocity is 4·6 ft. per sec.

Also we have $\frac{v}{8}=\frac{\sqrt{h}}{\sqrt{12}}$, $\quad\therefore\ v=\frac{8}{\sqrt{12}}\cdot\sqrt{h}$

$$\therefore\ \underline{v=2{\cdot}3\sqrt{h}.}$$

Example 2.—*The weights of a number of circular discs of the same material and thickness, but different diameters, are given in the following table:*

Diameter $=d$	2	3	4	5	6	7	8	*cm.*
Weight $=w$	20	45	80	125	180	245	320	*grams*

Show how the relation $w \propto d^2$ *may be established* (*i*) *graphically,* (*ii*) *by calculation.*

(i) The table connecting w and d^2 is

w	20	45	80	125	180	245	320
d^2	4	9	16	25	36	49	64

If we plot w against d^2 we shall obtain a straight line through the origin (the reader should do this); therefore $w \propto d^2$.

(ii) From the table we see that $\frac{w}{d^2}$ equals 5 for each set of corresponding values, for $\frac{20}{4}=5=\frac{45}{9}$, etc.

$$\therefore\ \frac{w}{d^2} \text{ is constant and } w \propto d^2.$$

Example 3.—*Find the law of variation between the volume and the surface area of a sphere.*

We have $\frac{A}{A_1}=\frac{r^2}{r_1{}^2}$. $\quad\therefore\ \frac{r}{r_1}=\frac{A^{\frac{1}{2}}}{A_1{}^{\frac{1}{2}}}$.

Also $\frac{V}{V_1}=\frac{r^3}{r_1{}^3}=\frac{A^{\frac{3}{2}}}{A_1{}^{\frac{3}{2}}}$. $\quad\therefore\ V \propto A^{\frac{3}{2}}$.

EXERCISE XXIV (b)

1. If $y \propto x^2$, what is the effect on y (i) of trebling x, (ii) of dividing x by 4? What is the effect on x (i) of multiplying y by 9, (ii) of dividing y by 100, (iii) of multiplying y by 2, (iv) of increasing y by 44 per cent.?

2. If $y \propto x^2$, and if $y=3$ when $x=2$, find y when $x=7$, and find x when $y=300$.

3. If $y \propto \sqrt{x}$, what is the effect on y (i) of multiplying x by 25, (ii) of dividing x by 16? What is the effect on x (1) of multiplying y by 9, (ii) of dividing y by 100?

4. If $y \propto x^2$, complete the following table:

$x=$	0	1	2	3	5	10
$y=$			12			

5. What is the law connecting the following variables? Write it with the symbol of variation.

(i) The area A of a circle and its radius r.
(ii) The speed v of a train and the time t taken to travel a given distance.
(iii) The area A of an equilateral triangle and its base b.
(iv) The volume V of a regular tetrahedron and its edge l.
(v) The area A of a circle and its circumference C.
(vi) The volume V of a jug of given shape and its height h.

6. Write the following laws in symbols:

(i) The force F between two magnetic poles varies inversely as the square of their distance d apart.
(ii) The time t of oscillation of a pendulum varies as the square root of the length l.
(iii) The area A of the surface of a solid of given shape varies as the square of its greatest girth l.
(iv) The volume V of a sphere varies as a cube of its radius r. What is the law of variation connecting V with the diameter d?
(v) The range of view r at sea varies as the square root of the height h of the observer above sea-level.
(vi) The square of the least electric current C which fuses a wire of given material varies as the cube of the diameter d of the wire.

7. Use the data of No. 6 (ii) to say what change must be made in the length of the pendulum in order to treble the time of oscillation.

8. Use No. 6 (iv) to say what change must be made in the radius to multiply the volume by 1000.

9. Use No. 6 (v) to say what change in range of view results from doubling the height of the observer above sea-level.

10. If the ratio of the areas of the surfaces of two spheres is 64 : 1, what is the ratio of (i) their diameters, (ii) their volumes?

11. Two jugs are similar in shape; one is 8 in. high and holds a quart. What is the volume of the other, which is 2 ft. high? What is the ratio of the areas of their bases?

12. It costs 15 shillings to gild a bowl whose height is 6 in. What should it cost to gild a similar bowl whose height is 12 in.?

13. A solid ball of Onazote whose diameter is $2\frac{1}{2}$ in. weighs $\frac{3}{8}$ oz. How much would a similar ball of diameter 5 in. weigh? If a ball of the same composition weighed 1 oz., what would you expect its diameter to be?

14. A cube of metal whose edge is x cm. weighs y grams: complete the table:

x	1	2	4	5
y				325

15. If y varies as $\sqrt{x}$, complete the table:

x	0	1	2	4	25	100
y				4		

16. The area S of the surface of a regular pyramid varies as the square of an edge, and the volume V varies as the cube of an edge. What is the variation relation between V and S? If S is increased in the ratio 25 : 1, what is the effect on V?

17. The tension of a wire of given material varies as the square of its length if it is to produce a given musical note. What must be the tension in a wire 1 ft. long to produce the same note as a wire 9 in. long at a tension of 11 lb.?

18. A triangle whose area is 40 sq. cm. has one of its sides 10 cm. The corresponding side of a similar triangle is 25 cm. Find its area.

19. A is 35 miles from B, and the distance between these places on a map is 2·5 in. The area of the whole map is 14·6 sq. in. Find the area of the country which the map covers.

20. The distance s which a body falls from rest varies as the square of the time it falls. If it falls 16 ft. in 1 second, how far does it fall in 4 seconds, and in the fourth second?

21. Two solid spheres of brass, whose diameters are 2·3 cm. and 4·6 cm., are melted down and cast into one sphere. What will be the diameter of this new sphere?

22. Six lb. of paint are required to cover the surface of a globe 8 ft. in diameter. How much would be required for a globe 12 ft. in diameter?

23. On a map an area of 10 million sq. metres is represented by 0·1 sq. metre. What is the scale of the map?

24. A solid expands so that every linear measurement in increased in the ratio 1·0054 : 1. In what ratio is the volume increased?

25. A cube weighs 24 lb., and the area of one of the faces is 36 sq. in.

What is the area of one of the faces of a cube of the same material weighing 375 lb. ?

26. A jug whose surface area is 2 sq. ft. contains 6 pints. What is the surface area of a similar jug which holds 6 gallons ?

27. The attractive force between two electrical charges is inversely proportional to the square of the distance between them. If it is 1·2 dynes when the distance is 2 ft., what will it be when the distance is 1·5 ft. ?

28. It is required to reduce the illumination received by a lamp 6 ft. away in the ratio 3 : 1. To what distance must it be removed ?

29. The force between two given magnetic poles varies inversely as the square of their distance apart. If the force is 4 dynes when they are 10 cm. apart, at what distance will it be 6 dynes ?

30. Test whether $y \propto x^2$ for the values given below, and, if so, find the variation constant.

y	1·56	2·05	3·06	4·13	6·37
x	2·16	2·48	3·03	3·52	4·37

31. Fill in the gaps in the following table, given that $y \propto \sqrt{x}$.

y	7·59		8·42	
x	5·63	6·28		7·6

32. The square of the time that a planet takes to revolve round the Sun varies as the cube of its distance from the Sun. If the Earth's distance is 93 million of miles, and Mercury's distance is 36 million miles, how long does Mercury take to revolve round the Sun ?

33. A halfpenny and a penny are respectively 2·54 cm. and 3·15 cm. in diameter. Find the relative thicknesses of the two coins, assuming a penny to weigh twice as much as a halfpenny.

34. The time of swing of a simple pendulum varies as the square root of its length. If a pendulum 1 metre in length swings once in a second, find the length of a pendulum that swings 75 times in 1 min.

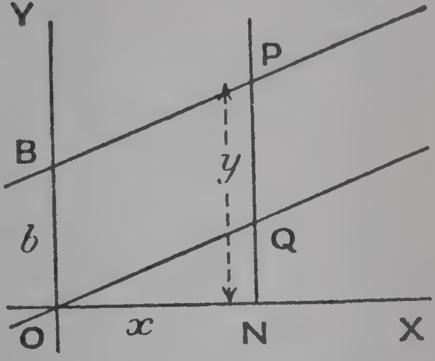
35. A uniform sphere weighs 136·8 grams, and its diameter is 2·62 cm. It is required to reduce the weight to 112·4 grams. How much must be taken off its diameter ?

36. A cone whose height is 12·7 cm. has a volume of 162 c.c. What is the height of a similar cone whose volume is 48·6 c.c. ?

The Linear Relation

The relation between the co-ordinates x and y of any point which lies on a line not passing through the origin is found thus :

Suppose the gradient of the line to be a, and the intercept OB which it makes on the axis OY to be b, and let $P(x, y)$ be any point on it. Through O draw OQ having the same gradient as BP. Then



$$\frac{QN}{ON}=a. \quad \therefore\ QN=ax.$$

But $$PQ=OB=b.$$

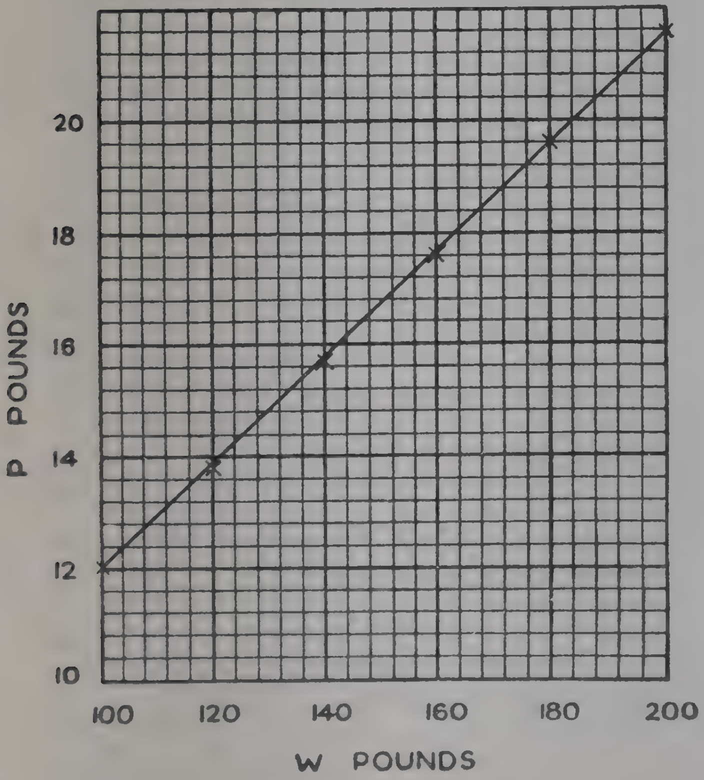
$$\therefore\ PN(y)=ax+b.$$

$y=ax+b$ is called a linear relation between x and y, because its graph is a straight line.

When points are plotted on squared paper, and it is found that they lie on a straight line, it follows that the relation between the co-ordinates of the points is an equation of the form $y=ax+b$.

Example.—An experiment with a screw-jack gives the effort P *pounds required to raise a load* W *pounds as follows:*

P	12·0	13·8	15·7	17·6	19·6	21·5 *lb.*
W	100	120	140	160	180	200 *lb.*



When these values are plotted on a graph, the points lie practically on a straight line whose equation must be

$$P=aW+b,$$

when

$P=12{\cdot}0$, $W=100$, and when $P=21{\cdot}5$, $W=200$.

$$\therefore\ 12=100a+b;$$
$$21{\cdot}5=200a+b.$$

Solving, we obtain

$$a=0{\cdot}095,\ b=2{\cdot}5.$$

$\therefore$ the relation is

$$P=0{\cdot}095W+2{\cdot}5.$$

Experimental errors can often be corrected from the graph by drawing a line which has as many points on one side as on the other; but if the points give a curve which shows a definite tendency to bend continuously, it must not be assumed that the points necessarily should lie on a straight line.

Law $y=ax^n$

It is often suspected that variables obey a law of the form $y=ax^n$; but a and n are unknown. These can be found by plotting the logarithms of y and x. If $y=ax^n$, then $\log y=\log a+n\log x$. If we call $\log y$, Y, and $\log x$, X, this equation reads $Y=\log a+nX$, which is a linear relation between Y and X, and will give a straight line graph.

Example.—The load W *lb. required to produce a given deflection in beams of length* L *in. is observed to be as follows*:

L	10	12	14	16	18	20
W	11·7	6·8	4·27	2·87	2·0	1·47

It is surmised that W and L obey a law of the form $WL^n=c$. Find n and c.

We plot log L against log W, the values being

log W	1·07	·83	·63	·46	·30	·17
log L	1	1·08	1·15	1·20	1·26	1·30

The reader should plot these values, he will then see that they lie approximately on a straight line; its equation must be of the form

$$\log W=a\log L+b.$$

$$\therefore\ 1{\cdot}07=a\,.\,1+b$$

$$\text{and } 0{\cdot}17=a(1{\cdot}3)+b.$$

$$\therefore\ a=-3,\ b=4{\cdot}07=\log 11700.$$

$$\therefore\ \log W=-3\log L+\log 11700.$$

$$\therefore\ W=L^{-3}(11700).$$

$$\therefore\ WL^3=1{\cdot}17\times 10^4.$$

EXERCISE XXIV (c)

1. Two variables x and y are related by the law $y=ax+b$. When $x=20{\cdot}2$, $y=16{\cdot}4$, and when $x=50{\cdot}8$, $y=23{\cdot}7$. Find a and b.

2. The effort P required to raise a load W is given by

P	60	66	75	86	100	lb. wt.
W	230	260	300	360	430	lb. wt.

Find the relation between P and W.

3. From experiments on a screw-jack the effort P required to raise a load W was given by

P	12·0	13·8	15·7	17·6	19·6	21·5	lb. wt.
W	100	120	140	160	180	200	lb. wt.

Show that P and W are related by an equation of the form $P=aW+b$, and find the constants.

4. The corresponding pressure and volume of a gas are given by

v	11	12	13	14	16	18	20
p	188	167	150	135	113	96	83

The law is of the form $pv^n=k$. Find n and k.

5. y and x are related by the law $y=kx^n$, given

x	2	3·5	4·2	7	9·1
y	20·8	195	407	3120	8960

Find n.

6. The volume and pressure of superheated steam are found to be

v	2	4	6	8	10
p	105	42·7	25·3	16·7	13

The law being of the form $pv^n=k$, find it.

7. A stretched wire always gave the same musical note when the tension and length were given by

w	5·5	10	15	20
l	23·6	31·8	38·9	44·9

Find the law connecting l and w by plotting their logarithms. Plot also l and $\sqrt{w}$.

8. The distance travelled by a falling body in time t is given by the following table:

t	1	2	3	4	5	sec.
d	16·1	64·4	145	258	403	ft.

Find the law connecting d and t.

9. Find the law connecting x and y, given

x	18	19·5	20·9	22	27
y	·73	·68	·63	·6	·49

10. Find the law connecting H, the head of water, and Q, the quantity flowing per second, given

H	1·2	1·4	1·6	1·8	2·0	2·4
Q	4·2	6·1	8·6	11·5	15·0	23·7

11. By plotting y and $\frac{y}{x}$, find the law connecting x and y when

x	1	2	3	4	5	7
y	·72	·59	·56	·54	·53	·52

12. Plot y and x^2 to find the relation between y and x, given

x	0	1	2	3	4	5
y	4·95	5·15	5·75	6·7	8·15	9·95

CHAPTER XXV

MISCELLANEOUS PROBLEMS

Relative Velocity

A TRAIN travelling at 40 mls./hr. overtakes a train travelling at 30 mls./hr. at the rate of 10 mls./hr.: this is called the relative velocity. If the trains were moving in opposite directions, their relative velocity would be 70 mls./hr. If they started 210 miles apart they would meet in 3 hours, since they approach 70 miles nearer one another in every hour.

Example (i).—*A train 250 ft. long, travelling at 40 mls./hr., passes another train 200 ft. long, which is travelling 20 mls./hr. in the opposite direction. How long do they take to pass one another?*

A diagram will show that, while passing, the engines have to separate a distance of $200+250=450$ ft., and since they are separating at the rate of 60 mls./hr., *i.e.* 88 ft./sec., they will take $\frac{450}{88}$ sec. $=5 \cdot 1$ sec.

Example (ii).—*At what times between 4 and 5 are the hands of a clock at right angles?*

While the hour hand moves from 4 to 5 (say 1 division), the minute hand travels 12 divisions. ∴ the latter gains 11 divisions on the former during the hour.

At 4 o'clock the minute hand is 4 divisions behind the hour hand, and when they are at right angles it will be either 3 divisions behind or 3 divisions in front; it therefore has to gain either 1 division or 7.

$$\therefore \text{ time taken} = \tfrac{1}{11} \text{ hr. or } \tfrac{7}{11} \text{ hr.}$$
$$= 5\tfrac{5}{11} \text{ min. or } 38\tfrac{2}{11} \text{ min.}$$

∴ the times are $5\frac{5}{11}$ min. past four or $38\frac{2}{11}$ min. past four.

Rate of Working

Example (iii).—*One tap fills a tank in 3 hours and another one would take 4 hours. How long will they take if both are turned on together?*

The first tap fills $\frac{1}{3}$ of the tank in 1 hour, and the second fills $\frac{1}{4}$. ∴ together they fill $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$ of the tank in 1 hour. ∴ they take $\frac{12}{7}$ hours to fill the whole tank, *i.e.* $1\frac{5}{7}$ hours.

Mixtures

Example (iv).—*The specific gravity of a liquid A is* 1·2 *and of B is* 0·62. *In what proportion by volume must they be mixed to make a solution of specific gravity* 0·8*?*

Every unit volume of A in the compound will be $1 \cdot 2 - 0 \cdot 8 = 0 \cdot 4$ over the required weight, whereas every unit volume of B will be $0 \cdot 8 - 0 \cdot 62 = 0 \cdot 18$ below.

∴ A and B must be mixed in the ratio 0·18 of A to 0·4 of B, *i.e.* 18 : 40, for $18(1 \cdot 2) + 40(0 \cdot 62) = 58(0 \cdot 8)$.

∴ required proportion of A to B = 18 : 40 = **9 : 20**.

It is sometimes preferable to use algebraic methods.

Suppose x cu. cm. of A are mixed with 1 cu. cm. of B. Then the mixture contains $(x+1)$ cu. cm. and weighs $1{\cdot}2x+0{\cdot}62$ gr.

$$\therefore\ 1{\cdot}2x+0{\cdot}62=0{\cdot}8\,(x+1)=0{\cdot}8x+0{\cdot}8.$$

$$\therefore\ 0{\cdot}4\,x=0{\cdot}18 \text{ or } x=\frac{0{\cdot}18}{0{\cdot}4}=\frac{18}{40}=\frac{9}{20}.$$

EXERCISE XXV (a)

Note.—The use of algebraic symbols is permitted.

1. How long does it take a train, 300 ft. long, travelling at 30 mls./hr., to pass another train, 250 ft. long, travelling at 20 mls./hr., (1) in the same direction, (2) in the opposite direction ?

2. How long would it take a train, 200 ft. long, travelling at 40 mls./hr., to pass a passenger in another train, 250 ft. long, travelling at 20 mls./hr., (1) in the same direction, (2) in the opposite direction ?

3. A man A, running at an average speed of 100 yd. in 10 sec., starts 10 yd. behind a boy B who runs at the rate of 120 yd. in 14 sec. How long will it take A to catch B ?

4. What do you understand by average speed ? A man goes a walk at 4 mls./hr. and returns at 5 mls./hr. Find his average speed for the whole journey.

5. A cyclist travelling 10 mls./hr. starts 2 hours after a friend who walks at 4 mls./hr. How long will it take the cyclist to overtake his friend ?

6. The cold-water tap will fill a bath in 12 min. and the hot-water tap takes 15 min. How long will it take to fill the bath if both are turned on together ?

7. The cold-water tap fills a bath in 16 min., and the hot-water tap fills it in 20 min. Five minutes after opening the cold tap, the bath is half full. How long was the hot tap running ?

8. When are the hands of a clock together between 6 and 7 ?

9. When are the hands of a clock in line between 4 and 5 ?

10. A is travelling at 20 mls./hr. and is 2 miles in front of B. If B passes A 10 min. later, find B's average speed.

11. A train leaves A for B at 10 a.m. and travels 40 mls./hr. ; at 10.30 a train leaves B for A, travelling at 20 mls./hr. If from A to B is 80 miles, find where they pass.

12. In what proportion must tea worth 1s. 8d. a lb. be mixed with tea worth 3s. a lb. to give a blend worth 2s. 2d. a lb. ?

13. I cycle downhill to the station at 12 mls./hr. and back again at 8 mls./hr. What is my average speed for the double journey ?

14. A motor mowing-machine will mow a cricket field in 4 hours. A man can mow $\frac{1}{20}$th of the field in an hour. How many men must be employed together with the motor to get the whole field finished in 2 hours ?

15. In what proportion must coffee costing 1s. 6d. per lb. be mixed with chicory worth 4d. per lb. so that there may be a profit of 25 per cent. by selling the mixture at 1s. 8d. a lb. ?

16. I go from A to B and back again. Part of the time I walk at 4 mls./hr., and the rest of the time I ride at 20 mls./hr. My average speed is 8 mls./hr. For what fraction of the distance did I walk ?

17. A train 240 ft. long overtook a workman who was walking along the line at 4 mls./hr. and passed him completely in 12 sec. : it then overtook another man, whom it passed in 10 sec. At what rate was the second man walking ?

18. Rowing at a constant average speed, a man can row 6 miles down-stream in 48 min. and back again in 80 min. Find the velocity of the current.

19. A can address 5 circulars per minute and B can address 8. How many will they address together in an hour ?

20. A man sculls at 6 m.p.h. in still water. Compare his time in sculling a mile up-stream and back when there is no current with the time he takes when the stream is running at 2 m.p.h.

21. A watch was 2 min. slow at 8 a.m. and 1 min. fast at 6 p.m. on the same day. When did it show correct time ?

22. A landowner lets land for grazing : one farmer puts in 50 cows for 30 days, another 80 cows for 25 days, and another 100 cows for 10 days. The landowner receives £30. What did each farmer pay ?

23. My train starts in 10 min. and the station is 1 mile away. I walk at 4 m.p.h. and run at 8 m.p.h.. How far must I run ?

24. A cyclist averages 6½ m.p.h. uphill and 19½ m.p.h. downhill. His average speed uphill and downhill is 16½ m.p.h. What is the ratio of the uphill to the downhill portion of his journey ?

25. What is the time at Boston, longitude 71° W., when it is 1 p.m. at Greenwich ?

26. What is the longitude of Calcutta, if the time in Calcutta is 6 hours ahead of Greenwich time ?

27. A train approaching a station at 30 m.p.h. passes over two fog signals placed half a mile apart. Find the interval between the times at which the reports are heard at the station, if sound travels at 1100 ft. per sec.

28. A goods train starts from A at 11.30 a.m., travels at 20 m.p.h. stops for 15 min. at B, 10 miles from A, and then proceeds a 20 m.p.h. A passenger train on a separate track leaves A at 12.10 p.m. and travels at 40 m.p.h., running through B without a stop When and where does the latter train pass the former ?

29. A man stands on a railway platform which is 220 yd. long and times a passing train. He finds that it takes 6 sec. to pass him, and 21 sec. to pass completely through the station. Find the length of the train and its speed in miles per hour.

30. A, B ride at the same rate, namely 12 m.p.h. on the level, 8 m.p.h. uphill, and 14 m.p.h. downhill. They start on the level 60 yd. apart. What is the greatest and least distance between them during a ride over hilly roads ?

31. Four merchants, A, B, C, D, co-operate in a business. A contributes £300 for 12 months, B £330 for 10 months, C £375 for 8 months, D £395 for 6 months. The profits are £818. How should they be divided ?

32. A man starts from A and rows up-stream : $1\frac{1}{2}$ miles above A he passes a bottle floating down ; after another 18 min. he turns back and arrives at A at the same moment as the bottle. If his speed through the water and the speed of the stream are both uniform, find the speed of the stream in miles per hour.

33. A can do a piece of work in 6 days, B in 8 days, C in 9 days, each working 9 hours a day. They all work together and finish the job in 3 days. How many hours a day do they work ? If the total payment is £14, 10s., how ought it to be divided between them ?

34. A submarine can travel at 20 knots on the surface and at 14 knots when submerged. A journey of 65 nautical miles was completed in 4 hours. For what fraction (i) of the distance, (ii) of the time, was the submarine submerged ?

35. The life of a tyre on the front wheel of a car may be taken as 10,000 miles. The tyres on the back wheels wear out one and a half times as fast as those on the front wheels. What is the life (in mileage) of a new set of four tyres, if they are changed round, when necessary, to produce even wear ?

36. In a game of fives—15 points, sudden death—the most level arrangement is for A to give B 7 points and to give C 3 points. Handicap B and C.

37. In a game of billiards, 200 up, the most suitable handicap is for A to give B 50 points and to receive from C 50 points. Handicap B and C.

38. A landing A is approached by two equal moving staircases BA, CA, each ascending. If a man stands still on either he takes

2 min. to get from the bottom to the top. If the staircase stops working, he takes 1 min. to walk from the bottom to the top or from the top to the bottom. How long will he take to go from B to C if he walks up to A and down to C when both staircases are ascending ?

39. A and B are two places 4 miles apart. Trams run regularly from A to B at 3-minute intervals at a uniform speed of 10 m.p.h. A motor cyclist starts from B and rides at a uniform rate to A. He meets 12 trams *between* B and A (not counting any which are going in the same direction as himself). What is (i) the greatest possible speed, (ii) the least possible speed, of the cyclist ?

40. The face of a clock shows all hours from 1 up to 24. At what time are the hands of the clock at right angles between 9 and 10 o'clock ?

Some of the examples in the next Exercise are more properly described as "puzzles" than as problems. It is suggested that these should be taken incidentally. rather than as part of a set course.

EXERCISE XXV (b)

1. A common type of error when using an automatic machine for adding sums of money is to record sums such as £7, 5s. as 7s. 5d., or £13, 9s. as 13s. 9d. If only one amount is recorded incorrectly, and if the error in the result is £x, ys. zd., prove that $x+y+z$ is either 11 or 30, and that the wrong entry is either xs. $(12-z)$d. or $(x+1)$s., $(12-z)$d.

2. Show that any exact number of lb. up to 13 can be weighed out if a 1-lb. weight, a 3-lb. weight, and a 9-lb. weight are available. What is the least number of weights necessary to weigh out any exact number of lb. up to 120 lb. ?

3. A pipe of diameter 16 cm. branches into three portions, each of diameter 9 cm. Is the rate of flow in the main pipe more or less than that in the branches, if each pipe is full ?

4. What is the product of the first hundred of the expressions $1-\frac{1}{2}$, $1-\frac{1}{3}$, $1-\frac{1}{4}$, $1-\frac{1}{5}$, etc. ?

5. Two rectangular sheets of paper are such that they can be bent without overlapping to form a closed paper box. One sheet is a in. by b in., the other is c in. by d in. What relation connects a, b, c, d ?

6. Write down a "reasonable" formula for the volume of an ellipsoid whose semi-axes are of lengths a, b, c ft. [*Note.*—If $a=b=c$, the ellipsoid is a sphere of radius a ft.]

7. In a billiard match of 9000 points up, A gives B a start of 2000.

When A has scored 4000, B's score reads 5190. Who has the better of the game so far, and by how much ?

8. A cistern can be emptied by 3 pipes in 3 hours : one pipe alone would take 6 hours, and another alone would take 9 hours. How long would the third pipe take by itself ?

9. There are two sets of pumping apparatus for pumping hydrogen into an airship : one set can fill it in 18 hours and the other in 12 hours. What is the least time in which the airship can be filled ?

10. A greengrocer buys bananas at a penny each and wishes to sell them at a profit of *about* 30 per cent. The selling price is to be quoted at so many for a shilling. What number should he sell for a shilling ?

11. A 6-in. diameter pipe takes the water from two other pipes of diameters 3, 5 in., in which the water is running at 2, 3·6 ft. per sec. respectively. If all the pipes are full, find the speed in the 6-in. pipe.

12. At a certain instant the minute and seconds hands of a watch are parallel. After how long will they be again parallel ?

13. A bath is filled in 39 min. by the hot-water tap A, and in 33 min. by the cold-water tap B. It is emptied by a pipe C in $35\frac{3}{4}$ min. If, when the bath is empty, both taps suddenly start running and cannot be turned off, owing to faulty washers, how soon will the bath overflow if C is (i) closed, (ii) open ?

14. What is the least multiple of 7 which has a remainder 1 when divided either by 2 or 3 or 4 or 5 or 6 ?

15. The gear of a bicycle is 70. Show that the number of revolutions of the pedals in $12\frac{1}{2}$ sec. gives the speed in miles per hour [Take $\pi=\frac{22}{7}$. The gear of a bicycle is said to be x if one revolution of the pedals advances the bicycle a distance equal to the circumference of a circle of diameter x in.]

16. A grocer buys teas at 2s. 2d. per lb. and 3s. 6d. per lb., and mixes them in such a proportion that he gains 10 per cent. by selling the mixture at 3s. 8d. per lb. In what proportion does he mix them ?

17. A tradesman marks his goods at 5 per cent. above cost price he uses a fraudulent balance which makes goods whose true weight is 15 oz. appear to weigh 1 lb. What is his gain per cent. for goods sold by weight ?

18. A number consists of 4 digits ; the first and last digits are equal, and the digit in the ten's place is 4 times the digit in the hundred's place. Show that the number must be divisible by 7 Can you draw any conclusion if the first and last digits are equal and if the digit in the ten's place is 3 times the digit in the hundred's place ?

19. A cricket ground 120 yd. by $\frac{1}{4}$ mile is to be mown by three machines : (1) a hand machine A, width 1 ft. ; (ii) a horse machine B

width 3 ft.; (iii) a motor machine C, width 3 ft. The speeds of A, B, C are 3, 3, 5 m.p.h. respectively: they all start together. When half the field is done, A gives up. How long will B, C take to finish the job? What is the total time taken to mow the whole field?

20. Two jars, each full, contain mixtures of rum and water, one in the proportion 4 : 11 and the other 3 : 5. When the contents of the two are mixed, the proportion becomes 1 : 2. Find the ratio of the capacities of the two jars.

21. A can plough a field in 60 hours of work, B in 48 hours, and C in 50 hours. How long will C take to finish it by himself, if he starts when A and B have each done 12 hours' work?

22. The total area of three fields is $25\frac{1}{2}$ acres, and the areas of the two smaller fields are respectively three-quarters and four-fifths of the largest. Find the size of each.

23. A circular running-track is 440 yards round, and two boys A and B start at the same time from the same point to run round the track in opposite directions. They pass one another when A has run 230 yards. By how much would A win in a quarter-mile race, if both continued to run at uniform speeds?

24. A town desired to raise a statue 12 ft. high to King Edward. The sculptor made a plaster model 2 ft. 6 in. high which weighed 36 pounds. If marble is 2·1 times as heavy as the plaster, find the weight of the statue to the nearest cwt.

25. (i) A housekeeper can buy candles, either $8\frac{1}{2}$ in. long at 9d. a half-dozen, or $10\frac{1}{4}$ in. long at 11d. a half-dozen, the quality and thickness being the same in each case. She decides the former is the cheaper. Is she right? (ii) Suppose now that on the average a length of candle $\frac{3}{4}$ in. at the bottom is wasted. Is she right in this case?

26. A takes 7 steps to 5 of B's, but 9 of B's steps cover the same length as 11 of A's. Which moves more quickly?

27. A man leaves in his will $\frac{1}{3}$ of his whole property to his wife and $\frac{1}{5}$ to each of two sons; other legacies amount to £375, and the residue goes to a daughter. She receives £929. What is the whole property?

28. The Field Service Regulations state that "250 pack ponies pass a given point in 10 minutes, 100 transport carts pass a given point in 10 minutes; each pack pony takes up 4 yd. and each transport cart takes up 7 yd., including clearance." (i) Find the speed at which ponies travel and carts travel. (ii) Find how long it would take a column of pack ponies to catch up a column of transport carts two miles ahead.

29. A flask is fitted with an air-tight cap. When full of air it weighs 72·43 gr. Some of the air is sucked out and it now weigh 72·27 gr. The volume of air withdrawn is now determined by opening the tap just under water, when it is found that 125 c.c. of wate run in. Find to two significant figures the weight of 1 c.c. of air.

30. In an examination, the marks for each subject were scale so that the top boy should have 100 and the others in proportion There were three candidates, A, B, C. In French, before scaling A had 96, B had 60, C had 32; in Mathematics C was top with 8 while A had 48. After scaling it was found that in the totals B wa bracketed top. How many marks (unscaled) did he get for Mathematics ?

31. A destroyer passes a lighthouse at 20 miles per hour; a gu is fired from the lighthouse at this moment and at intervals of on minute afterwards. At what intervals will the reports of the gu be heard on the deck of the destroyer, whose course is straight. [Tak the velocity of sound as 1100 ft. per second.]

32. A car uses petrol at the rate of 18 miles to the gallon whe travelling at 20 m.p.h. and 15 miles to the gallon at a speed of 36 m.p.h. a journey of 153 miles is made on 9½ gallons, part being run at 2 m.p.h. and the rest at 36 m.p.h. How long did the journey take ?

33. A reservoir for a town of 120,000 people is 1200 yd. long 110 yd. wide, and of average depth 6 ft. The springs which feed i are just sufficient in ordinary times. In a dry season the water i supplied by the springs at only ¼ of the ordinary rate, and it is foun that if the reservoir starts full it is emptied in 200 days. Calculat the number of gallons used daily per head of the population. [1 cu. ft =6¼ gallons.]

34. The Musketry Regulations lay down that (*a*) for every inc the barometer rises or falls above 30 in., you should add or deduc 1½ yd. per 100 yd. of range; (*b*) for every degree the temperatur rises or falls above 60° F., you should add or subtract $\frac{1}{10}$ yd. pe 100 yd. of range.

(i) If the barometer is at 31·5 in. and the temperature is 100° F calculate how you should sight your rifle for a range of 1200 yd.

(ii) For the same temperature and range, calculate the height o the barometer if the rifle should be sighted for 1220 yd.

35. A customer was about to buy 3 articles at 3s. 7½d. each, bu the shopkeeper persuaded him to take 5 of the articles for 15s Assuming that this gave the shopkeeper a larger profit, find th greatest possible cost of each article.

36. A liner 814 ft. long travelling at 20 m.p.h. is overtaken b a torpedo boat 154 ft. long. A passenger watching from a porthol on the liner notices that it takes 7 seconds for the torpedo boat to pass him. How long would it take to pass the liner completely, if going in the opposite direction ?

37. On estates exceeding £400,000 and not exceeding £600,000 the estate duty is 12 per cent. ; but for estates from £600,000 to £800,000 it is 13 per cent. If an estate is worth £524,000 after duty is paid, what was it worth before, correct to nearest £1000. Show that there are two answers. Find also the greatest and least values of an estate, after duty is paid, so that two answers may be possible.

38. In a large two-bladed pocket knife of ordinary pattern the distance between the tips of the two blades is (i) 6 cm. when both are closed, (ii) 30 cm. when both are fully open, (iii) 13 cm. when both are at right angles to the frame. Find the distance between the pins about which the blades pivot, and the length of each blade.

39. Abbas II., the deposed Khedive of Egypt, succeeded to the throne on January 7, 1892. The Egyptian law says that a Khedive becomes of age to rule on the eighteenth anniversary of his birth. According to the Christian calendar, Abbas would have been 6 months 7 days too young to rule. But according to the Mohammedan calendar he was 14 days over the required age. When was he born, and how many days are there in the Mohammedan year ?

40. A slide rule is graduated from 1 to 10 so that any graduation x is at a distance from 1 which is proportional to log x. If the whole scale from 1 to 10 is 10 in. long, find the distance between the graduations 6·5 and 7. If another precisely similar scale is placed alongside, pointing the same way, so that 3·6 on the second is opposite 6·5 on the first, what graduation on the second will be opposite 7 on the first ?

MISCELLANEOUS EXAMPLES

M (VIII)

1. Show that $\dfrac{\frac{1}{70}-\frac{1}{99}}{1+\frac{1}{6930}}$ equals $\dfrac{\frac{6}{7}+\frac{2}{17}-\frac{115}{119}}{\frac{5}{3}+\frac{122}{357}}$.

2. One-sixth of the contents of a cask, originally full, evaporated, and then 8 gallons were drawn off. If the cask was still two-thirds full, how much could it hold ?

3. Write down the cost of a gross of envelopes at $4\frac{3}{4}$d. per dozen.

4. Find the square root of 0·01841449.

5. Simplify $\frac{7}{8}\div 1\frac{5}{16}-\frac{2}{3}\times\dfrac{\frac{4}{5}-\frac{1}{3}}{1-\frac{8}{15}}$.

6. A man buys 5000 shares when they stand at 6s. 3d. How many more could he have bought for the same money, if he had waited until they fell to 5s. 6d. ?

7. The speed of a car increased at a uniform rate from 20 m.p.h. to 35 m.p.h. in 3 min. What was its speed after 40 sec. ? Answer in ft./sec.

8. A man purchases a $\frac{2}{5}$ interest in a business for £3500 : the average annual profits of the business are £525. What percentage does he receive on his investment ?

9. Take any perfect square number : divide it by 0·01 ; add the square root of the quotient to the original number ; increase the result by 25. Is the final number a perfect square ? Give a general proof.

10. A certain liquid expands 10 per cent. when it freezes. How much per cent. does it contract when it melts ?

11. In a school of 500 boys, 350 learn Latin, 250 learn Greek, and 450 learn French. How many at least learn all three languages ?

12. In a mixed school there were twice as many boys as girls in 1920. By 1925 the number of boys had fallen by 20 per cent. and the number of girls increased by 20 per cent. Find the ratio of boys to girls in 1925. Find also the percentage change in the total numbers of the school from 1920 to 1925.

13. An inch scale and a centimetre scale are placed side by side, so that the 2-inch mark coincides with the 3-cm. mark and the scales run the same way. What graduations on the cm. scale fall between the 7-inch and 8-inch graduations on the inch scale ?

14. The volume of a gas is inversely proportional to the pressure if the temperature is constant. If the volume is 650 c.c. when the pressure is 15 lb. per square inch, what will be the volume when the pressure is 25 lb. per square inch ?

15. A man sailing along the Equator notices that his chronometer appears to gain 20 min. a day. At what rate is he travelling ? [Take the circumference of the Earth to be 24,900 miles.]

16. There are 3 exits from a concert hall by means of which the room can be cleared in 5 min., 8 min., 10 min. respectively. How soon could it be cleared if all were used at once ?

17. If $x \propto \dfrac{1}{\sqrt{y}}$, find the effect on y of doubling x, and the effect on x of doubling y.

18. A square wooden trencher, standing in a vertical plane, with one side on a flat table, is rolled in its own plane until this same side is again in contact with the table. If the side of the trencher is 8 in. long, how far does each corner of it travel ?

19. A cylindrical tank has a diameter of 4 ft. Find to the nearest inch the depth of water in the tank when the weight of water is a ton.

20. A man's annual income from a loan was increased by £1, 7s. 6d., owing to the rate of interest rising from 3 per cent. to $3\frac{1}{2}$ per cent. How much was the loan ?

21. The depth of a standard bushel measure is 8 in., and its diameter is $18\frac{1}{2}$ in. What must be the diameter of another cylindrical measure of the same capacity, and depth 18 in. ?

22. The area that can be enclosed by a given length of fencing is greatest when the area is circular. If the length of fencing available is 400 yd., what is the maximum area that can be enclosed ? Answer in acres to three significant figures.

23. Two cyclists, A and B, are riding together. A stops to adjust his saddle, while B rides on at 10 mls./hr. A is delayed for 5 min., and then overtakes B after another 10 min. At what speed does A ride ?

24. According to the Registrar-General's tables, out of 10,000 persons born, 6652 live to be 35 years of age, 4418 live to be 60, and 2279 live to be 70. What percentage of persons 35 years old may expect to survive to 60 ? What percentage of persons of 60 may expect to die before 70 ?

25. I raise £350 by selling $2\frac{1}{2}$ per cent. Consols at 55. How much stock do I sell, to nearest £ stock, and what is the consequent loss of income ?

26. A tradesman sells $\frac{3}{4}$ of his stock of an article at a profit of 35 per cent. At what percentage below cost can he mark the remainder at a sale so as to secure a profit of 25 per cent. on the whole stock ?

27. A rides 12 miles from Bristol to Clevedon at 10 mls./hr., and, starting at 12 o'clock, he stays there 1 hour and returns at 9 mls./hr. B starts at 2 o'clock from Bristol and walks to Clevedon at 3 mls./hr. How far from Bristol does B meet A ?

28. A motorist approaching a fort at the rate of 30 mls./hr. sees the flash of a gun fired from the fort and hears the report 16 sec. later. Taking the velocity of sound to be 1120 ft./sec., find how far the motorist was from the fort when the gun was fired.

29. In a certain business A invests £20.000, B £3000, C £2000. In any year half the profits are divided equally between B and C for managing the concern; the other half is distributed between A, B, C in proportion to their capital invested. In a year in which C receives £870, what will each of the others receive ?

30. A contractor employs 14 men, each working 8 hours a day, to do a job in 19 days. At the end of 10 days the work is suspended for 2 days owing to an accident in which 4 of the men are disabled. How many more men must he engage to finish the job in the required time if all the men now work 9 hours a day?

31. To what depth will 500 spherical drops of water, each $\frac{1}{10}$ inch in diameter, fill a conical wine-glass, the height of the cone being equal to the diameter of the base?

32. Seven families occupy the different storeys of a seven-storied house, the lowest storey being on the ground floor. The annual charge for keeping the common staircase clean is £9, 2s. What is the share of the family on the top storey?

33. What is the least possible difference between two unequal numbers if one is a multiple of 105 and the other a multiple of 504?

34. A body consists of a circular cylinder with a hemisphere at one end and a cone at the other end, fitting these ends. If the surfaces of these three portions are of equal area, show that the volumes of these portions are respectively in the ratio $3 : 2 : \sqrt{3}$.

35. A clock is put right at 6 o'clock, and at 5 minutes past 7 by the correct time the hands of the clock are observed to be again in a straight line. How much is the clock losing or gaining per day?

36. The following observations were made of the length and time of vibration of a pendulum:

l	0·5	1·0	1·5	2·0	2·5	3·0	ft.
t	0·79	1·11	1·36	1·58	1·76	1·94	sec.

The relation between l and t being of the form $t=kl^n$, find n from these figures.

37. One gallon of wine containing 11 per cent. of water is added to 3 gallons of water containing 7 per cent. of wine, and to this mixture ½ a gallon of water is added. What is the percentage of water in the whole?

38. If $N=\dfrac{2\pi^2e^4m}{h^3}$ where $h=6{\cdot}545\times 10^{-27}$,

$$e=4{\cdot}774\times 10^{-10},\quad \frac{e}{m}=1{\cdot}767\times 10^{7},$$

find N to three significant figures.

39. On a certain steamer the expense of coaling per mile is found to be directly proportional to the square of the speed maintained, being 2s. 6d. per mile when the speed is 30 mls./hr., the other expenses being £1, 16s. per hour at all speeds. Express in £ the cost of a

journey of 100 miles at x mls./hr., and draw a graph showing the expense at any speed from 10 to 35 mls./hr. Find the most economical speed and the expense at that speed.

40. The surfaces of two spheres are in the ratio 2 : 3. What is the ratio of the volume of the less to the volume of the greater in the form 1 : n?

41. A cylindrical pan with a radius of a cm. contains water to a depth of x cm. Show that if another cylindrical pan of the same height, but with a radius of b cm., is placed within it the water rises $\frac{b^2x}{a^2-b^2}$ cm. Evaluate when $a=10{\cdot}5$, $b=8{\cdot}5$, $x=5{\cdot}5$.

42. Three metals which weigh p, q, r lb. per cubic foot are united in the respective proportions of $a : b : c$ by weight to form an alloy, no change of volume taking place through mixing. Show that the volume in cubic inches of 1 lb. of alloy is $\frac{1728}{a+b+c}\left(\frac{a}{p}+\frac{b}{q}+\frac{c}{r}\right)$. Find this volume if the proportions by weight are 21 : 6 : 5 and their weights in lb./cu. ft. are 1206, 656, 556.

43. If 215542 is divided by a certain number by long division, the successive remainders in the process are 436, 353, and 94. Find the quotient.

44. A gas company is not allowed to pay a dividend of more than 7 per cent. on its (£1) shares until the price of 1000 cu. ft. of gas falls below 3s. 9d., but it is allowed to add 3s. 6d. per 100 shares for every penny that the price of gas falls below 3s. 9d. per 1000 cu. ft. A householder who has shares in the company finds that when the price of gas is lowered 3d. per 1000 cu. ft., the money that he receives in dividends is increased by 5 per cent.; also that he can now get an extra 9000 cu. ft. of gas per year without increasing his gas bill. Find how much gas he consumed before the change, assuming the company always pays the full dividend permitted.

45. In 1891 the average price of wheat was said to be 37s., and 18 per cent. less than in 1881: in 1901 it was 26s. 9d., and 41 per cent. less than in 1881. If each percentage is correct to the nearest whole number, find whether these statements can be consistent.

46. A publisher spends £300 on producing a book and £5 a year on advertising it. He pays the author 6d. a copy on all copies sold after the first 1000. He sells it at 2s. 6d. a copy, accounts being made up at the end of each year. After a certain number of years he is £160 to the bad, but if he had sold twice as many copies he would be just free of loss. Find the number of years.

47. The inhabitants of a Swiss town speak either French or

German, or both: 73 per cent. speak French and 87 per cent. speak German. What percentage speak both?

48. A penny and a halfpenny have diameters 1·2 in. and 1 in. respectively: 1 lb. avoirdupois of bronze can be coined into 48 pennies or into 80 halfpennies. Express the mean thickness of a halfpenny as a decimal of the mean thickness of a penny.

49. Two casks, A, B, contained originally 40 gallons of wine and 20 gallons of water respectively. Eight gallons are transferred from A to B, then 8 gallons from B to A, and finally 8 gallons from A to B, the liquids being thoroughly mixed. Express in gallons and decimals of a gallon the quantity of wine now in each cask.

50. A lends B a certain sum: at the same time he insures B's life for £737, 12s. 6d., paying annual premiums of £20. At the end of 3 years, and just before the fourth premium is to be paid, B dies, having never repaid anything. What must A have lent B in order that he may have just enough to recoup himself, together with 5 per cent. per annum compound interest on the sum lent and on the premiums?

51. The engine of a car makes $4\frac{1}{2}$ revolutions to each revolution of the road wheel; the diameter of the road wheel is 875 mm. Calculate the number of revolutions per minute of the engine for a speed of 40 m.p.h. [1 ft. = 30·5 cm.]

52. 230,400 grains of gold are needed to coin 1869 sovereigns. During the War £5,000,000 was sent on a ship to America. Find its weight to the nearest ton. [7000 grains = 1 lb. avoirdupois.]

53. Southampton Water is a channel about $8\frac{1}{2}$ miles long, with an average breadth of $1\frac{2}{3}$ miles. If the difference between the high-water and low-water mark is 6 ft., find to the nearest million tons the number of tons of water that enter and leave the channel at each tide. Take the water as weighing 5 per cent. more than fresh water does.

54. The bulb of a thermometer is a sphere of 0·85 cm. inside diameter. The stem is graduated in degrees, and the distance between the graduations is 2·2 mm. Find the cross-section of the stem, given that mercury expands 0·000182 of its volume when heated one degree, neglecting the expansion of the glass.

55. The muzzle-velocity of a shell of diameter D in., weight w lb., is $\sqrt{\left\{\frac{64\cdot4\times2240\times3\cdot142\times D^2\,PL}{4w}\right\}}$ feet per second, where P = pressure in tons per sq. in. and L = travel of shell in feet. Calculate the velocity if D = 12, w = 850, L = 45, P = 11·5.

56. A piece of a paper a mile long is rolled round so as to form a solid cylinder of paper. Find its diameter, given that 300 such sheets make a pile an inch thick.

57. A certain tooth-paste is sold in a collapsible cylinder $3\frac{1}{2}$ in. long and of internal diameter 0·9 in. The paste is squeezed out through a cylindrical nozzle 0·18 in. in diameter. Assuming that, each time it is used, a length of 0·35 in. is squeezed out, find how many days it lasts, using it twice a day.

58. A hollow sphere of specific gravity s will just float if its thickness is $1-\left(1-\frac{1}{s}\right)^{\frac{1}{3}}$ times its external radius. Find the maximum thickness permissible if $s=7·8$ and the external radius is 10 cm.

59. A certain article on its way from the manufacturer to the user passes through the hands of three middlemen, each of whom makes a profit of 20 per cent. When the price to the user is raised £1, 16s., what increase of price has been made by the manufacturer ?

60. A solenoid for magnetisation is constructed by winding two layers of wire on a glass rod 1 ft. long, of external diameter 0·4 in. The thickness of the first layer and the necessary insulation necessitates the second layer being wound on a cylinder of diameter 0·5 in. The cross-section of the wire is 0·024 in. in diameter. Find the number of turns of wire in each layer and the total length of wire required, to the nearest foot.

61. A man saves 20 per cent. of his income. If his expenditure increases by 35 per cent., by how much per cent. must his income be increased, so that he may still save 10 per cent. of it ?

62. Four vertical posts, each 5 ft. high, are placed so that their feet form the corners of a square of side 8 ft., in a horizontal plane ; the posts are the framework of a wire entanglement. The top of each post is joined by wire to the foot of each other post. Find to the nearest foot the length of wire required.

63. When treated with sufficient acid, it is found that 65·4 gr. of zinc give 22·9 litres of hydrogen (barometric height 760 mm.) ; but this volume is inversely proportional to the height of the barometer. How much zinc will be required to give 3·67 litres of hydrogen when the barometer stands at 785 mm., temperature constant ?

64. My coal bill for last year was £48 ; ten years ago it was £36. If my annual coal consumption during the interval has diminished by 15 per cent., find by how much per cent. the price of coal has increased in the same time ?

65. APOS is a square whose diagonals intersect at T. The circle, centre O, radius OP, cuts AO at K. Taking $\pi=\frac{22}{7}$ and $\sqrt{2}=1·4$, show that to this approximation the triangle APK and the circular half-segment PKT are equal in area.

REVISION PAPERS

(R 51–60)

R 51

1. What is the volume correct to the nearest cubic inch of the largest circular cylinder that can be cut from a rectangular block 8 in. by 10 in. by 10 in. ?

2. If I am losing 3 per cent., what did I pay for goods which I am selling for £x ?

3. Evaluate to three significant figures $a\sqrt{\left(\frac{b}{c^3}\right)}$ where $a=47{\cdot}2$, $b=0{\cdot}3413$, $c=0{\cdot}265$.

4. In coaling a ship 2114 bags of coal were taken on board : the average weight of a bag was obtained from weighing ten full bags chosen at random ; these were found to weigh 214, 218, 232, 208, 212, 238, 226, 256, 210, 242 lb. An empty bag weighs 12½ lb. Find the number of tons of coal received, correct to three significant figures.

5. 3½ per cent. Conversion Loan can be bought at 74⅞. Allowing ¼ per cent. for charges, find the cost of £550 stock and the income it yields.

6. A can do a piece of work in 7 days which A and B together can do in 4 days. How long would B take to do it alone ?

R 52

1. Taking 1 yard as ·9144 metre, express in metres a length of 72 yd. 2 ft. 6 in.

2. What sum of money diminished by 6 per cent. amounts to £1222 ?

3. Calculate the radius of a sphere whose volume is 112·7 cu. in.

4. Find the simple interest on £240, 16s. 8d. for 2½ years at 4 per cent.

5. The foot of a ladder is 15 ft. from a wall, and the top reaches a point 40 ft. from the ground. How high will the top be when the foot is 20 ft. from the wall ?

6. The cross-section of a trough is an isosceles triangle and the water in it is 1 ft. deep. How deep will the water be if the volume of water in the trough is doubled ?

R 53

1. Simplify (i) $(\frac{51}{15}-\frac{15}{51})\div\frac{22}{17}+\frac{17}{22}$; (ii) $\frac{51}{15}-\frac{15}{51}\div\frac{22}{17}-\frac{17}{22}$.

2. A train 66 yd. long overtakes a man walking along the line at 4 m.p.h. and passes him in 7½ seconds. Find the speed of the train in miles per hour.

3. Oranges are bought at three for 2d. How many must be sold for a shilling to gain 80 per cent.?

4. Evaluate to three significant figures $\dfrac{mv^2}{gr}$ where $m=3{\cdot}6$, $v=44$, $g=32{\cdot}2$, $r=1270$.

5. Find correct to the nearest penny the compound interest on £245 for 3 years at 4½ per cent. per annum.

6. What are the proceeds resulting from the sale of 80 Imperial Tobacco shares at $5\frac{5}{16}$ and 120 Bleachers at $3\frac{7}{16}$, if 1½ per cent. must be deducted from the cash receipts to cover the cost of the sales?

R 54

1. A gold mine produced in 1 year 6260 oz. of gold valued at £26,338. Find, without logs, the value per ounce to the nearest penny.

2. A beam with a square section is 9 ft. long and weighs 3½ cwt. The density of the wood is 32 lb. per cubic foot. Find the thickness of the beam.

3. If the interest on £162, 13s. 8d. for 3½ years is £25, 12s. 5d., find what rate of interest was charged.

4. If a varies inversely as b^2, find in what ratio a must be increased to make b diminish in the ratio 1 : 5.

5. Find the compound interest on £216, 16s. 8d. for 2½ years at 4 per cent., paid half-yearly.

6. If I invest in 4 per cent. stock at 81, what rate of interest do I obtain on my money?

R 55

1. A small heavy sphere of lead is suspended by a fine thread and swings freely as a pendulum. The time of 20 swings is observed for different lengths of thread, and is recorded as follows:

Length of thread, l in.	8	12	22	32	48	62	72	83
Time of 20 swings, t sec.	9	11		18		25		29

Show that these observations are consistent with $l \propto t^2$, and estimate the missing observations.

2. How long will it take a battalion of 1200 men in fours to pass a given point? The distance between two fours is 54 in., and the normal rate of marching is 120 paces of 30 in. each per minute.

3. Evaluate as accurately as four-figure tables permit

$$\frac{(1\cdot065)^{21}-1}{1\cdot065-1}.$$

✓ 4. A man bought 80 lb. of fish for 12 shillings. He had to throw away 35 lb. of it as bad. At what price per lb. must he sell the rest to gain 50 per cent.?

✓ 5. A customer borrows £243, 7s. 6d. from his bank for 60 days at $5\frac{1}{2}$ per cent. What interest to the nearest penny will the bank charge?

6. What is the yield per cent. on a two-shilling share quoted at 6s. 9d. and paying 45 per cent.?

R 56

1. Express $\frac{77}{321}$ as a decimal (to three places).

2. Find the area of the ring between two concentric circles whose diameters are 7·28 in. and 10·72 in.

3. Find the prime factors common to 374 and 204.

4. £520 amounts to £551, 4s. in 2 years at simple interest. What sum would have amounted to twice as much in the same time at the same rate?

5. A club deducts 5 per cent. every year from the book-value of its furniture as depreciation. If at the end of 4 years its book-value stands at £240, what was its cost?

6. A litre of a certain liquid weighs 1·340 Kg. When mixed with water the solution weighs 1·270 Kg. per litre. Find the volume of the liquid in a litre of the mixture.

R 57

1. One rupee=16 annas=64 pice, and 1 rupee is worth 1s. $7\frac{1}{2}$d. Express 62 rupees 12 annas 2 pice as a decimal of £1, correct to three places of decimals.

2. A sports ground has a circular track of the same width all round. The area of the ground, including the track, is 12 acres, and excluding the track is 10 acres. Find the width of the track. [Take $\pi=3\frac{1}{7}$.]

3. Some Christmas cards are bought at 10d. a dozen; half of them are sold at 2d. each, one-quarter at $1\frac{1}{2}$d. each, and the rest remain unsold. Find the profit per cent.

4. Evaluate to two significant figures $e^{2\pi\mu n}$ where $e=2{\cdot}718$, $\pi=3{\cdot}142$, $\mu=0{\cdot}35$, $n=2\frac{3}{4}$.

5. Find correct to the nearest penny what sum of money invested now at $4\frac{3}{4}$ per cent. per annum simple interest will amount 8 months hence to £437, 10s.

6. The Bovril Australian Estates have issued £125,940 6 per cent. debentures and £195,530 7 per cent. cumulative preference shares. What profit was made if £7328 was put to reserve after paying the interest on the debentures and the preference shares?

R ·58

1. Simplify $\dfrac{(\frac{4}{9})^2-(\frac{5}{27})^2}{(\frac{3}{5})^2-(\frac{2}{15})^2}$.

2. A frustum of a cone is 1 ft. high, the radii of the two ends being 9 in. and 16 in. Find the height of the complete cone and the volume of the frustum.

3. Two lead spheres of radii 1 in. and 1·5 in. are melted down and cast into a single sphere. Find its radius.

4. The resistance to an electric current of a steel wire of given length varies inversely as the square of its diameter. If the resistance is 3·5 ohms when the diameter is ·025 in., find the resistance of the same length of wire whose diameter is ·035 in.

5. What difference would it make in a man's income if he invested £1000 in a 4 per cent. stock at 125 or a $2\frac{3}{4}$ stock at 88. [Neglect brokerage and charges.]

6. A road up a hill has a gradient of 1 in 4. A cyclist zigzags up so that his track makes an angle of 60° to the line of greatest slope. What is the gradient of his track?

R 59

1. Simplify $\dfrac{1\frac{1}{7}-\frac{3}{11}}{\frac{7}{22}+\frac{3}{35}}\div 1\frac{2}{3}$.

2. A room is 31 ft. long and 23 ft. wide. Find the cost of carpeting it, leaving a border 2 ft. 6 in. wide all round, if the carpet is 27 in. wide and costs 8s. 9d. per yard.

3. A merchant makes a profit of 15 per cent. by selling an article for £8, 12s. 6d. What selling price would double his profit?

4. Evaluate by logarithms $(0{\cdot}623)^{0\cdot4}$.

5. Which stock gives the bigger yield, 4½ per cent. at 87 or 5½ per cent. at 106 ?

6. Five miles of paper are wound on to a roller of diameter 6 in., the complete roll being a cylinder 9 ft. long, 3 ft. in diameter. Calculate the thickness of the paper as the decimal of an inch, correct to two significant figures.

R 60

1. Find the H.C.F. of 1768, 4862, and 11271.

2. What will it cost to carpet a room 35 ft. 4 in. long and 27 ft. wide with carpet 27 in. wide at 5s. 9d. per yard.

3. Find the value of $(1{\cdot}34)^{-1{\cdot}4}$.

4. A man saves £40 every year for 4 years, investing it at the beginning of each year. If he receives compound interest at 4 per cent., how much money has he at the end of 4 years, to the nearest shilling ?

5. The bore of a pipe is 4·20 in. and the thickness of metal is ·50 in. Find the weight of a foot length if the specific gravity of the iron is 7·2.

6. 49 per cent. of a heap containing 68,900 copper coins are English pennies. The coins are roughly sorted into two heaps, one containing all the English pennies, which form 91 per cent. of that heap. How many coins were in that heap ?

PROBLEM PAPERS

(P 51–60)

P 51

1. Given that the logarithms of 2 and 3 to base 10 are respectively 0·30103 and 0·47712, find without using tables the logarithms of (i) 1½, (ii) 120, (iii) 5, (iv) $\sqrt{0{\cdot}6}$.

2. A man borrows £45 and discharges his debt at the end of 8 months by repaying £54. What rate of interest has the money-lender charged ?

3. A is 20 per cent. heavier than B, and C is 20 per cent. lighter than B. Compare the weights and A and C.

4. The gasbag of an airship is in the form of a cylinder with hemispherical ends. If its diameter is 40 ft., and if it can hold 285,000 cu. ft. of gas, find its extreme length.

5. A person puts £160 in the bank at the beginning of a year: he adds to it £75 at the beginning of each of the three succeeding years, and is allowed $2\frac{1}{2}$ per cent. per annum interest on his money, the interest being added at the end of each year. Find the amount he will have in the bank at the end of 4 years.

P 52

1. When a certain number is multiplied by 284, the result is 2,545,236 greater than it would have been if the multiplier were 176. How much greater still would the result be if the multiplier were 316 instead of 284?

2. Two equal solid metal cylinders are melted down and recast into a single cylinder, whose height is to that of the original as 5 : 3. By how much per cent. has the radius been increased?

3. A procession takes 10 min. 10 sec. to pass me, and I then immediately mount my bicycle and, riding at 10 mls./hr., I reach the head of the procession in 5 min. Find the length of the procession and the rate it walked.

4. A house costs £460 to build. At how much per week must it be let in order to pay 6 per cent. per annum, allowing 10 per cent. of the rent for repairs?

5. A car deteriorates 40 per cent. in value during its first year's use, and during each subsequent year by a certain other fixed percentage of its value at the beginning of each year. A car bought 4 years ago for £350 would now fetch £153, 2s. What will it be worth in another 3 years?

P 53

1. What is the average of the prime numbers between 30 and 50?

2. An Englishman holds \$6000 of 4 per cent. American Railway Bonds. What is the change in the yearly dividend if the American exchange alters from 4·50 to 4·35 dollars to the £?

3. A man's weekly wages were increased by 50 per cent. and the number of hours he worked per week was reduced by 20 per cent. By how much per cent. was his rate of pay per hour increased?

4. Starting in each case from the beginning, the sum of any number of terms of the set of numbers 3, 21, 57, 111, 183, 273, . . . varies as the cube of the number of terms taken. Find (i) the sum of the first ten terms, (ii) the value of the seventh term.

5. A manufacturer sells a car to a dealer at a profit of 20 per cent.;

the dealer sells it to a customer at a profit of 25 per cent.; the customer sells it to a friend for £350 and thereby loses 30 per cent. What did the car cost to make?

P 54

1. In an examination for which the maximum was 6000, A got 12 per cent. more than B, B got 16 per cent. more than C, and C got 20 per cent. more than D. A's mark was 4872. Find what percentage of the maximum was obtained by D.

2. One pendulum oscillates 6 times in 3·2 sec., and another 8 times in 3·6 sec. If they start simultaneously, how often will they tick together in 1 hour?

3. On the average, rainfall in March is the same as in April. In 1923 the rainfall was 20 per cent. above average in March and 11 per cent. below average in April. If it had been 8 per cent. below average in May, the rainfall for the 3 months would have been normal. Find whether May is, on the average, a drier or wetter month than April, and by how much per cent.

4. A man borrows £2500, which he is to repay in 3 equal yearly instalments, starting one year after the loan is made. Compound interest at 5 per cent. per annum is to be paid, calculated on the amount owing at the time of each payment. Find the amount of each instalment.

5. A passenger on a vessel 250 yards long, steaming at 20 mls./hr., notices that waves travelling in the same direction as the steamer pass the stern at intervals of 35 sec., and that a wave travels the length of the boat in 10 sec. Find the distance between the crests of successive waves.

P 55

1. Find the least number which, when divided by 56, 66, or 385, leaves no remainder.

2. A and B invest £170, 10s. and £232, 10s. respectively at the same date in War Savings Certificates costing 15s. 6d. each. At a certain subsequent date A's holding has a value of £200. What is the value of B's holding at that date, correct to the nearest shilling?

3. A cylindrical tin holds 1 pint. A second tin is half the height of the first, but its diameter is $1\frac{1}{2}$ times that of the first. How much does it hold?

4. A train leaves a terminus A at 9 a.m., and its first stopping-place is at B, half a mile away. A man's house is 600 yd. from A and 800 yd. from B. The train travels at 12 m.p.h. and the man

walks at 4 m.p.h. Can he catch the train at A or at B if he starts at 8.55 a.m.?

5. The profits of a boarding establishment in 1912 were 40 per cent. of the gross yearly expenses. In 1920 the expenses had increased 80 per cent., and the profits were 15 per cent. of the expenses. In addition, the purchasing value of money was 55 per cent. less in 1920 than in 1912. What was the decrease per cent. in the purchasing value of the profits derived from the business during these dates?

P 56

1. A tank is emptied in 30 min. by a pipe whose bore is 1·2 in. How long would it have taken if the bore had been 2 in., assuming a constant rate of discharge in each case?

2. One vessel contains 20 gallons of liquid A, and another contains 8 gallons of liquid B. One gallon is transferred from the first vessel to the second and then one gallon from the second to the first. Find the proportion of A to B in the first vessel after the operation has been performed.

3. A body of troops $\frac{1}{4}$ mile long, marching along a road at 3 mls./hr., is passed by cyclists in single file. A cyclist takes $2\frac{1}{2}$ min. to pass the troops, and successive cyclists arrive at the rear of the troops at intervals of 5 sec. Find the distance between the cyclists.

4. A bridge is in the form of an arc of a circle: its span is 234 ft. and its height 13 ft. Find the radius of the circle.

5. Three sledges, A, B, C, each taking provisions for 40 days, set off from their base together. A goes to a point where it can fill up B and C and keep enough food to return to the base. B goes on with C to a point where it can fill up C and return to the base. A and B on reaching the base at once fill up and set off to meet the returning sledges. How many days' journey from the base can C travel? [Note the total consumption of food and solve mentally.]

If A and B each wait 20 days at the base before setting out again, show that C can then reach a maximum distance from the base, and find that distance. Draw a diagram showing the position of each sledge when 40, 60, 70 days have elapsed.

P 57

1. A salesman finds that his profits amount to 50 per cent. of his expenditure. What percentage are they of his turnover (*i.e.* the sum of the receipts and expenditure)?

2. If one of the following numbers, 153, 171, 172, 180, 182, 198, 200, 208, 229, 253, 254, 271, 345, 425, 4395, is crossed out, the remaining numbers can be arranged in two groups such that the sum of the numbers in one group is eight times the sum of the numbers in the other group. Which number must be crossed out? [You are **not** asked to say how the two groups are composed.]

3. In a certain year, 2785 tons of ore were taken from a tin mine and yielded 7328 lb. of tin. In the following year the output of ore was increased by 15 per cent., and the yield of tin per ton of ore was increased by 10 per cent. Find the amount of tin obtained, to the nearest 100 lb.

4. A metal sphere whose external diameter is 6 in. weighs 20 lb. ; the metal weighs 448 lb. per cubic foot. Show that the sphere is not solid, and find the volume of the hollow in it.

5. A's income is $\frac{3}{4}$ of B's income; A's expenditure is $\frac{1}{2}$ of B's income; B's expenditure is equal to the whole of A's income. Which saves most?

P 58

1. In a certain district there are two evening schools. In the first, 20 per cent. of the students are adults and 50 per cent. are boys, the rest are girls. The second school contains boys and girls only. If the two schools were amalgamated there would be 8 per cent. adults and 56 per cent. boys. Find the percentage of boys and girls in the second school.

2. Driving to the station every morning at 20 mls./hr., I generally pass at the same place a man who is walking at 4 mls./hr. One day when I am 5 min. late I pass the same man $\frac{1}{4}$ mile beyond the usua place. Is he early or late, and by how many minutes?

3. The surface of a reservoir is evenly frozen over. If the uppe surface of the ice is $\frac{3}{10}$ in. higher than that of the water before the frost (there having been no loss of water), find the thickness of the ice. Density of ice ·917.

4. A rectangular cistern has a base 4 ft. 6 in. by 3 ft. 6 in. and is 2 ft. 6 in. high. It is placed on a slope, so that one of the shorte edges of the base is 1 ft. above the other. How much water will i now hold?

5. [To be solved mentally.] A and B start simultaneously t meet one another from places 100 miles apart, riding at 25 mls./hr At the moment of starting a fly leaves A and travels continuousl backwards and forwards between A and B during their journey If the fly travels 35 mls./hr., how far has it flown when A meets B?

P 59

1. A population increases at "compound interest," rising from b to c in m years. What will it be at the end of a further period of $2m$ years?

2. The girth of the trunk of an oak tree varies as the square root of the number of years it has lived. The girth of an oak 36 years old is 3 ft. What is the girth of an oak 100 years old?

3. A hollow spherical copper ball of radius 12 cm. floats in water, so that its highest point is 16 cm. above the surface of the water. What fraction of the whole surface is immersed?

4. Criticise the following advertisement:

"Our prices are 30 per cent. below those of any other firm. Moreover, until June 30, we will give you 20 per cent. off these prices. That is to say, your money now will go twice as far with us as with anyone else."

What further discount must be given to substantiate the claim made in the last sentence?

5. For the year 1923–4 the standard rate of income tax was 4s. 6d. in the £. But certain deductions are made from a man's total income in order to fix the amount which is liable to income tax, viz. (i) one-tenth of the income which comes from salary is deducted; (ii) £225 for a man and his wife and £27 for each child is deducted. Also the first £225 of the taxable income is only charged at half the standard rate. What income tax is paid by a man with wife and 3 children if his salary is £1250 a year, and if he has £9750 of 5 per cent. War Loan?

P 60

1. In a shooting competition A hits the bull every 2 shots out of 3, B hits 3 times out of 4, and C 4 times out of 5. The number of bulls was 931. If each fired the same number of shots, find the number of bulls each scored.

2. Air is a mixture by weight of approximately 77 per cent. nitrogen and 23 per cent. oxygen. Find the percentage composition by volume, given that the densities of nitrogen and oxygen are as 7 : 8.

3. A and B can do a piece of work in $5\frac{2}{3}$ days, and A does twice as much as B in a given time. How long would it take A alone to do the work?

4. A number of 4 digits is divisible by 33. The number is *1*4. Fill in the missing digits. Is there more than one answer?

5. Find five consecutive integers, none of which are prime.

EXERCISES IN RAPID COMPUTATION

(½-hour Papers)

(A–J)

A

1. $936{,}217-17{,}843-234{,}613-42{,}738$.

2. $647{,}839\times 27$.   3. $426{,}718\div 113$.

4. £100 − £2, 7s. $8\frac{1}{2}$d. × 12.   5. $623\times 3\frac{1}{7}$.

6. Cost of 564 things at 4s. 7d. each.

7. What number, increased by 4 per cent., equals 648,362 ?

8. Express £11, 17s. $8\frac{1}{4}$d. as a decimal of £1 to four places.

9. Evaluate $\frac{3{\cdot}1416\times(0{\cdot}43)^2\times 880}{144}$, given that the significant figures in the answer are 355.

10. Simple interest on £427, 5s. 6d. for 2 years at $3\frac{1}{2}$ per cent.

11. Factors of 2805.

12. Sum of all odd numbers between 10 and 100 which are multiples of 3.

B

1. $28{,}614{,}319+17{,}824{,}150-9{,}623{,}174-3{,}149{,}518$.

2. 1,643,872 inches to miles, etc.   3. $814\div 6\frac{2}{3}$.

4. $719{,}623\div 137$.

5. Cost of 627 things at £3, 11s. 9d. each.

6. 1 divided by an integer gives a quotient 0·01923 . . . Find the integer.

7. Two carpets are the same shape; one is 12 ft. 10 in. long and 10 ft. wide; the other is 8 ft. 9 in. wide: how long is it ?

8. After a reduction of 1s. 8d. in the £, a sum of £588, 19s. 2d. is left. What was it originally ?

9. Compound interest on £632, 14s. for $2\frac{1}{2}$ years at 3 per cent., paid yearly.

10. 6,217,843 tons are worth £4,917,825, 16s. Find value per ton to nearest penny.

11. If 92 per cent. of a number is 70,288, what is 104 per cent. of it ?

12. Evaluate $\frac{1}{11}+\frac{1}{11^2}+\frac{1}{11^3}+\ldots$ to three decimal places.

C

1. $605-1749-(87-3624)+99$.
2. Quotient and remainder when 30,071 is divided by 59.
3. $4\frac{4}{39}-2\frac{15}{26}$.
4. $2\frac{1}{9}\div 12\frac{2}{3}$.
5. $0{\cdot}0212925\div 425$.
6. 87 articles at 6s. 9d. per half-dozen.
7. $6\frac{1}{4}$ per cent. of £4, 3s. 4d.
8. $\frac{567}{x}=\frac{7}{27}$. What is x?
9. Express as a percentage a rate of 9d. in the £.
10. Express 10,000 oz. in cwt., lb.
11. The cost of 3 tons 12 cwt. 2 qr. at £1, 14s. 2d. per ton.
12. 19 articles cost £1, 18s. $9\frac{1}{2}$d. What do 48 cost?

D

1. $3872-(999+1869)-1001$.
2. $18{,}075\times 995$.
3. 1000 articles at 18s. 4d. each.
4. $10{,}101\div 99$.
5. $12\frac{7}{12}\times 12\frac{7}{12}$.
6. Express 17s. $8\frac{3}{4}$d. as a decimal of £1 to three figures.
7. Find the rent of 17 acres 2 roods 24 poles at £2, 7s. 6d. per acre.
8. Divide 0·01 by 7·07 to three figures.
9. The simple interest on £62, 4s. for 9 months at $5\frac{1}{2}$ per cent., to the nearest penny.
10. 17 per cent. of a mile and a quarter in yards and inches.
11. Arrange in order of magnitude, $\frac{5}{7}$, $\frac{11}{14}$, $\frac{19}{35}$, $\frac{15}{28}$, $\frac{13}{21}$.
12. How many yards of linen at 4s. 1d. per yard cost as much as 126 yards of cloth at 13s. $1\frac{1}{2}$d. per yard?

E

1. $64{,}218+7192\times 37-3596\times 53$.
2. $536{,}297\times 379$.
3. £250 − £1, 19s. $7\frac{1}{2}$d. × 17.
4. $\cdot 0101\div 97$ to 3 figures.
5. A gross at 13s. $3\frac{1}{2}$d. each.
6. $\dfrac{3\frac{3}{4}+2\frac{5}{8}-4\frac{1}{16}}{3\frac{1}{12}\times 2\frac{2}{5}\div 2\frac{1}{2}}$.
7. £27, 19s. 8d. ÷ $7\frac{2}{3}$.
8. A debtor pays 15s. 8d. in the £. How much does he owe a man to whom he pays £376, 18s. 9d.?

9. Find the volume of a pyramid on a square base of side 18 in., the length of the slant edges being 2 ft.

10. Evaluate $\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\ldots$ correct to three significant figures.

11. The population of a town decreased 4 2 per cent. between 1891 and 1901, increased 5·3 per cent. between 1901 and 1911, and increased 6 6 per cent. between 1911 and 1921. What was the percentage increase for the 30 years ?

12. What sum of money amounts to £285, 10s. in 3 years at 4 per cent. compound interest ?

F

1. Multiply 7624 by 789.

2. 62·758 ÷ ·003678 to three significant figures.

3. Cost of 547 at £1, 18s. 4d. each.

4. $361 \div 3\frac{9}{13}$.

5. What decimal of £2, 17s. 7d. is £1, 11s. 5½d. ? (Three places.)

6. Cost of 2 miles 5 furlongs 55 yards at £19, 8s. 6d. a mile.

7. Find the value per cwt. to nearest penny if 6,827,319 cwt. are worth £4,761,825, 15s.

8. A manufacturer wishes to gain 20 per cent. on his goods and to offer 25 per cent. trade discount to retailers. At what price must he list goods which cost him £1, 4s. 2d. ?

9. Compound interest on £186, 17s. at 3½ per cent. for 4 years.

10. L.C.M. of 594, 616, 2079.

11. Find in acres the area of a district covering 26 sq. in. on a map of 6 in. to the mile.

12. Find the radius of a sphere whose volume is 892 c.c.

G

1. (i) $4\frac{11}{24}+9\frac{7}{36}$; (ii) $11\frac{2}{21}-8\frac{3}{14}$. 2. 0·3321 ÷ 73·8.

3. Value of 2 tons 9 cwt. 2 qr. 8 lb. at £4, 13s. 4d. per ton.

4. How much per cent. of £8, 15s. is 15s. 9d. ?

5. Goods worth £1 are increased in price to £1, 1s. 7d. What was the original value of goods now worth £121, 18s. 11d. ?

6. Find the area of a path 6½ ft. wide round a rectangular grass plot 27 ft. by 42½ ft.

7. A cylinder which holds 46 gallons is 2 ft. 6 in. high. Find the internal radius of the base.

8. How long will it take a cyclist riding at 10 mls./hr. to catch up a man who is $\frac{3}{4}$ mile in front of him and walking $4\frac{1}{2}$ mls./hr. ?

9. What sum of money will amount to £411, 18s. 4d. in 3 years at 4 per cent. simple interest ?

10. How much per cent. on my money do I get by investing in $3\frac{1}{2}$ per cent. stock at 74 ?

11. Evaluate $\sqrt[5]{\cdot 00623}$.

12. A sale price of £18, 8s. gives a profit of 15 per cent. What profit per cent. would be got by selling for £18 ?

H

1. Calculate exact value of $\dfrac{417\cdot 197}{\cdot 0253 \times 164\cdot 9}$.

2. Express as a vulgar fraction in lowest terms 0·96875.

3. Find to nearest unit, without logs, the average number of people per square mile, given population 315,086,372, and area of country 1,802,577 square miles.

4. Find cost of 61·273 Kg. at £12, 19s. 4d. per Kg.

5. $7\frac{1}{3}$ per cent. of £396, 15s. 8d., to nearest penny.

6. Volume of metal in 40 ft. of iron tube, internal radius 4·60 in. and thickness of metal 0·40 in.

7. A bankrupt owes £726, 13s. 8d., and his assets are £246, 7s. 9d. How much can he pay in the £ ?

8. If 5 sovereigns weigh 41 grams, how many sovereigns will weigh 2 Kg. 50 grams ?

9. Find the area of the walls of a hall 26 ft. 3 in. wide, 42 ft. 9 in. long, and 18 ft. 4 in. high.

10. What sum of money, when decreased by 6 per cent., comes to £35 ?

11. On a map a road 1320 yards is represented by $18\frac{3}{4}$ in. What area on this map would represent $\frac{1}{10}$ square mile ?

12. A barometer tube 125 cm. long has an internal diameter of 24 mm. Find the cost of filling it with mercury at 16s. 3d. a Kg. [Specific gravity of mercury 13·6.]

I

1. $634{,}279 \times 749$.
2. £118, 19s. 3d. $\div 4\frac{3}{7}$.

3. Evaluate to nearest integer without using logs :

$$3\cdot 1416 \times 242\cdot 376 \times \cdot 00634.$$

4. Show that $\sqrt{8}$ lies between $\frac{17}{6}$ and $\frac{82}{29}$.

5. Divide £9, 8s. 6d. in the ratio $\frac{2}{3} : \frac{5}{7}$.

6. A pond of 8 acres is frozen over with ice 3 in. thick. Find th weight of ice, given 1 cu. ft. weighs 896 oz.

7. Evaluate $3{\cdot}142(7{\cdot}86^2 - 5{\cdot}93^2)$.

8. Find in yards, etc., the equivalent of 62·43 metres, give 1 metre = 39·37011 inches.

9. Find the circumference of a circle whose area is 64·28 sq. in.

10. If 10 per cent. is gained by selling at 2s. 3d. per lb., wha selling price would represent a loss of 5 per cent.?

11. Out of a circular disc of metal 140 equal circular holes ar punched, and the weight of metal thus removed is to the weight o the original disc as 45 : 112. Find the ratio of the diameter of th disc to that of each hole.

12. Compound interest on £126, 17s. 9d. at 4 per cent. for $1\frac{1}{2}$ years paid half-yearly.

J

1. $6328 + 473 \times 968$.

2. £621, 17s. 9d. + £8342, 17s. 7d. + £3216, 13s. 10d. + £8476, 9s. 8d

3. 29 tons 6 cwt. 3 qr. 11 lb. ÷ 218.

4. If $\frac{2}{3}$ of a sum of money is £1697, 12s. 6d., what is the value o $\frac{5}{9}$ of the sum?

5. Cost of 5 miles 3 fur. 44 yd. at £12, 7s. 4d. per mile.

6. $57{\cdot}528 \div 0{\cdot}0376$.

7. A brick with mortar is 9 in. by $4\frac{1}{2}$ in. by 3 in. How many wil be required for the 3 nine-inch walls of a garage 10 ft. high, tw of them 17 ft. 6 in. wide, with a back wall 12 ft. wide, interna dimensions, allowing for two windows each 3 ft. by 3 ft. in one o the side walls?

8. A man buys 12 boxes of oranges, each containing 120, at 8s. 6d per box. What profit per cent. does he make by selling them a $1\frac{1}{2}$d. each?

9. A box with a lid measures externally 30 in. by 28 in. by 9 in. and the wood is $\frac{3}{4}$ in. thick. Find the volume of wood.

10. A solid sphere has a volume of 74·6 cu. in. What would be it volume if the radius were doubled?

11. Five per cent. of the value at the beginning of each year i deducted from goods originally worth £250. What is their value a the end of 4 years?

12. Evaluate $\sqrt[5]{{\cdot}07648}$.

R. B. A. N. M'S EDUCATIONAL CHARITIES
No.........................
Date.........................
BANGALORE.

TABLES

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

RECIPROCALS OF NUMBERS. FROM 1 TO 5·5

[*Numbers in difference columns to be subtracted, not added.*]

	0	1	2	3	4	5	6	7	8	9	Mean Differences.							
											1	2	3	4	5	6	7	8
1·0	1·000	9901	9804	9709	9615	9524	9434	9346	9259	9174								
1·1	·9091	9009	8929	8850	8772	8696	8621	8547	8475	8403								
1·2	·8333	8264	8197	8130	8065	8000	7937	7874	7813	7752								
1·3	·7692	7634	7576	7519	7463	7407	7353	7299	7246	7194								
1·4	·7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5	10	14	19	24	29	33	38
1·5	·6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	4	8	13	17	21	25	29	33
1·6	·6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	7	11	15	18	22	26	29
1·7	·5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	6	10	13	16	20	23	26
1·8	·5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3	6	9	12	15	17	20	23
1·9	·5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3	5	8	11	13	16	18	21
2·0	·5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19
2·1	·4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	7	9	11	13	15	17
2·2	·4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	12	14	16
2·3	·4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2	4	5	7	9	11	13	14
2·4	·4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	3	5	7	8	10	12	13
2·5	·4000	3984	3968	3953	3937	3922	3906	3891	3876	3861	2	3	5	6	8	0	11	12
2·6	·3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1	3	4	6	7	8	10	11
2·7	·3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	3	4	5	7	8	9	11
2·8	·3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	1	2	4	5	6	7	9	10
2·9	·3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	2	3	5	6	7	8	9
3·0	·3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1	2	3	4	5	6	7	9
3·1	·3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1	2	3	4	5	6	7	8
3·2	·3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8
3·3	·3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	4	5	6	7
3·4	·2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	1	2	3	3	4	5	6	7
3·5	·2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1	2	2	3	4	5	6	6
3·6	·2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	2	3	4	5	5	6
3·7	·2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	1	2	3	4	4	5	6
3·8	·2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1	1	2	3	3	4	5	5
3·9	·2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	1	2	3	3	4	4	5
4·0	·2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5
4·1	·2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	1	2	2	3	3	4	5
4·2	·2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	4
4·3	·2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4
4·4	·2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1	1	2	2	3	3	4	4
4·5	·2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	0	1	1	2	2	3	3	4
4·6	·2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	1	1	2	2	3	3	4
4·7	·2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	3	3	4
4·8	·2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	0	1	1	2	2	3	3	3
4·9	·2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3
5·0	·2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	2	2	2	3	3
5·1	·1961	1957	1953	1949	1946	1942	1938	1934	1931	1927	0	1	1	2	2	2	3	3
5·2	·1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0	1	1	1	2	2	3	3
5·3	·1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3
5·4	·1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0	1	1	1	2	2	2	3

[*Numbers in difference columns to be subtracted, not added.*]

	0	1	2	3	4	5	6	7	8	9	Mean Differences.								
											1	2	3	4	5	6	7	8	9
5·5	·1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0	1	1	1	2	2	2	3	3
5·6	·1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0	1	1	1	2	2	2	3	3
5·7	·1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	1	2	2	2	3
5·8	·1724	1721	1718	1715	1712	1709	1706	1704	1701	1698	0	1	1	1	1	2	2	2	3
5·9	·1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0	1	1	1	1	2	2	2	3
6·0	·1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	1	2	2	2	3
6·1	·1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
6·2	·1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
6·3	·1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0	0	1	1	1	1	2	2	2
6·4	·1562	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	0	1	1	1	1	2	2	2
6·5	·1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	0	0	1	1	1	1	2	2	2
6·6	·1515	1513	1511	1508	1506	1504	1502	1499	1497	1495	0	0	1	1	1	1	2	2	2
6·7	·1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	1	1	1	1	2	2	2
6·8	·1471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	0	1	1	1	1	2	2	2
6·9	·1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	0	1	1	1	1	2	2	2
7·0	·1429	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	0	1	1	1	1	1	2	2
7·1	·1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0	0	1	1	1	1	1	2	2
7·2	·1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	0	1	1	1	1	1	2	2
7·3	·1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	0	1	1	1	1	1	2	2
7·4	·1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	0	1	1	1	1	1	1	2
7·5	·1333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0	0	1	1	1	1	1	1	2
7·6	·1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	1	1	1	1	1	1	2
7·7	·1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	1	1	1	1	1	1
7·8	·1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	0	0	1	1	1	1	1	1
7·9	·1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	0	0	1	1	1	1	1	1
8·0	·1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	1	1	1	1	1	1
8·1	·1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	0	0	1	1	1	1	1	1
8·2	·1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
8·3	·1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0	0	0	1	1	1	1	1	1
8·4	·1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	1	1	1	1	1	1
8·5	·1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	0	0	1	1	1	1	1	1
8·6	·1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
8·7	·1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	0	0	1	1	1	1	1	1
8·8	·1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	0	0	1	1	1	1	1	1
8·9	·1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	0	0	1	1	1	1	1	1
9·0	·1111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0	0	0	1	1	1	1	1	1
9·1	·1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0	0	0	0	1	1	1	1	1
9·2	·1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	0	0	0	1	1	1	1	1
9·3	·1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	0	0	0	1	1	1	1	1
9·4	·1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	1	1	1	1	1
9·5	·1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
9·6	·1042	1041	1039	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	1	1	1	1	1
9·7	·1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0	0	0	0	1	1	1	1	1
9·8	·1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	1	1	1	1	1
9·9	·1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	0	1	1	1	1

	0	1	2	3	4	5	6	7	8	9	Mean Differences.							
											1	2	3	4	5	6	7	8
10	1000	1020	1040	1061	1082	1103	1124	1145	1166	1188	2	4	6	8	10	13	15	17
11	1210	1232	1254	1277	1300	1323	1346	1369	1392	1416	2	5	7	9	11	14	16	18
12	1440	1464	1488	1513	1538	1563	1588	1613	1638	1664	2	5	7	10	12	15	17	20
13	1690	1716	1742	1769	1796	1823	1850	1877	1904	1932	3	5	8	11	13	16	19	22
14	1960	1988	2016	2045	2074	2103	2132	2161	2190	2220	3	6	9	12	14	17	20	23
15	2250	2280	2310	2341	2372	2403	2434	2465	2496	2528	3	6	9	12	15	19	22	25
16	2560	2592	2624	2657	2690	2723	2756	2789	2822	2856	3	7	10	13	16	20	23	26
17	2890	2924	2958	2993	3028	3063	3098	3133	3168	3204	3	7	10	14	17	21	24	28
18	3240	3276	3312	3349	3386	3423	3460	3497	3534	3572	4	7	11	15	18	22	26	30
19	3610	3648	3686	3725	3764	3803	3842	3881	3920	3960	4	8	12	16	19	23	27	31
20	4000	4040	4080	4121	4162	4203	4244	4285	4326	4368	4	8	12	16	20	25	29	33
21	4410	4452	4494	4537	4580	4623	4666	4709	4752	4796	4	9	13	17	21	26	30	34
22	4840	4884	4928	4973	5018	5063	5108	5153	5198	5244	4	9	13	18	22	27	31	36
23	5290	5336	5382	5429	5476	5523	5570	5617	5664	5712	5	9	14	19	23	28	33	38
24	5760	5808	5856	5905	5954	6003	6052	6101	6150	6200	5	10	15	20	24	29	34	39
25	6250	6300	6350	6401	6452	6503	6554	6605	6656	6708	5	10	15	20	25	31	36	41
26	6760	6812	6864	6917	6970	7023	7076	7129	7182	7236	5	11	16	21	26	32	37	42
27	7290	7344	7398	7453	7508	7563	7618	7673	7728	7784	5	11	16	22	28	33	38	44
28	7840	7896	7952	8009	8066	8123	8180	8237	8294	8352	6	12	17	23	29	34	40	46
29	8410	8468	8526	8585	8644	8703	8762	8821	8880	8940	6	12	18	24	30	35	41	47
30	9000	9060	9120	9181	9242	9303	9364	9425	9486	9548	6	12	18	24	31	37	43	49
31	9610	9672	9734	9797	9860	9923	9986				6	13	19	25	31	38	45	52
31								1005	1011	1018	1	1	2	3	3	4	5	5
32	1024	1030	1037	1043	1050	1056	1063	1069	1076	1082	1	1	2	3	3	4	5	5
33	1089	1096	1102	1109	1116	1122	1129	1136	1142	1149	1	1	2	3	3	4	5	5
34	1156	1163	1170	1176	1183	1190	1197	1204	1211	1218	1	1	2	3	3	4	5	6
35	1225	1232	1239	1246	1253	1260	1267	1274	1282	1289	1	1	2	3	4	4	5	6
36	1296	1303	1310	1318	1325	1332	1340	1347	1354	1362	1	1	2	3	4	4	5	6
37	1369	1376	1384	1391	1399	1406	1414	1421	1429	1436	1	2	2	3	4	5	5	6
38	1444	1452	1459	1467	1475	1482	1490	1498	1505	1513	1	2	2	3	4	5	5	6
39	1521	1529	1537	1544	1552	1560	1568	1576	1584	1592	1	2	2	3	4	5	6	6
40	1600	1608	1616	1624	1632	1640	1648	1656	1665	1673	1	2	2	3	4	5	6	6
41	1681	1689	1697	1706	1714	1722	1731	1739	1747	1756	1	2	2	3	4	5	6	7
42	1764	1772	1781	1789	1798	1806	1815	1823	1832	1840	1	2	3	3	4	5	6	7
43	1849	1858	1866	1875	1884	1892	1901	1910	1918	1927	1	2	3	3	4	5	6	7
44	1936	1945	1954	1962	1971	1980	1989	1998	2007	2016	1	2	3	4	5	5	6	7
45	2025	2034	2043	2052	2061	2070	2079	2088	2098	2107	1	2	3	4	5	5	6	7
46	2116	2125	2134	2144	2153	2162	2172	2181	2190	2200	1	2	3	4	5	6	7	7
47	2209	2218	2228	2237	2247	2256	2266	2275	2285	2294	1	2	3	4	5	6	7	8
48	2304	2314	2323	2333	2343	2352	2362	2372	2381	2391	1	2	3	4	5	6	7	8
49	2401	2411	2421	2430	2440	2450	2460	2470	2480	2490	1	2	3	4	5	6	7	8
50	2500	2510	2520	2530	2540	2550	2560	2570	2581	2591	1	2	3	4	5	6	7	8
51	2601	2611	2621	2632	2642	2652	2663	2673	2683	2694	1	2	3	4	5	6	7	8
52	2704	2714	2725	2735	2746	2756	2767	2777	2788	2798	1	2	3	4	5	6	7	8
53	2809	2820	2830	2841	2852	2862	2873	2884	2894	2905	1	2	3	4	5	6	7	9

	0	1	2	3	4	5	6	7	8	9	Mean Differences.								
											1	2	3	4	5	6	7	8	9
54	2916	2927	2938	2948	2959	2970	2981	2992	3003	3014	1	2	3	4	6	7	8	9	10
55	3025	3036	3047	3058	3069	3080	3091	3102	3114	3125	1	2	3	4	6	7	8	9	10
56	3136	3147	3158	3170	3181	3192	3204	3215	3226	3238	1	2	3	5	6	7	8	9	10
57	3249	3260	3272	3283	3295	3306	3318	3329	3341	3352	1	2	3	5	6	7	8	9	10
58	3364	3376	3387	3399	3411	3422	3434	3446	3457	3469	1	2	4	5	6	7	8	9	11
59	3481	3493	3505	3516	3528	3540	3552	3564	3576	3588	1	2	4	5	6	7	8	10	11
60	3600	3612	3624	3636	3648	3660	3672	3684	3697	3709	1	2	4	5	6	7	8	10	11
61	3721	3733	3745	3758	3770	3782	3795	3807	3819	3832	1	2	4	5	6	7	9	10	11
62	3844	3856	3869	3881	3894	3906	3919	3931	3944	3956	1	3	4	5	6	8	9	10	11
63	3969	3982	3994	4007	4020	4032	4045	4058	4070	4083	1	3	4	5	6	8	9	10	11
64	4096	4109	4122	4134	4147	4160	4173	4186	4199	4212	1	3	4	5	6	8	9	10	12
65	4225	4238	4251	4264	4277	4290	4303	4316	4330	4343	1	3	4	5	7	8	9	10	12
66	4356	4369	4382	4396	4409	4422	4436	4449	4462	4476	1	3	4	5	7	8	9	11	12
67	4489	4502	4516	4529	4543	4556	4570	4583	4597	4610	1	3	4	5	7	8	9	11	12
68	4624	4638	4651	4665	4679	4692	4706	4720	4733	4747	1	3	4	5	7	8	10	11	12
69	4761	4775	4789	4802	4816	4830	4844	4858	4872	4886	1	3	4	6	7	8	10	11	13
70	4900	4914	4928	4942	4956	4970	4984	4998	5013	5027	1	3	4	6	7	8	10	11	13
71	5041	5055	5069	5084	5098	5112	5127	5141	5155	5170	1	3	4	6	7	9	10	11	13
72	5184	5198	5213	5227	5242	5256	5271	5285	5300	5314	1	3	4	6	7	9	10	11	13
73	5329	5344	5358	5373	5388	5402	5417	5432	5446	5461	1	3	4	6	7	9	10	12	13
74	5476	5491	5506	5520	5535	5550	5565	5580	5595	5610	1	3	4	6	7	9	10	12	13
75	5625	5640	5655	5670	5685	5700	5715	5730	5746	5761	2	3	5	6	8	9	11	12	14
76	5776	5791	5806	5822	5837	5852	5868	5883	5898	5914	2	3	5	6	8	9	11	12	14
77	5929	5944	5960	5975	5991	6006	6022	6037	6053	6068	2	3	5	6	8	9	11	12	14
78	6084	6100	6115	6131	6147	6162	6178	6194	6209	6225	2	3	5	6	8	9	11	13	14
79	6241	6257	6273	6288	6304	6320	6336	6352	6368	6384	2	3	5	6	8	10	11	13	14
80	6400	6416	6432	6448	6464	6480	6496	6512	6529	6545	2	3	5	6	8	10	11	13	14
81	6561	6577	6593	6610	6626	6642	6659	6675	6691	6708	2	3	5	7	8	10	11	13	15
82	6724	6740	6757	6773	6790	6806	6823	6839	6856	6872	2	3	5	7	8	10	12	13	15
83	6889	6906	6922	6939	6956	6972	6989	7006	7022	7039	2	3	5	7	8	10	12	13	15
84	7056	7073	7090	7106	7123	7140	7157	7174	7191	7208	2	3	5	7	8	10	12	14	15
85	7225	7242	7259	7276	7293	7310	7327	7344	7362	7379	2	3	5	7	9	10	12	14	15
86	7396	7413	7430	7448	7465	7482	7500	7517	7534	7552	2	3	5	7	9	10	12	14	16
87	7569	7586	7604	7621	7639	7656	7674	7691	7709	7726	2	4	5	7	9	11	12	14	16
88	7744	7762	7779	7797	7815	7832	7850	7868	7885	7903	2	4	5	7	9	11	12	14	16
89	7921	7939	7957	7974	7992	8010	8028	8046	8064	8082	2	4	5	7	9	11	13	14	16
90	8100	8118	8136	8154	8172	8190	8208	8226	8245	8263	2	4	5	7	9	11	13	14	16
91	8281	8299	8317	8336	8354	8372	8391	8409	8427	8446	2	4	5	7	9	11	13	15	16
92	8464	8482	8501	8519	8538	8556	8575	8593	8612	8630	2	4	6	7	9	11	13	15	17
93	8649	8668	8686	8705	8724	8742	8761	8780	8798	8817	2	4	6	7	9	11	13	15	17
94	8836	8855	8874	8892	8911	8930	8949	8968	8987	9006	2	4	6	8	9	11	13	15	17
95	9025	9044	9063	9082	9101	9120	9139	9158	9178	9197	2	4	6	8	10	11	13	15	17
96	9216	9235	9254	9274	9293	9312	9332	9351	9370	9390	2	4	6	8	10	12	14	15	17
97	9409	9428	9448	9467	9487	9506	9526	9545	9565	9584	2	4	6	8	10	12	14	16	18
98	9604	9624	9643	9663	9683	9702	9722	9742	9761	9781	2	4	6	8	10	12	14	16	18
99	9801	9821	9841	9860	9880	9900	9920	9940	9960	9980	2	4	6	8	10	12	14	16	18

SQUARE ROOTS. From 1 to 5·5

											Mean Differences						
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7
1·0	1·000	1·005	1·010	1·015	1·020	1·025	1·030	1·034	1·039	1·044	0	1	1	2	2	3	3
1·1	1·049	1·054	1·058	1·063	1·068	1·072	1·077	1·082	1·086	1·091	0	1	1	2	2	3	3
1·2	1·095	1·100	1·105	1·109	1·114	1·118	1·122	1·127	1·131	1·136	0	1	1	2	2	3	3
1·3	1·140	1·145	1·149	1·153	1·158	1·162	1·166	1·170	1·175	1·179	0	1	1	2	2	3	3
1·4	1·183	1·187	1·192	1·196	1·200	1·204	1·208	1·212	1·217	1·221	0	1	1	2	2	2	3
1·5	1·225	1·229	1·233	1·237	1·241	1·245	1·249	1·253	1·257	1·261	0	1	1	2	2	2	3
1·6	1·265	1·269	1·273	1·277	1·281	1·285	1·288	1·292	1·296	1·300	0	1	1	2	2	2	3
1·7	1·304	1·308	1·311	1·315	1·319	1·323	1·327	1·330	1·334	1·338	0	1	1	2	2	2	3
1·8	1·342	1·345	1·349	1·353	1·356	1·360	1·364	1·367	1·371	1·375	0	1	1	1	2	2	3
1·9	1·378	1·382	1·386	1·389	1·393	1·396	1·400	1·404	1·407	1·411	0	1	1	1	2	2	3
2·0	1·414	1·418	1·421	1·425	1·428	1·432	1·435	1·439	1·442	1·446	0	1	1	1	2	2	2
2·1	1·449	1·453	1·456	1·459	1·463	1·466	1·470	1·473	1·476	1·480	0	1	1	1	2	2	2
2·2	1·483	1·487	1·490	1·493	1·497	1·500	1·503	1·507	1·510	1·513	0	1	1	1	2	2	2
2·3	1·517	1·520	1·523	1·526	1·530	1·533	1·536	1·539	1·543	1·546	0	1	1	1	2	2	2
2·4	1·549	1·552	1·556	1·559	1·562	1·565	1·568	1·572	1·575	1·578	0	1	1	1	2	2	2
2·5	1·581	1·584	1·587	1·591	1·594	1·597	1·600	1·603	1·606	1·609	0	1	1	1	2	2	2
2·6	1·612	1·616	1·619	1·622	1·625	1·628	1·631	1·634	1·637	1·640	0	1	1	1	2	2	2
2·7	1·643	1·646	1·649	1·652	1·655	1·658	1·661	1·664	1·667	1·670	0	1	1	1	2	2	2
2·8	1·673	1·676	1·679	1·682	1·685	1·688	1·691	1·694	1·697	1·700	0	1	1	1	1	2	2
2·9	1·703	1·706	1·709	1·712	1·715	1·718	1·720	1·723	1·726	1·729	0	1	1	1	1	2	2
3·0	1·732	1·735	1·738	1·741	1·744	1·746	1·749	1·752	1·755	1·758	0	1	1	1	1	2	2
3·1	1·761	1·764	1·766	1·769	1·772	1·775	1·778	1·780	1·783	1·786	0	1	1	1	1	2	2
3·2	1·789	1·792	1·794	1·797	1·800	1·803	1·806	1·808	1·811	1·814	0	1	1	1	1	2	2
3·3	1·817	1·819	1·822	1·825	1·828	1·830	1·833	1·836	1·838	1·841	0	1	1	1	1	2	2
3·4	1·844	1·847	1·849	1·852	1·855	1·857	1·860	1·863	1·865	1·868	0	1	1	1	1	2	2
3·5	1·871	1·873	1·876	1·879	1·881	1·884	1·887	1·889	1·892	1·895	0	1	1	1	1	2	2
3·6	1·897	1·900	1·903	1·905	1·908	1·910	1·913	1·916	1·918	1·921	0	1	1	1	1	2	2
3·7	1·924	1·926	1·929	1·931	1·934	1·936	1·939	1·942	1·944	1·947	0	1	1	1	1	2	2
3·8	1·949	1·952	1·954	1·957	1·960	1·962	1·965	1·967	1·970	1·972	0	1	1	1	1	2	2
3·9	1·975	1·977	1·980	1·982	1·985	1·987	1·990	1·992	1·995	1·997	0	1	1	1	1	2	2
4·0	2·000	2·002	2·005	2·007	2·010	2·012	2·015	2·017	2·020	2·022	0	0	1	1	1	1	2
4·1	2·025	2·027	2·030	2·032	2·035	2·037	2·040	2·042	2·045	2·047	0	0	1	1	1	1	2
4·2	2·049	2·052	2·054	2·057	2·059	2·062	2·064	2·066	2·069	2·071	0	0	1	1	1	1	2
4·3	2·074	2·076	2·078	2·081	2·083	2·086	2·088	2·090	2·093	2·095	0	0	1	1	1	1	2
4·4	2·098	2·100	2·102	2·105	2·107	2·110	2·112	2·114	2·117	2·119	0	0	1	1	1	1	2
4·5	2·121	2·124	2·126	2·128	2·131	2·133	2·135	2·138	2·140	2·142	0	0	1	1	1	1	2
4·6	2·145	2·147	2·149	2·152	2·154	2·156	2·159	2·161	2·163	2·166	0	0	1	1	1	1	2
4·7	2·168	2·170	2·173	2·175	2·177	2·179	2·182	2·184	2·186	2·189	0	0	1	1	1	1	2
4·8	2·191	2·193	2·195	2·198	2·200	2·202	2·205	2·207	2·209	2·211	0	0	1	1	1	1	2
4·9	2·214	2·216	2·218	2·220	2·223	2·225	2·227	2·229	2·232	2·234	0	0	1	1	1	1	2
5·0	2·236	2·238	2·241	2·243	2·245	2·247	2·249	2·252	2·254	2·256	0	0	1	1	1	1	2
5·1	2·258	2·261	2·263	2·265	2·267	2·269	2·272	2·274	2·276	2·278	0	0	1	1	1	1	2
5·2	2·280	2·283	2·285	2·287	2·289	2·291	2·293	2·296	2·298	2·300	0	0	1	1	1	1	2
5·3	2·302	2·304	2·307	2·309	2·311	2·313	2·315	2·317	2·319	2·322	0	0	1	1	1	1	2
5·4	2·324	2·326	2·328	2·330	2·332	2·335	2·337	2·339	2·341	2·343	0	0	1	1	1	1	1

											Mean Differences.								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	3·162	3·178	3·194	3·209	3·225	3·240	3·256	3·271	3·286	3·302	2	3	5	6	8	9	11	12	14
11	3·317	3·332	3·347	3·362	3·376	3·391	3·406	3·421	3·435	3·450	1	3	4	6	7	9	10	12	13
12	3·464	3·479	3·493	3·507	3·521	3·536	3·550	3·564	3·578	3·592	1	3	4	6	7	8	10	11	13
13	3·606	3·619	3·633	3·647	3·661	3·674	3·688	3·701	3·715	3·728	1	3	4	5	7	8	10	11	12
14	3·742	3·755	3·768	3·782	3·795	3·808	3·821	3·834	3·847	3·860	1	3	4	5	7	8	9	11	12
15	3·873	3·886	3·899	3·912	3·924	3·937	3·950	3·962	3·975	3·987	1	3	4	5	6	8	9	10	11
16	4·000	4·012	4·025	4·037	4·050	4·062	4·074	4·087	4·099	4·111	1	2	4	5	6	7	9	10	11
17	4·123	4·135	4·147	4·159	4·171	4·183	4·195	4·207	4·219	4·231	1	2	4	5	6	7	8	10	11
18	4·243	4·254	4·266	4·278	4·290	4·301	4·313	4·324	4·336	4·347	1	2	3	5	6	7	8	9	10
19	4·359	4·370	4·382	4·393	4·405	4·416	4·427	4·438	4·450	4·461	1	2	3	5	6	7	8	9	10
20	4·472	4·483	4·494	4·506	4·517	4·528	4·539	4·550	4·561	4·572	1	2	3	4	6	7	8	9	10
21	4·583	4·593	4·604	4·615	4·626	4·637	4·648	4·658	4·669	4·680	1	2	3	4	5	6	8	9	10
22	4·690	4·701	4·712	4·722	4·733	4·743	4·754	4·764	4·775	4·785	1	2	3	4	5	6	7	8	9
23	4·796	4·806	4·817	4·827	4·837	4·848	4·858	4·868	4·879	4·889	1	2	3	4	5	6	7	8	9
24	4·899	4·909	4·919	4·930	4·940	4·950	4·960	4·970	4·980	4·990	1	2	3	4	5	6	7	8	9
25	5·000	5·010	5·020	5·030	5·040	5·050	5·060	5·070	5·079	5·089	1	2	3	4	5	6	7	8	9
26	5·099	5·109	5·119	5·128	5·138	5·148	5·158	5·167	5·177	5·187	1	2	3	4	5	6	7	8	9
27	5·196	5·206	5·215	5·225	5·235	5·244	5·254	5·263	5·273	5·282	1	2	3	4	5	6	7	8	9
28	5·292	5·301	5·310	5·320	5·329	5·339	5·348	5·357	5·367	5·376	1	2	3	4	5	6	7	7	8
29	5·385	5·394	5·404	5·413	5·422	5·431	5·441	5·450	5·459	5·468	1	2	3	4	5	5	6	7	8
30	5·477	5·486	5·495	5·505	5·514	5·523	5·532	5·541	5·550	5·559	1	2	3	4	4	5	6	7	8
31	5·568	5·577	5·586	5·595	5·604	5·612	5·621	5·630	5·639	5·648	1	2	3	3	4	5	6	7	8
32	5·657	5·666	5·675	5·683	5·692	5·701	5·710	5·718	5·727	5·736	1	2	3	3	4	5	6	7	8
33	5·745	5·753	5·762	5·771	5·779	5·788	5·797	5·805	5·814	5·822	1	2	3	3	4	5	6	7	8
34	5·831	5·840	5·848	5·857	5·865	5·874	5·882	5·891	5·899	5·908	1	2	3	3	4	5	6	7	8
35	5·916	5·025	5·933	5·941	5·950	5·958	5·967	5·975	5·983	5·992	1	2	2	3	4	5	6	7	8
36	6·000	6·008	6·017	6·025	6·033	6·042	6·050	6·058	6·066	6·075	1	2	2	3	4	5	6	7	7
37	6·083	6·091	6·099	6·107	6·116	6·124	6·132	6·140	6·148	6·156	1	2	2	3	4	5	6	7	7
38	6·164	6·173	6·181	6·189	6·197	6·205	6·213	6·221	6·229	6·237	1	2	2	3	4	5	6	6	7
39	6·245	6·253	6·261	6·269	6·277	6·285	6·293	6·301	6·309	6·317	1	2	2	3	4	5	6	6	7
40	6·325	6·332	6·340	6·348	6·356	6·364	6·372	6·380	6·387	6·395	1	2	2	3	4	5	6	6	7
41	6·403	6·411	6·419	6·427	6·434	6·442	6·450	6·458	6·465	6·473	1	2	2	3	4	5	5	6	7
42	6·481	6·488	6·496	6·504	6·512	6·519	6·527	6·535	6·542	6·550	1	2	2	3	4	5	5	6	7
43	6·557	6·565	6·573	6·580	6·588	6·595	6·603	6·611	6·618	6·626	1	2	2	3	4	5	5	6	7
44	6·633	6·641	6·648	6·656	6·663	6·671	6·678	6·686	6·693	6·701	1	2	2	3	4	5	5	6	7
45	6·708	6·716	6·723	6·731	6·738	6·745	6·753	6·760	6·768	6·775	1	1	2	3	4	4	5	6	7
46	6·782	6·790	6·797	6·804	6·812	6·819	6·826	6·834	6·841	6·848	1	1	2	3	4	4	5	6	7
47	6·856	6·863	6·870	6·877	6·885	6·892	6·899	6·907	6·914	6·921	1	1	2	3	4	4	5	6	7
48	6·928	6·935	6·943	6·950	6·957	6·964	6·971	6·979	6·986	6·993	1	1	2	3	4	4	5	6	6
49	7·000	7·007	7·014	7·021	7·029	7·036	7·043	7·050	7·057	7·064	1	1	2	3	4	4	5	6	6
50	7·071	7·078	7·085	7·092	7·099	7·106	7·113	7·120	7·127	7·134	1	1	2	3	4	4	5	6	6
51	7·141	7·148	7·155	7·162	7·169	7·176	7·183	7·190	7·197	7·204	1	1	2	3	4	4	5	6	6
52	7·211	7·218	7·225	7·232	7·239	7·246	7·253	7·259	7·266	7·273	1	1	2	3	3	4	5	6	6
53	7·280	7·287	7·294	7·301	7·308	7·314	7·321	7·328	7·335	7·342	1	1	2	3	3	4	5	5	6
54	7·348	7·355	7·362	7·369	7·376	7·382	7·389	7·396	7·403	7·409	1	1	2	3	3	4	5	5	6

											Mean Differences						
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7
5·5	2·345	2·347	2·349	2·352	2·354	2·356	2·358	2·360	2·362	2·364	0	0	1	1	1	1	1
5·6	2·366	2·369	2·371	2·373	2·375	2·377	2·379	2·381	2·383	2·385	0	0	1	1	1	1	1
5·7	2·387	2·390	2·392	2·394	2·396	2·398	2·400	2·402	2·404	2·406	0	0	1	1	1	1	1
5·8	2·408	2·410	2·412	2·415	2·417	2·419	2·421	2·423	2·425	2·427	0	0	1	1	1	1	1
5·9	2·429	2·431	2·433	2·435	2·437	2·439	2·441	2·443	2·445	2·447	0	0	1	1	1	1	1
6·0	2·449	2·452	2·454	2·456	2·458	2·460	2·462	2·464	2·466	2·468	0	0	1	1	1	1	1
6·1	2·470	2·472	2·474	2·476	2·478	2·480	2·482	2·484	2·486	2·488	0	0	1	1	1	1	1
6·2	2·4900	2·492	2·494	2·496	2·498	2·500	2·502	2·504	2·506	2·508	0	0	1	1	1	1	1
6·3	2·510	2·512	2·514	2·516	2·518	2·520	2·522	2·524	2·526	2·528	0	0	1	1	1	1	1
6·4	2·530	2·532	2·534	2·536	2·538	2·540	2·542	2·544	2·546	2·548	0	0	1	1	1	1	1
6·5	2·550	2·551	2·553	2·555	2·557	2·559	2·561	2·563	2·565	2·567	0	0	1	1	1	1	1
6·6	2·569	2·571	2·573	2·575	2·577	2·579	2·581	2·583	2·585	2·587	0	0	1	1	1	1	1
6·7	2·588	2·590	2·592	2·594	2·596	2·598	2·600	2·602	2·604	2·606	0	0	1	1	1	1	1
6·8	2·608	2·610	2·612	2·613	2·615	2·617	2·619	2·621	2·623	2·625	0	0	1	1	1	1	1
6·9	2·627	2·629	2·631	2·632	2·634	2·636	2·638	2·640	2·642	2·644	0	0	1	1	1	1	1
7·0	2·646	2·648	2·650	2·651	2·653	2·655	2·657	2·659	2·661	2·663	0	0	1	1	1	1	1
7·1	2·665	2·666	2·668	2·670	2·672	2·674	2·676	2·678	2·680	2·681	0	0	1	1	1	1	1
7·2	2·683	2·685	2·687	2·689	2·691	2·693	2·694	2·696	2·698	2·700	0	0	1	1	1	1	1
7·3	2·702	2·704	2·706	2·707	2·709	2·711	2·713	2·715	2·717	2·718	0	0	1	1	1	1	1
7·4	2·720	2·722	2·724	2·726	2·728	2·729	2·731	2·733	2·735	2·737	0	0	1	1	1	1	1
7·5	2·739	2·740	2·742	2·744	2·746	2·748	2·750	2·751	2·753	2·755	0	0	1	1	1	1	1
7·6	2·757	2·759	2·760	2·762	2·764	2·766	2·768	2·769	2·771	2·773	0	0	1	1	1	1	1
7·7	2·775	2·777	2·778	2·780	2·782	2·784	2·786	2·787	2·789	2·791	0	0	1	1	1	1	1
7·8	2·793	2·795	2·796	2·798	2·800	2·802	2·804	2·805	2·807	2·809	0	0	1	1	1	1	1
7·9	2·811	2·812	2·814	2·816	2·818	2·820	2·821	2·823	2·825	2·827	0	0	1	1	1	1	1
8·0	2·828	2·830	2·832	2·834	2·835	2·837	2·839	2·841	2·843	2·844	0	0	1	1	1	1	1
8·1	2·846	2·848	2·850	2·851	2·853	2·855	2·857	2·858	2·860	2·862	0	0	1	1	1	1	1
8·2	2·864	2·865	2·867	2·869	2·871	2·872	2·874	2·876	2·877	2·879	0	0	1	1	1	1	1
8·3	2·881	2·883	2·884	2·886	2·888	2·890	2·891	2·893	2·895	2·897	0	0	1	1	1	1	1
8·4	2·898	2·900	2·902	2·903	2·905	2·907	2·909	2·910	2·912	2·914	0	0	1	1	1	1	1
8·5	2·915	2·917	2·919	2·921	2·922	2·924	2·926	2·927	2·929	2·931	0	0	1	1	1	1	1
8·6	2·933	2·934	2·936	2·938	2·939	2·941	2·943	2·944	2·946	2·948	0	0	1	1	1	1	1
8·7	2·950	2·951	2·953	2·955	2·956	2·958	2·960	2·961	2·963	2·965	0	0	1	1	1	1	1
8·8	2·966	2·968	2·970	2·972	2·973	2·975	2·977	2·978	2·980	2·982	0	0	1	1	1	1	1
8·9	2·983	2·985	2·987	2·988	2·990	2·992	2·993	2·995	2·997	2·998	0	0	1	1	1	1	1
9·0	3·000	3·002	3·003	3·005	3·007	3·008	3·010	3·012	3·013	3·015	0	0	0	1	1	1	1
9·1	3·017	3·018	3·020	3·022	3·023	3·025	3·027	3·028	3·030	3·032	0	0	0	1	1	1	1
9·2	3·033	3·035	3·036	3·038	3·040	3·041	3·043	3·045	3·046	3·048	0	0	0	1	1	1	1
9·3	3·050	3·051	3·053	3·055	3·056	3·058	3·059	3·061	3·063	3·064	0	0	0	1	1	1	1
9·4	3·066	3·068	3·069	3·071	3·072	3·074	3·076	3·077	3·079	3·081	0	0	0	1	1	1	1
9·5	3·082	3·084	3·085	3·087	3·089	3·090	3·092	3·094	3·095	3·097	0	0	0	1	1	1	1
9·6	3·098	3·100	3·102	3·103	3·105	3·106	3·108	3·110	3·111	3·113	0	0	0	1	1	1	1
9·7	3·114	3·116	3·118	3·119	3·121	3·122	3·124	3·126	3·127	3·129	0	0	0	1	1	1	1
9·8	3·130	3·132	3·134	3·135	3·137	3·138	3·140	3·142	3·143	3·145	0	0	0	1	1	1	1
9·9	3·146	3·148	3·150	3·151	3·153	3·154	3·156	3·158	3·159	3·161	0	0	0	1	1	1	1

											Mean Differences.								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7·416	7·423	7·430	7·436	7·443	7·450	7·457	7·463	7·470	7·477	1	1	2	3	3	4	5	5	6
56	7·483	7·490	7·497	7·503	7·510	7·517	7·523	7·530	7·537	7·543	1	1	2	3	3	4	5	5	6
57	7·550	7·556	7·563	7·570	7·576	7·583	7·589	7·596	7·603	7·609	1	1	2	3	3	4	5	5	6
58	7·616	7·622	7·629	7·635	7·642	7·649	7·655	7·662	7·668	7·675	1	1	2	3	3	4	5	5	6
59	7·681	7·688	7·694	7·701	7·707	7·714	7·720	7·727	7·733	7·740	1	1	2	3	3	4	4	5	6
60	7·746	7·752	7·759	7·765	7·772	7·778	7·785	7·791	7·797	7·804	1	1	2	3	3	4	4	5	6
61	7·810	7·817	7·823	7·829	7·836	7·842	7·849	7·855	7·861	7·868	1	1	2	3	3	4	4	5	6
62	7·874	7·880	7·887	7·893	7·899	7·906	7·912	7·918	7·925	7·931	1	1	2	3	3	4	4	5	6
63	7·937	7·944	7·950	7·956	7·962	7·969	7·975	7·981	7·987	7·994	1	1	2	3	3	4	4	5	6
64	8·000	8·006	8·012	8·019	8·025	8·031	8·037	8·044	8·050	8·056	1	1	2	2	3	4	4	5	6
65	8·062	8·068	8·075	8·081	8·087	8·093	8·099	8·106	8·112	8·118	1	1	2	2	3	4	4	5	6
66	8·124	8·130	8·136	8·142	8·149	8·155	8·161	8·167	8·173	8·179	1	1	2	2	3	4	4	5	5
67	8·185	8·191	8·198	8·204	8·210	8·216	8·222	8·228	8·234	8·240	1	1	2	2	3	4	4	5	5
68	8·246	8·252	8·258	8·264	8·270	8·276	8·283	8·289	8·295	8·301	1	1	2	2	3	4	4	5	5
69	8·307	8·313	8·319	8·325	8·331	8·337	8·343	8·349	8·355	8·361	1	1	2	2	3	4	4	5	5
70	8·367	8·373	8·379	8·385	8·390	8·396	8·402	8·408	8·414	8·420	1	1	2	2	3	4	4	5	5
71	8·426	8·432	8·438	8·444	8·450	8·456	8·462	8·468	8·473	8·479	1	1	2	2	3	4	4	5	5
72	8·485	8·491	8·497	8·503	8·509	8·515	8·521	8·526	8·532	8·538	1	1	2	2	3	3	4	5	5
73	8·544	8·550	8·556	8·562	8·567	8·573	8·579	8·585	8·591	8·597	1	1	2	2	3	3	4	5	5
74	8·602	8·608	8·614	8·620	8·626	8·631	8·637	8·643	8·649	8·654	1	1	2	2	3	3	4	5	5
75	8·660	8·666	8·672	8·678	8·683	8·689	8·695	8·701	8·706	8·712	1	1	2	2	3	3	4	5	5
76	8·718	8·724	8·729	8·735	8·741	8·746	8·752	8·758	8·764	8·769	1	1	2	2	3	3	4	5	5
77	8·775	8·781	8·786	8·792	8·798	8·803	8·809	8·815	8·820	8·826	1	1	2	2	3	3	4	4	5
78	8·832	8·837	8·843	8·849	8·854	8·860	8·866	8·871	8·877	8·883	1	1	2	2	3	3	4	4	5
79	8·888	8·894	8·899	8·905	8·911	8·916	8·922	8·927	8·933	8·939	1	1	2	2	3	3	4	4	5
80	8·944	8·950	8·955	8·961	8·967	8·972	8·978	8·983	8·989	8·994	1	1	2	2	3	3	4	4	5
81	9·000	9·006	9·011	9·017	9·022	9·028	9·033	9·039	9·044	9·050	1	1	2	2	3	3	4	4	5
82	9·055	9·061	9·066	9·072	9·077	9·083	9·088	9·094	9·099	9·105	1	1	2	2	3	3	4	4	5
83	9·110	9·116	9·121	9·127	9·132	9·138	9·143	9·149	9·154	9·160	1	1	2	2	3	3	4	4	5
84	9·165	9·171	9·176	9·182	9·187	9·192	9·198	9·203	9·209	9·214	1	1	2	2	3	3	4	4	5
85	9·220	9·225	9·230	9·236	9·241	9·247	9·252	9·257	9·263	9·268	1	1	2	2	3	3	4	4	5
86	9·274	9·279	9·284	9·290	9·295	9·301	9·306	9·311	9·317	9·322	1	1	2	2	3	3	4	4	5
87	9·327	9·333	9·338	9·343	9·349	9·354	9·359	9·365	9·370	9·375	1	1	2	2	3	3	4	4	5
88	9·381	9·386	9·391	9·397	9·402	9·407	9·413	9·418	9·423	9·429	1	1	2	2	3	3	4	4	5
89	9·434	9·439	9·445	9·450	9·455	9·460	9·466	9·471	9·476	9·482	1	1	2	2	3	3	4	4	5
90	9·487	9·492	9·497	9·503	9·508	9·513	9·518	9·524	9·529	9·534	1	1	2	2	3	3	4	4	5
91	9·539	9·545	9·550	9·555	9·560	9·566	9·571	9·576	9·581	9·586	1	1	2	2	3	3	4	4	5
92	9·592	9·597	9·602	9·607	9·612	9·618	9·623	9·628	9·633	9·638	1	1	2	2	3	3	4	4	5
93	9·644	9·649	9·654	9·659	9·664	9·670	9·675	9·680	9·685	9·690	1	1	2	2	3	3	4	4	5
94	9·695	9·701	9·706	9·711	9·716	9·721	9·726	9·731	9·737	9·742	1	1	2	2	3	3	4	4	5
95	9·747	9·752	9·757	9·762	9·767	9·772	9·778	9·783	9·788	9·793	1	1	2	2	3	3	4	4	5
96	9·798	9·803	9·808	9·813	9·818	9·823	9·829	9·834	9·839	9·844	1	1	2	2	3	3	4	4	5
97	9·849	9·854	9·859	9·864	9·869	9·874	9·879	9·884	9·889	9·894	1	1	1	2	3	3	4	4	5
98	9·899	9·905	9·910	9·915	9·920	9·925	9·930	9·935	9·940	9·945	0	1	1	2	2	3	3	4	4
99	9·950	9·955	9·960	9·965	9·970	9·975	9·980	9·985	9·990	9·995	0	1	1	2	2	3	3	4	4

ANSWERS. PART I

Exercise I (a) (p. 3)

1. 1694. **2.** 1520. **3.** 2156. **4.** 2454. **5.** 10,316.
6. 16,565. **7.** 20,053. **8.** 19,756. **9.** 95,552. **10.** 212,103.
11. 81,386. **12.** 689,408. **13.** 1546. **14.** 2610. **15.** 1755.
16. 1913. **17.** 7684. **18.** 18,068. **19.** 18,805. **20.** 22,133.
21. 158,881. **22.** 193,062. **23.** 47,547. **24.** 678,959. **25.** 1,152,963.
26. 61,083. **27.** 121,726. **28.** 243,551. **29.** 503,801. **30.** 756,532.
31. 1,550,382. **32.** 1,022,245. **33.** 1,514,023. **34.** 1,874,901.
35. 3,153,284. **36.** 129,816. **37.** 68,604. **38.** 542,595.
39. 201,414. **40.** 744,264. **41.** 1,643,475. **42.** 833,229.
43. 2,406,131. **44.** 2,438,047. **45.** 1,793,953. **Total,** 10,801,528.
46. 10,264,917. **47.** 8,140,483. **48.** 459,085,496. **49.** 102,449,580.
50. 92,960,242. **51.** 79,634,798. **52.** 76,375,748. **53.** 65,183,253.
54. 60,887,275. **Total,** 477,490,896.
55. Men, 13,662,200. Women, 14,857,113. **Total,** 28,519,313.

Exercise I (b) (p. 5)

1. 32. **2.** 112. **3.** 733. **4.** 85. **5.** 588.
6. 2852. **7.** 10,409. **8.** 162,892. **9.** 194,389. **10.** 109.
11. 14,413. **12.** 86,924. **13.** 21. **14.** 56. **15.** 3525.
16. 83,681. **17.** 24,000. **18.** 25,205. **19.** 101,935. **20.** 13,576
21. 5929. **22.** 605. **23.** 6462. **24.** 13,051. **25.** 1372.
26. 40,566. **27.** 10,394. **28.** 66,795. **29.** 16,397. **30.** 58,554.
31. 1298. **32.** 11. **33.** 397. **34.** 565. **35.** 2188.
36. 8567. **37.** 55,092.
38. −72,876 ; −159,234 ; −197,812 ; +39,234 ; +37,922 ; +192,750.
39. 639,362; 85,381; 724,743. **40.** 215,069,286. **41.** Decrease £643,129,481.

Exercise I (c) (p. 7)

1. 8, 7. **2.** 4, −4, −1. **3.** 3180, 3800, 620 ft. **4.** 51°. **5.** 68°.
6. 24°, 13°. **7.** +2°, −14°. **8.** −200 ft. **9.** −24, 30 ft. **10.** 3, 6 sec.

Exercise I (d) (p. 8)

1. 3, 1, 8. **4.** 0, 8, 16, 24. **5.** 4, 8, 0. **6.** 2, −2, −5.

Exercise II (a) (p. 10)

1. 170, 1700, 170,000. **2.** 2030, 203,000, 4060. **3.** 14, 28, 56, 112

4. 6, 18, 54, 162. **5.** 38, 3800, 570, 760, 9500.

6. 39, 52, 65, 91, 104, 130, 156, 7800, 1430.

7. 515, 721, 9270, 1236, 1339, 1751. **8.** 1881; 14,630; 2299; 2508; 62,700.

9. 720, 288, 7200, 3600. **10.** 26, 52, 104, 208, 416.

11. 91, 9100, 9100, 182. **12.** 1000, 4000, 5000.

13. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 169, 196, 225, 256, 289, 324, 361, 400.

14. 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000. **15.** 30, 210, 220, 1080.

16. 2, 3, 6, 7, 14, 21 ; 3, 5, 9, 15 ; 7 ; 5, 19 ; 7, 13.

17. 3, 5, 7, 15, 21, 35 ; 11, 13 ; 2, 3, 6, 7, 9, 14, 18, 21, 42, 63 ; 5, 7, 25, 35 ; 3, 9, 37, 111. **18.** 400, 1000. **19.** 1300 ; 3500 ; 27,000.

20. 198, 1089, 6993, 3663. **21.** 20, 16, 12, 64. **22.** 6, 27, 9, 34, 32, 12, 10, 333.

23. 18, 2, 13, 5, −5, 25, 1, 234. **24.** 12, 4, 3, 34. **25.** 6138. **26.** 326,673.

27. 16. **28.** 205. **29.** 0. **30.** 0. **31.** 0.

Exercise II (b) (p. 12)

1. 544. **2.** 855. **3.** 621. **4.** 3692. **5.** 4706.

6. 6802. **7.** 9702. **8.** 13,176. **9.** 17,131. **10.** 31,217.

11. 103,532. **12.** 535,472. **13.** 84,728. **14.** 172,061.

15. 847,406. **16.** 518,364. **17.** 6,335,304. **18.** 4,681,410.

19. 2,161,334. **20.** 3,914,648. **21.** 5,375,643. **22.** 1,412,472.

23. 5,513,701. **24.** 20,020,404. **25.** 3,021,521. **26.** 9,039,030.

27. 56,492,540. **28.** 197,367,300. **29.** 7,232,940. **30.** 5,999,994.

Exercise II (c) (p. 12)

1. 37,275. **2.** 18,713,461. **3.** 1288.. **4.** £354,606.

5. 19,764 ; 276,696. **6.** £42,500. **7.** £254,904. **8.** 669,600,000 mi.

9. 218,240. **10.** 388,129. **11.** 32,768. **12.** 503,040.

13. £4912. **14.** 230,132. **15.** 6,832,996. **16.** 2416 ft.

17. 8672 yd. **18.** 782. **19.** 114 ft. **20.** 1272 ft.

21. 1776 yd. **22.** 10,184. **23.** 7293. **24.** $2ax+2bx$; $3ax+2ay$; $4a-8b$.

25. $ax+ay+bx+by$; $2ax+6bx+ay+3by$. **26.** 676, 1089, 3721, 5625.

27. 87, 336, 2499, 6396. **28.** 9604, 9216.

Exercise II (d) (p. 14)

1. 62. **2.** 2563. **3.** 403. **4.** 2364. **5.** 4829.

6. 324. **7.** 548. **8.** 214. **9.** 5324. **10.** 2139.

11. 2414. **12.** 1862. **13.** 962. **14.** 73, 4 ; 432, 11 ; 47, 23 ; 22, 193.

15. 3. **16.** 37. **17.** 173. **18.** 42. **19.** 3436.

20. 1612. **21.** 94,152. **22.** 362. **23.** 264, 209. **24.** 3423.

25. 262. **26.** 16. **27.** 9072. **28.** 9078. **29.** 58.

30. 174, 9. **31.** 703. **32.** 48. **33.** 92,880. **34.** 18, 9, 42, 33.
35. 110. **36.** 27. **37.** 24. **38.** 240. **39.** 240.
40. 17 ft. **41.** 531 ; 66, 3 ; 38. **42.** 1458, 208, 7.

Exercise II (e) (p. 16)

1. 1318, 20 ; 1171, 50 ; 494, 52. **2.** 464, 26 ; 345, 41 ; 852, 38.
3. 11, 84 ; 23, 243 ; 15, 339. **4.** 315, 0 ; 63, 0 ; 31, 125.
5. 198, 212 ; 118, 260 ; 90, 176. **6.** 2301, 319 ; 1634, 442 ; 229, 1696.
7. 8587, 46 ; 384, 1281 ; 276, 321. **8.** 17. **9.** 236 ; 41 lb.
10. 474 ; 389 yd. **11.** 671. **12.** 31. **13.** £26. **14.** 15 ; 6 in.
15. 4824. **16.** 24 min. **17.** 6944 ; 64. **18.** £200. **19.** £124, 16s.
20. 28 yr. **21.** 410 ; 98,400 ; 240 wk. **22.** 54. **23.** 65.
24. 203. **25.** 79. **26.** £2. **27.** 16. **28.** 241 in.

Exercise M (I) (p. 18)

1. 54,902. **2.** 161 mi. **3.** 17 hr. **4.** 12 yr. **5.** 2016.
6. 46. **7.** 298. **8.** 220 d. **9.** 56 in. **10.** 1351, 1141 cm.
11. 28, 22, 11 cm. **12.** 6 mi. **13.** £500. **14.** 11. **15.** 30.
16. 1s. 8d. **17.** 35 wk. **18.** 4s. **19.** 146,097. **20.** 3840.
21. −308 ft. **22.** 180, 144. **23.** 73,263. **24.** 27. **25.** 55.
26. 2420 lb. **27.** −900 ft. **28.** 79 ac. **29.** 540 yd. **30.** 3·36 p.m.
31. £8950; £11,448. **32.** 8929. **33.** 23 ft. **34.** 32 ; 400 sq. yd.
35. 826 yd. **36.** 8448. **37.** 4851. **38.** 866,475. **39.** 6 lb.
40. 1320 cu. ft., 158,400 lb. **41.** 65 ft. **42.** 1,200,898 increase.
43. D receives £5 from A and £25 from B. **44.** 10 lines.
45. 8 hr. 22 min. **46.** 30. **47.** 114 min. **48.** £660, £660.
49. 302 in. **50.** 20. **51.** 40, 80 yd. **52.** 30 steps.
53. (i) 11, 31, 27 ; 39, 23, 7 ; 19, 15, 35 : (ii) 16, 51, 44 ; 65, 37, 9 ; 30, 23, 58.

Exercise III (a) (p. 23)

1. 16, 27, 45, 62, 54, 96, 99, 29, 111, 124. **2.** 40, 70, 85, 63.
3. 1s. 2d., 1s. 6d., 2s. 2d., 8s. 4d., £1, 5s., £1, 12s., £7, 10s., £1, 0s. 10d., £1, 5s. 10d., £2, 1s. 8d.
4. 8, 20, 25. **5.** 44, 87, 35. **6.** 18s. 6d. **7.** 8s. 6d. **8.** £4, 11s. 8d.
9. 8$\frac{3}{4}$d., 4$\frac{1}{2}$d., 7$\frac{1}{4}$d., 2$\frac{1}{2}$d., 6$\frac{3}{4}$d. **10.** 3s. 10$\frac{1}{2}$d., 1s. 5d., 2s. 0$\frac{1}{2}$d., 4s. 0$\frac{1}{4}$d., 1s. 1$\frac{1}{2}$d.
11. 12s. 1d., 6s. 2$\frac{1}{2}$d., 9s. 7d., 2s. 9$\frac{3}{4}$d., 15s. 1$\frac{1}{4}$d. **12.** 50 ; £12, 10s.

Exercise III (b) (p. 23)

1. 282, 536, 814, 2909, 3453, 3766.
2. £6, 7s. 2d., £26, 0s. 8d., £30, 9s. 11d., £35, 8s. 1d., £98, 8s. 2d.
3. 3196, 4179, 6092, 7623, 13,433. **4.** 1228, 1065.

5. 45, 945, 21. 6. 42. 7. £4166, 13s. 4d. 8. 800.
9. £15, 7s. 10½d. 10. £56,495. 11. £21,250,000. 12. £5360, 14s
13. 2172. 14. £31, 5s. 15. £50,000. 16. 114 yr.
17. £131,400. 18. 60. 19. 11. 20. £6500.

Exercise III (c) (p. 27)

1. 4820, 29,828. 2. 5574, 1989.
3. 2374, 2492, 7540. 4. 2660, 65,597, 26,652.
5. 24 t. 10 cwt. 2 qr.; 1 t. 15 cwt. 3 qr. 16 lb. 8 oz.; 33 t. 18 cwt. 3 qr. 7 lb
6. 49 mi. 904 yd.; 46 mi. 683 yd.; 41 mi. 613 yd. 6 in.
7. 151, 120. 8. 203 gal. 2 qt. 1 pt.; 1046 gal. 3 qt. 1 pt.; 2705 gal. 1 qt. 1 pt
9. $240x$, $12y$, $240x+12y$; $\frac{x}{240}$, $\frac{y}{20}$, $\frac{x}{240}+\frac{y}{20}$; $2240x$, $112y$, $2240y+112x$.
10. 16 mi. 11. 41 yd. 2 ft. 12. 5 mi. 200 yd. 13. 44 t. 2 cwt. 6 lb
14. 96. 15. 960. 16. 26 t. 5 cwt. 17. 6.
18. 74,880. 19. 2 hr. 56 min. 20. 15 in. 21. 160. 22. 5596

Exercise III (d) (p. 28)

1. £23, 11s. 1d. 2. £16, 18s. 8d. 3. £43, 7s.
4. £86, 7s. 3¾d. 5. £156, 10s. 6¼d. 6. £223, 10s. 11d.
7. £1309, 0s. 5¾d. 8. £2251, 17s. 5d. 9. £1789, 11s. 2¼d.
10. £3082, 14s. 6d. 11. £1638, 11s. 8¼d. 12. £2433, 3s. 5d.
13. £4, 7s. 3d. 14. £2, 2s. 11d. 15. £2, 12s. 10d.
16. £1, 16s. 8¾d. 17. £12, 18s. 8½d. 18. £27, 18s. 8½d.
19. 10 yd. 1 ft. 8 in. 20. 10 t. 4 cwt. 3 qr. 21. 15 gal. 1 qt.
22. 10 t. 3 cwt. 1 qr. 8 lb. 23. 43 hr. 31 min. 57 sec. 24. 7 mi. 3 fur. 149 yd
25. 14 cwt. 3 qr. 26. 3 qt. 1 pt. 27. 6 fur. 209 yd.
28. 15 cwt. 23 lb. 29. 2 yd. 8 in. 30. 1342 yd. 1 ft.
31. £1, 4s. 9½d. 32. £95, 6s. 3d. 33. £41, 16s. 7d. incr.
34. £132, 0s. 5d. 35. 3 t. 19 cwt. 14 lb. 36. 1 hr. 57 min. 8 sec
37. 2 gal. 1 qt. 1 pt. 38. 17 ft. 3 in. 39. 557 yd. 1 ft.
40. 172 mi. 6 fur. 140 yd. 41. 12 hr. 50 min. 42. 21 t. 7 cwt. 1 qr.
43. 309 yd. 1 ft. 6 in. 44. 17 hr. 34 min. 14 sec. 45. £27, 14s. 6d.

Exercise III (e) (p. 31)

1. £883,254, 8s. 11d.; *Cr.* £274,193, 12s. 7d.
2. £1,154,700, 3s. 3d.; *Cr.* £118,847, 5s. 3d.
3. £834,650, 5s. 5d.; *Cr.* £219,401, 14s. 5d.
4. £226,753, 9s. 7d.; *Cr.* £46,603, 8s. 8d.
5. £57,349, 7s. 4d.; *Cr.* £9865, 13s. 7d.
6. £303,966, 0s. 7d.; *Cr.* £40,104, 19s. 6d.

Exercise III (f) (p. 33)

1. £70, 17s. 11d. ; £99, 5s. 1d. ; £127, 12s. 3d. ; £170, 3s.
2. £194, 16s. 8d. ; £292, 5s. ; £501.
3. £103, 14s. 9d. ; £933, 12s. 9d. ; £1728, 19s. 2d.
4. £308, 0s. 1¼d. ; £402, 15s. 6¼d. ; £544, 18s. 7¾d.
5. £2405, 2s. 5½d. ; £3670, 19s. 6½d. ; £3924, 2s. 11½d.
6. 23 yd. 1 ft. 4 in. ; 52 yd. 2 ft. 3 in. ; 70 yd. 1 ft.
7. 181 yd. 8 in. ; 271 yd. 2 ft. 6 in. ; 543 yd. 2 ft.
8. 2746 yd. 1 ft. 4 in. ; 3069 yd. 1 ft. 8 in. ; 5008 yd. 8 in.
9. 14 t. 1 cwt. 3 qr. 14 lb. ; 19 t. 14 cwt. 2 qr. 14 lb. ; 31 t. 14 lb.
10. 59 t. 14 cwt. 16 lb. ; 89 t. 11 cwt. 24 lb. ; 119 t. 8 cwt. 1 qr. 4 lb.
11. 186 t. 2 cwt. 2 qr. 16 lb. ; 243 t. 8 cwt. 8 lb. ; 272 t. 3 qr. 4 lb.
12. 39 d. 4 hr. ; 91 d. 9 hr. 20 min. ; 156 d. 16 hr.
13. 29 gal. ; 116 gal. ; 163 gal. 1 pt.   **14.** £40, 19s. 10d.   **15.** £153, 6s.
16. £2, 13s. 1d.   **17.** 5 cwt. 2 qr. 3 lb. 12 oz.   **18.** £3, 16s. 6d.
19. 12 lb. 1 oz.   **20.** £19, 16s. 8d.   **21.** £33, 2s. 6d.
22. 22 t. 9 cwt. 3 qr.   **23.** £17, 18s. 4d.   **24.** 232 yd. 1 ft.
25. 20 gal.   **26.** £5, 10s.   **27.** £8, 2s. 8d.
28. 120 mi.   **29.** £177, 13s. 4d.   **30.** 47 hr.

Exercise III (g) (p. 34)

1. £4, 8s. 9d. ; £2, 13s. 3d. ; £2, 4s. 4½d. ; 19s. 8d. ; rem. 1s. 6d.
2. £9, 9s. 4d. ; £2, 7s. 4d. ; £2, 3s. 3d., rem. 11d.
3. £15, 6s. 2d., rem. 7d. ; £5, 19s., rem. 2s. 11d. ; £5, 2s., rem. 2s. 11d.
4. £36, 2s., rem. 8d. ; £19, 15s. 11d., rem. 1s. 3d. ; £13, 1s. 1d., rem. 3s. 9d.
5. £6, 18s. 7d., rem. 3s. 3d. ; £3, 1s. 4d., rem. 1s. 9d. ; £1, 1s. 7d., rem. 4s. 2d.
6. 1 mi. 6 fur. 77 yd. ; 6 fur. 33 yd. ; 4 fur. 172 yd., rem. 3 yd. ; 3 fur. 129 yd., rem. 3 yd.
7. 5 fur. 30 yd., rem. 4 yd. ; 3 fur. 93 yd., rem. 16 yd. ; 2 fur. 205 yd., rem. 34 yd.
8. 478 yd., rem. 34 yd. ; 374 yd., rem. 24 yd.
9. 1 ft. 1 in., rem. 4 in. ; 9 in., rem. 1 ft. 6 in. ; 6 in., rem. 3 in.
10. 13 cwt. 1 qr. 17 lb. ; 6 cwt. 2 qr. 22 lb. 8 oz. ; 3 cwt. 3 qr. 8 lb., rem. 12 lb. ; 2 cwt. 2 qr. 20 lb., rem. 4 lb.
11. 10 cwt. 1 qr. 1 lb., rem. 20 lb. ; 5 cwt. 3 qr. 13 lb., rem. 2 lb. ; 5 cwt. 1 qr. 25 lb., rem. 11 lb.
12. 1 t. 1 cwt. 3 qr. 13 lb., rem. 7 lb. ; 16 cwt. 2 qr. 25 lb., rem. 3 lb. ; 14 cwt. 3 qr. 24 lb.
13. £2, 14s. ; £2, 8s. 9d. ; £31, 0s. 6d. ; 3 cwt. 2 qr. 19 lb.
14. 5s. 0½d.   **15.** 9s. 10d.   **16.** £1, 15s. 6d.   **17.** 40 lb.   **18.** 14 ft.
19. 16.   **20.** 149.   **21.** 3d.   **22.** 1320 ft.   **23.** 240 yd.
24. £56, 11s.   **25.** 28 ; 2s.   **26.** 42.   **27.** 5280.   **28.** 55 yd.
29. 1048.   **30.** 10 ; 11s.

Exercise III (h) (p. 37)

1. 85 ; 114. 2. 19 mi. 3. £12, 8s. 6d. 4. 209 yd
5. 12 st. 3 lb. 6. £39. 7. 39. 8. 63.
9. 17 mi. 10. 14 yr. 10 mo. 11. 20 m.p.h. 12. 8 m.p.
13. 2s. 11½d. 14. 286. 15. 165 cm. 16. 3700.
17. £127, 14s. 18. £1460. 19. 35, 1015. 20. 34s.
21. 2s. 4d. 22. $\frac{px}{x+1}$. 23. $\frac{px+qy}{x+y}$ lb. 24. Both 7

Exercise IV (a) (p. 39)

1. 1, 4, 3, 2, 8, 9, 10. 2. 1, 2, 3, 4, 8, 9. 3. 5, 4, 2½, 7
4. 880, 440, 220, 11. 5. 56, 1120, 280, 1680.
6. $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{240}$, $\frac{1}{10}$, $\frac{1}{6}$, $\frac{7}{36}$, $\frac{2}{15}$, $\frac{3}{5}$, $\frac{1}{4}$, $\frac{1}{60}$.
7. 8d., 15s., 7s. 6d., 12 oz., 8 cwt., 45 min., £2, 10s., £3, 15s., 1 hr. 20 min
1 ft. 3 in.

Exercise IV (b) (p. 39)

1. 14s. 2. £3. 3. £5, 5s. 4. £1, 1s. 5. 1 ft. 8 in.
6. 10s. 4d. 7. 1320 yd. 8. 2 yd. 9. 2 qt. 10. 1 qr. 14 lb. 3 oz
11. $\frac{9}{10}$. 12. $\frac{2}{3}$. 13. $\frac{7}{15}$.

Exercise IV (c) (p. 40)

1. $\frac{1}{2}$. 2. $\frac{1}{2}$. 3. $\frac{2}{5}$. 4. $\frac{6}{7}$. 5. $\frac{3}{4}$. 6. $\frac{4}{5}$. 7. $\frac{11}{12}$. 8. $\frac{13}{14}$
9. $\frac{4}{5}$. 10. $\frac{22}{35}$. 11. 4. 12. 2. 13. 25. 14. 45. 15. 169. 16. 4.
17. $\frac{10}{12}$, $\frac{9}{12}$, $\frac{8}{12}$, $\frac{6}{12}$; $\frac{22}{48}$, $\frac{21}{48}$, $\frac{20}{48}$, $\frac{18}{48}$; $\frac{51}{60}$, $\frac{28}{60}$, $\frac{26}{60}$, $\frac{25}{60}$. 18. $\frac{a}{b}$, $\frac{b}{2a}$, $\frac{a}{b}$, $\frac{a}{2b^2}$, $\frac{2a}{3x}$.
19. $\frac{18xy}{6x^2y^2}$, $\frac{3y}{6x^2y^2}$, $\frac{xy^3}{6x^2y^2}$, $\frac{xy^2}{6x^2y^2}$. 20. $\frac{9ac^2}{12a^2b^2c^2}$, $\frac{16a^3c}{12a^2b^2c^2}$, $\frac{2b^3}{12a^2b^2c^2}$.

Exercise IV (d) (p. 41)

1. $\frac{1}{20}$, $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$. 2. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$. 3. $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{12}$. 4. $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{12}$

Exercise IV (e) (p. 42)

1. £126, 10s. 2. £409, 4s. 3. £379, 10s. 4. £502, 14s.
5. £1690, 6s. 8d. 6. £382, 9s. 3d. 7. £1022, 5s. 8. £456, 16s. 10½d.
9. £356, 12s. 8½d. 10. £83, 14s. 11. £547, 11s. 1½d. 12. £181, 17s. 2¼d.
13. 517 t. 5 cwt. 2 qr. 24 lb. 14. 324 t. 16 cwt. 15. 394 t. 12 cwt. 16 lb.
16. £72, 16s. 17. £291, 3s. 6d. 18. £50, 14s. 4d. 19. £308, 0s. 9d.
20. £656, 8s. 1½d. 21. £45, 6s. 7d. 22. £132, 0s. 11d. 23. £189, 16s. 8d.
24. £61, 10s. 6d. 25. £81, 13s. 2d. 26. £501, 13s. 4d. 27. £76, 13s. 4d.

Exercise IV (f) (p. 43)

1. 19s. 8½d. 2. 10s. 6½d. 3. £2, 5s. 2½d. 4. £5, 0s. 9d.
5. £5, 15s. 6d. 6. £2, 6s. 9d. 7. £66, 7s. 4½d. 8. £67, 18s. 10d.

9. £201, 14s. 7d. **10.** £8, 13s. 4d. **11.** £61, 17s. $2\frac{1}{4}$d. **12.** £6, 11s. $3\frac{3}{4}$d.
13. £942, 5s. $7\frac{1}{2}$d. **14.** £6691, 16s. **15.** £13, 14s. 1d. **16.** £5, 1s. 1d.
17. £839, 18s. 9d. **18.** £138, 12s. $1\frac{1}{2}$d. **19.** 81 tons. **20.** £15, 16s. 3d.
21. £5140, 0s. $1\frac{1}{2}$d. **22.** 187 mi. 1650 yd. **23.** £121,339, 8s. 1d.
24. £47,909, 9s. **25.** £203, 15s. 2d.

Exercise IV (g) (p. 45)

1. £20, 15s. 4d. **2.** £109, 19s. **3.** £4, 4s. $1\frac{3}{4}$d. **4.** £4, 1s. 10d.
5. £1745, 13s. 6d. **6.** £63, 16s. **7.** £1, 15s. $6\frac{3}{4}$d. **8.** £15, 11s. $10\frac{3}{4}$d.
9. £51, 6s. $11\frac{3}{4}$d. **10.** £26, 1s. 1d. **11.** £2, 0s. $8\frac{3}{4}$d. **12** £2, 15s. 4d.
13. £3, 11s. 3d. **14.** £233, 9s. 2d.

Exercise M (II) (p. 46)

1. 25. **2.** 9, 189, 21. **3.** 316,800, 88. **4.** 5 mi. 866 yd. 2 ft.
5. 145 t. 5 cwt. **6.** £6, 5s. $4\frac{1}{2}$d. **7.** 2 mi. 6 fur. **8.** 2 mi. 396 yd.
9. £9205, 7s. **10.** £198, 18s. **11.** £24, 5s. **12.** 18.
13. £9, 1s. 3d. **14.** 982 yd. 8 in. **15.** 2 cwt. 25 lb. **16.** £10, 13s.
17. 181 in. **18.** 1 in 19, 1 in 28, A to B. **19.** £41,545, 17s. 6d.
20. Friday midnight. **21.** 168. **22.** 88. **23.** 32 days. **24.** 3 oz.
25. £36, 4s. 6d. **26.** £541,666, 13s. 4d. **27.** £2, 4s. 2d.; 1s. 8d.
28. 135. **29.** £200,000,000. **30.** 4s. 2d.
31. £4, 13s. 6d.; 2s. 3d. per hr. **32.** 15 mi. 800 yd. **33.** 132.
34. 73 yd. $4\frac{1}{2}$ in. **35.** 8s. 4d. **36.** £44, 8s. 8d. **37.** £148, 7s. 8d.
38. 80. **39.** 4 lb. per bag. **40.** 1792. **41.** 344, 317 yd.
42. 225 wk. 5 days.

Exercise V (a) (p. 50)

1. 15 sq. in. **2.** 9 sq. in. **3.** 100.
5. 30 sq. in., 22 in. **6.** 60 sq. yd., 32 yd. **7.** 8 sq. ft., 12 ft.
8. 8250 sq. yd., 370 yd. **9.** 15 sq. ft., 16 ft. **10.** 7 sq. yd., 16 yd.
11. 360. **12.** 15. **13.** 6 ft., 28 ft.
14. 6 sq. ft., 0. **15.** 72 tiles. **16.** 94 sq. ft.
17. 100, 36 sq. in. **18.** 10, 6, 20, 12 sq. cm; 42 cm. **19.** equal.
20. 5 in. square. **21.** 18 sq. cm. **22.** 10.
23. 16. **24.** 488 sq. yd. **25.** 242 yd.
26. 864 sq. yd. **27.** 96 sq. in. **28.** $6x^2$ sq. ft.; $10x$ ft.
29. 1 in.; $2x+2$ in. **30.** $4x^2$ sq. ft. **31.** 16 sq. in.
32. 13 sq. in. **33.** 8 sq. in. **34.** 44 sq. in.
35. 13 sq. in. **36.** 22 sq. in.

Exercise V (b) (p. 53)

1. 15, 4, 60; 60 cu. in. **2.** 96, 10, 960, 960 cu. ft.
3. 60 cu. in., 150 cu. in.; 6, 5, 5 in.

4. 1 dm., 1 dm., 1 cm.; 1 dm., 1 cm., 1 cm.; 100; 1 cm. cube, 1000; 100
5. 60 cu. in.; 40 cu. cm.; 4 cu. ft.; 36 cu. in.
6. 1440 cu. ft. 7. 2,400,000 cu. ft. 8. 8 ft. 9. 8 ft.
10. 3 ft. 11. 1000; 1000. 12. 120. 13. 180 lb.
14. 360 sq. ft. 15. 127 cu. in. 16. 720 cu. ft. 17. 96 cu. i
18. 84 cu. in. 19. 12 lb. 20. 316 cu. ft. 21. $6x^3$ cu. in.; $22x^2$ sq. i
22. $19x^3$ cu. in. 23. xy cu. ft.; xyz cu. ft. 24. $2y$ ft.

REVISION PAPERS 1–10 (p. 55)

R 1. 1. 6,234,506. 2. 349. 3. £3, 5s. 4. 4471. 5. 20, 19. 6. 97, 0, 106
R 2. 1. 10,780, 1393. 2. 7, 5, 7, 5; −2, −8, −2, −8. 3. 783,216, 78
4. 754. 5. £2, 5s. 4¾d. 6. £7, 13s. 10d.
R 3. 1. 140, 14, 140, 1400. 2. £29,470, 14s. 0½d. 3. 54,054. 4. 620
5. 22 t. 14 cwt. 2 qr. 6. 5 mi. 5 fur. 100 yd.
R 4. 1. 48,099; £80, 10s. 11½d. 2. 36°. 3. 6. 4. £4, 16

5. 529. 6. £324, 4s. 6d.
R 5. 1. 528,560. 2. 56 qr. 6 bush. 2 pk. 3. 1 t. 10 cwt. 98 lb. 4.
5. 15°. 6. 12 yr. 9 mo.
R 6. 1. 424, 334. 2. 2s. 6½d. 3. 749,749; $4x^2-9y^2$. 4. 22, 1
5. 19s. 2¾d. 6. 18; 18.
R 7. 1. 452, 20. 2. £1835, 3s. 4d. 3. 147; 16,821. 4. £4, 6s. 8d
5. 2082; 104; 83; 83. 6. 968.
R 8. 1. 126. 2. 52,350,000 sq. mi. 3. 23, 2800; 23, 280; 23, 2
4. 37; 10s. 3d. 5. £4, 7s. 6. 141 t. 8 cwt. 64 lb.
R 9. 1. 1088. 2. 44 yd. 2 ft. 9 in. 3. 7 mi. 180 yd. 4. 960, 120, 8, 96
5. 6 m.p.h. 6. £5 gain.
R 10. 1. £266. 2. 470,086; 245,975; 76,824; 792,885. 3. 9527. 4. 4 mi
5. 44; 2 lb. 6. £19, 12s.

PROBLEM PAPERS 1–10 (p. 59)

P 1. 1. 6. 2. 11 ft. 3. £1026, 17s. 6d. 4. 70 mi. 5. 10
P 2. 1. £6, 19s. 4d. 2. 1924. 3. £2, 16s. 7d. 4. 96, 85. 5. 14, 78 l
P 3. 1. 50,161. 2. £641. 3. £236,160. 4. 17. 5. 12 min.
P 4. 1. 25. 2. 413 mi., 19 hr. 10 min. 3. £2, 15s. 8d. 4. 2373, £10
5. 6900 yd.
P 5. 1. 722,938. 2. £3440. 3. 104 boys, 46 girls. 4. £33, 14s. 8d
5. 338.
P 6. 1. 3725 increase. 2. £62, 4s. 3d. 3. 224. 4. 395, 51. 5. £77
P 7. 1. 15. 2. 12. 3. 80 ft. 4. Wednesday. 5. 6 wk.
P 8. 1. £12, 5s. 4d. 2. 1s. 3. 2 t. 8 cwt. 24 lb. 4. 19. 5. £48, 17s. 6d
P 9. 1. 563. 2. 156 lb. 3. 55. 4. £300, 12s; £50, 2s
5. 4 cwt. 1 qr. 7 lb.; 1 cwt. 1 qr. 21 lb.
P 10. 1. 2, 5, −7, 1d.; 1d. 2. 234 t. 12 cwt. 96 lb. 3. 4s. 5. 17s. 1d

ANSWERS. PART II

Exercise VI (a) (p. 66)

2. 0·3, 0·02, 0·007, 0·23, 0·037, 13·7, 24·89, 0·201, 30·3, 5·061, 203·09, 300·3.
3. 2·37, 24·501, 130·37, 0·405, 0·078, 10·07.
4. 2·6, 11·2, 0·67, 1·05, 0·304, 0·0013.
5. $\frac{3}{10}$, $\frac{7}{100}$, $2\frac{1}{200}$, $\frac{17}{100}$, $10\frac{1}{25}$, $4\frac{637}{1000}$, $3\frac{9}{10}$, $3\frac{9}{100}$, $\frac{3}{100}$, $41\frac{8603}{10,000}$.
8. 2, 23, 23, 0·024, 1·24, 0·02, 0·0002, 1·04, 0·073, 0·09.
9. 40, 62, 53, 7, 113. **10.** 300, 420, 216, 1137, 8, 503, 70.
11. 0·624, 0·0624, 0·00624, 0·000624 ; 62·4, 624, 6240, 62,400.
12. 0·052, 0·0052, 0·00052 ; 5·2, 52, 520.
13. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{20}$, $\frac{3}{40}$, $\frac{3}{5}$, $\frac{6}{25}$, $\frac{3}{400}$, $\frac{1}{8}$, $\frac{3}{8}$. **14.** 5·1, 10, 370, 100. **15.** 100.
16. 100. **17.** 0·63, 0·007, 78·3, 19·24, 0·0013.
18. 0·2 by 0·1 ; 0·5 by 0·05 ; 0·025 by 0·0001.

Exercise VI (b) (p. 69)

1. 7. **2.** 8. **3.** 0·8. **4.** 0·9. **5.** 0·08.
6. 0·09. **7.** 0·0009. **8.** 0·001. **9.** 9000. **10.** 9000.
11. 20. **12.** 100. **13.** 3·14. **14.** 10·1. **15.** 86,800.
16. 1·74. **17.** 0·835. **18.** 25,000. **19.** 0·0679. **20.** 0·00550.
21. 18·637, 18·64. **22.** 0·048, 0·05. **23.** 0·046, 0·05. **24.** 0·005, 0·01.
25. 89,700, 90,000. **26.** 9·0, 8·97. **28.** 3·465, 3·475 in.

Exercise VI (c) (p. 70)

1. 0·0057, 0·000057. **2.** 3120.
3. 6·04, 604, 0·604, 0·00604. **4.** 8·712, 8712.
5. 572, 57·2, 0·572, 0·000572. **6.** 19·034, 1903·4, 0·019034.
7. 2·04003, 204·003, 2040·03. **8.** 0·305007, 305·007, 30500·7
21. 0·4. **22.** 0·03. **23.** 0·012. **24.** 1·37. **25.** 0·3.
26. 0·112. **27.** 0·0134. **28.** 64·37. **29.** 1·03. **30.** 20·14.
31. 2000·017. **32.** 0·327. **33.** 700·46. **34.** 230·05. **35.** 190·27.
36. 2303·7. **37.** 4. **38.** 8600. **39.** 4302·8. **40.** 0·812.
41. 570·02. **42.** 10·426. **43.** 6·085, 608,500.
44. 5·408, 540,800. **45.** 0·0816, 8160. **46.** 0·0432, 4320.
47. 0·000056, 5·6. **48.** 0·0000032, 0·32. **49.** 5 Km. 2 Hm. 6 Dm. 4 m.
50. 13 Km. 2 Dm. **51.** 1 Dm. 8 m. 3 cm. **52.** 8 Dm. 4 m. 3 cm.
53. 8 Hm. 7 Dm. 4 m. 2 cm. **54.** 1 Km. 3 Dm. 1 dm. 2 cm.
55. 7 Dm. 1 dm. 3 cm. 4 mm. **56.** 1 Dm. 5 cm. **57.** 2 Hm. 9 Dm. 7 dm. 1 cm.

58. $\frac{1}{10}$, 0·1. 59. $\frac{1}{100}$, 0·01. 60. $\frac{1}{1000}$, 0·001. 61. $\frac{2}{5}$, 0·4.
62. $\frac{3}{1000}$, 0·003. 63. $\frac{3}{125}$, 0·024. 64. $\frac{4}{25}$, 0·16. 65. $\frac{14}{25}$, 0·56.
66. $\frac{37}{100,000}$, 0·00037. 67. $\frac{1}{2}$, 0·5. 68. $\frac{1}{50}$, 0·02. 69. $\frac{1}{10,000}$, 0·000
70. $\frac{27}{10,000}$, 0·0027. 71. $\frac{2}{25}$, 0·08. 72. $\frac{3}{10}$, 0·3. 73. 3000, 3200, 4
74. 27,000, 2710, 190,000. 75. 300, 2800, 1004.

Exercise VI (d) (p. 72)

1. 6·104. 2. 4·302. 3. 371·66. 4. 413·6$\dot{3}$. 5. 16·92.
6. 6·022. 7. 11·889. 8. 3·888. 9. 271·876. 10. 3·548.
11. 811·37. 12. 63·27. 13. 1. 14. 4. 15. 0·1.
16. 0·1. 17. 4·1. 18. 15. 19. 1. 20. 0·64.
21. 1. 22. 10·1. 23. 0·1. 24. 2·101. 25. 55·15
26. 300·66. 27. 10·9071. 28. 16·9125. 29. 11·111. 30. 967·22
31. 0·45. 32. 3·9. 33. 0·064. 34. 9·13. 35. 83·6.
36. 1·2. 37. 0·2. 38. 0·251. 39. 900·1. 40. 1·11.
41. 7·007. 42. 3·079. 43. 0·99. 44. 12·1. 45. 1·9.
46. ·08, 0·004. 47. 1, 0·02. 48. Equal. 49. ·007, 0·0006
50. 1·3, 0·012. 51. 4·4, 0·03. 52. 0·1, 0·09. 53. 0·3, 0·21.
54. 0·003, 0·007. 55. 0·02, 0·08. 56. 14·61. 57. 0·449.
58. 13·52. 59. 740·86. 60. 0·0721. 61. 1·597.
62. 86·6 m. 63. 0·006 Dm. 64. 0·906 Km. 65. 0·271 m.
66. 0·901 Km. 67. 22·1 dm. 68. 999·992 m. 69. 19·63 m.
70. 0·00009 Km.

Exercise VI (e) (p. 74)

1. 2·22. 2. 5·16. 3. 6·2. 4. 0·96. 5. 0·144. 6. 8·89.
7. 0·0249. 8. 0·504. 9. 15. 10. 0·0324. 11. 31·13. 12. 203·7
13. 2·22, 0·222, 0·0222, 22·2, 0·00222. 14. 5·04, 0·0504, 504, 0·00504, 0·504
15. 0·1065, 10·65, 106·5, 0·001065, 1·065.
16. 0·00546, 54·6, 5·46, 0·0546, 0·546.
17. 0·00144, 1·44, 0·144, 0·000144, 0·0144.
18. 0·00162, 0·162, 0·0162, 0·000162, 0·0000162.
19. 24·99, 249·9, 0·2499, 0·002499. 20. 18·9, 0·189, 0·0189.
21. 14,800, 14·8, 1·48, 1480. 22. 2708·1, 27·081, 0·27081, 2·7081.

Exercise VI (f) (p. 75)

1. 0·07. 2. 7. 3. 0·00007. 4. 77. 5. 1·2.
6. 0·006. 7. 100. 8. 0·6. 9. 0·000004. 10. 0·2.
11. 5. 12. 1,000,000. 13. 1·0584. 14. 0·010584. 15. 0·00010584.
16. 3·2604. 17. 0·32604. 18. 0·00032604. 19. 0·1725.
20. 0·1725. 21. 0·01725. 22. 0·10011. 23. 184·0284.

24. 1·055706. 25. 364·77. 26. 0·015656. 27. 0·0002266.
28. 0·006889. 29. 0·025921. 30. 1·4014. 31. 330·077.
32. 0·003111. 33. 920·2. 34. 0·000286. 35. 0·000001728.
36. 5·04. 37. 81·6714. 38. 0·0384. 39. 0·0112211.
40. 4·2018. 41. 5·70614. 42. 0·00363.

Exercise VI (g) (p. 76)

1. 0·02, 0·2, 2, 0·002. 2. 0·04, 0·004, 4, 0·0004.
3. 0·17, 1·7, 0·017, 170. 4. 0·012, 1·2, 0·00012, 1·2.
5. 0·27, 27, 27, 0·00027. 6. 0·8, 800, 0·0008, 800.
7. 0·7. 8. 0·6. 9. 0·35. 10. 0·72. 11. 0·75.
12. 0·0075. 13. 0·375. 14. 0·04. 15. 0·00875. 16. 1·625.

Exercise VI (h) (p. 78)

1. 0·03. 2. 0·002. 3. 0·2. 4. 0·06. 5. 1800.
6. 160,000. 7. 0·0002. 8. 0·03. 9. 1200. 10. 3·4.
11. 0·0034. 12. 34,000. 13. 0·53. 14. 0·0053. 15. 53.
16. 77·8. 17. 7·78. 18. 0·00778. 19. 0·63. 20. 0·000063.
21. 6300. 22. 0·0125. 23. 12,500. 24. 0·000125. 25. 275.
26. ·00322. 27. 3·25. 28. 1·92. 29. 0·072. 30. 11,900.
31. 0·318. 32. 0·000318. 33. 137. 34. 13·9. 35. 0·598.
36. 0·0124. 37. 8·65. 38. 7·21. 39. 0·0758. 40. 16·1.
41. 0·00545. 42. 0·0531. 43. 0·00911. 44. 9·23. 45. 35·8.
46. 12s., 16s., 9s. 47. 9d., 7½d., 4½d. 48. 8 cwt.; 3 qr.; 6 oz.; 220 yd.
49. 0·8, 0·85, 0·6. 50. 73 cwt.; 13·02 ft.; 162·8s.; 63·36d.

Exercise VI (j) (p. 79)

1. 1 cm. = 0·394 in.; 1 in. = 2·54 cm. 6. 3·94 in.
7. 8 Km., 32 Km., 160 Km. 8. 6¼ mi., 31 mi., 62 mi.
9. 100 m. = 109·4 yd. 11. 1 sq. cm. = 0·155 sq. in.; 1 sq. in. = 6·45 sq. cm.
15. 250 (247). 16. 2·8, 2·27, 4·54 litres. 17. 2, ½, 1000.
18. 17·6, 0·35, 5·3. 19. 570. 20. 11. 21. 51, 6·35, 1016.
22. 1 Kg., 1000 Kg. 23. 50 c.c.; 2 cu. ft. 24. 5 lb. 25. 5.
26. 900 gr.; 9 lb.; 900 gr.; 56 lb. 27. 247. 28. 7·87.
29. 2000. 30. 13,600.

Exercise VI (k) (p. 81)

1. 2·675. 2. 1·8687(5). 3. 1·5760. 4. 1·0104. 5. 0·9156.
6. 0·8937(5). 7. 1·5635. 8. 1·9896. 9. 3·2479. 10. 3·3469.
11. 1·1312(5). 12. 1·6719. 13. 9d. 14. 2s. 6d. 15. £3, 4s. 4d.
16. £1, 18s. 7d. 17. £1, 0s. 7d. 18. £2, 0s. 2d. 19. £3, 5s. 20. 2d.

21. £1, 1s. 7d. **22.** £1, 16s. **23.** £1, 11s. 4d. **24.** £2, 1s. 7d. **25.** 2·787(5)
26. 1·725. **27.** 14·661. **28.** 12·889. **29.** 15·861. **30.** 1·352.
31. 16·7. **32.** 12·606. **33.** 3·625. **34.** 3·705. **35.** 16 yd. 1 ft. 3·55 in
36. 1 t. 3 qr. 24 lb. **37.** 3 t. 4 cwt. 3 qr. 24 lb. **38.** 6 ac. 1 r. 29·9 p.
39. 7 m. 4 dm. 3 cm. 6 mm. **40.** 15 hr. 14 min. **41.** 3 qr. 14 lb. 3 oz
42. 1s. 2d. **43.** 0·1 oz. **44.** 0·87. **45.** 0·22. **46.** 2·204.
47. 1·093. **48.** 0·4733. **49.** 0·7303(5). **50.** 0·5, 0·397.

Exercise VI (l) (p. 82)

1. 5·94. **2.** 13·07. **3.** 28·53. **4.** 11·7. **5.** 11·7
6. 5·606. **7.** 2·654. **8.** 241·4. **9.** 0·5417. **10.** 8·24
11. £7, 6s. 3d. **12.** £48, 2s. 5¼d.

Exercise VI (m) (p. 83)

1. 36, 9. **2.** 0·09, 0·3. **3.** 5·2. **4.** 70, 0·0000
5. 8·63. **6.** 1·42 in. **7.** 4·44 in. **8.** 51·1 yd.
9. 2·23. **10.** 27·87 cm. **11.** 13·762. **12.** 14·88 fr.
13. 74·85 gr., 74·85 c.c. **14.** 1·12 Km. **15.** 1·1 cm. **16.** 72.
17. 519·06 fr. **18.** 264. **19.** 1524·05 gr. **20.** 240,000.
21. 45. **22.** 2·95. **23.** 15·6. **24.** 10·4 cm.
25. 0·39, 2·5. **26.** 50·8. **27.** 82 min. **28.** £20, 9s 10
29. 28·9, 4·90 in. **30.** 535 lb. **31.** 1·30 sq. dm. **32.** 0·0946.
33. 0·12. **34.** 165,000,000; 21,000,000 mi. **35.** 15·17 lb.
36. −2 + ·61, −1 + ·46, −3 + ·16, −2 + ·93, −5 + ·872. **37.** 12; 2·6 in
38. 175·25 gr. **39.** 3. **40.** 19·05, 18·75 cm.
41. 0·9142 Km. **42.** 4·869 Kg.

Exercise VII (a) (p. 86)

1. 23, 29. **2.** 19, 29. **3.** $2 \times 14, 4 \times 7$; $2 \times 18, 3 \times 12, 4 \times 9, 6 \times 6$.
4. 24, 30, 36. **5.** 18, 48, 150. **6.** 2, 3. **7.** 2, 3, 7. **8.** Yes.
10. 2, 3, 5, 7, 11, 13.

Exercise VII (b) (p. 87)

1. $2^3.\,3^2$. **2.** $3^4.\,5^3$. **3.** $2^4.\,3^2.\,5$. **4.** 2^5. **5.** 3^5.
6. $2^7.\,3^2$. **7.** 2^2. **8.** 3^5. **9.** 2^5. **10.** 3.
11. $2^3.\,3^2$. **12.** $2^3.\,5^3$. **13.** $2^4.\,3^6$. **14.** $2^4.\,3^2$. **15.** 2^8.
16. 10^3. **17.** 10^5. **18.** a^5. **19.** 1. **20.** a^4.
21. a. **22.** 2^3. **23.** 2^4. **24.** 6^4. **25.** $3^2.\,5 \cdot 11$.
26. 12^2. **27.** $4 \cdot 3^6$. **28.** $2^2.\,3^2$. **29.** $2^3.\,3 \cdot 5$. **30.** $2^4.\,3^2$.
31. $2^2.\,7^2$. **32.** $2 \cdot 3^3$. **33.** $5 \cdot 13$. **34.** $2 \cdot 53$. **35.** $2^2.\,3^2.\,5$.
36. $3 \cdot 37$. **37.** $2^3.\,3^3$. **38.** $2^2.\,3^2.\,7$. **39.** $2^5.\,3^3$. **40.** $3^2.\,5 \cdot 7$.
41. 7^3. **42.** $7 \cdot 11 \cdot 13$. **43.** $2 \cdot 5 \cdot 11 \cdot 13$. **44.** $2^4.\,3^2.\,11$. **45.** $2^5.\,3 \cdot 7 \cdot 13$

46. $3 \cdot 7 \cdot 11 \cdot 13 \cdot 37$. 47. $2^2 \cdot 3^2 \cdot 11^2$. 48. 3^3. 49. 3^2.

50. $2^4 \cdot 3^4$; $2^6 \cdot 3^4$; $2^8 \cdot 3^4 \cdot 5^2$; 2^{10}. 51. $2^6 \cdot 3^6$; $2^9 \cdot 3^3$; 5^{12} ; $2^6 \cdot 3^3$.

52. $3^2 \cdot 5$; 2^8 ; $7^2 \cdot 11$; 3^5. 53. $2 \cdot 3$; 2^2 ; $5^2 \cdot 7 \cdot 11$; 2^9.

54. 34, 21, 132, 150. 55. 12, 30, 24, 72. 56. 15, 50.

57. 4^4 ; 27^9 ; $(x^2y^2)^4$. 58. a ; $2a^2b$. 59. x^9 ; x^8. 60. 1 ft.

Exercise VII (c) (p. 90)

1. $2^2 \cdot 3^2$; $2^2 \cdot 3 \cdot 5$. 2. 2 ; $2 \cdot 3$; $2 \cdot 3^2$. 5. 12. 6. 72.

7. 6. 8. 12. 9. $2 \cdot 3^2$. 10. $2^2 \cdot 3^2 \cdot 5$.

11. 15 ; $2 \cdot 3^3 \cdot 5^2 \cdot 7^2$. 12. 4 ; $2^3 \cdot 3$. 13. 7 ; $3 \cdot 5 \cdot 7$. 14. 18 ; $2^2 \cdot 3^2 \cdot 5$.

15. 14 ; $2 \cdot 3^2 \cdot 5^2 \cdot 7$. 16. 48 ; $2^5 \cdot 3^2$. 17. 45 ; $2^3 \cdot 3^2 \cdot 5$. 18. 7 ; $2^2 \cdot 5 \cdot 7 \cdot 13$.

19. 240 ; $2^4 \cdot 3^2 \cdot 5^2$. 20. 8 ; $2^7 \cdot 41$. 21. 45 ; $2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11$.

22. 11 ; $11 \cdot 17 \cdot 101$. 23. 65 ; $2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 13$. 24. 8 ; $2^4 \cdot 3 \cdot 5$.

25. 14 ; $2^3 \cdot 5 \cdot 7^2$. 26. 18 ; $2^2 \cdot 3^2 \cdot 5^2 \cdot 7$. 27. 25 ; $2^2 \cdot 3 \cdot 5^3$.

28. 48 ; $2^5 \cdot 3^2 \cdot 5$. 29. 2 ; $2^5 \cdot 7$. 30. 26 ; $2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 13$.

31. 23 ; $2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 23$. 32. $2a^5b^2c^3$; $2^4 \cdot 3 \cdot 17 \cdot a^6b^4c^5$.

33. $3a^2bc$; $18a^3b^3c^3$. 34. x^2y ; $10x^4y^3z^2$. 35. $3x$; $60x^3y^3z^2$.

36. $2^2 \cdot 3^2 \cdot 5$; $2^5 \cdot 3^4 \cdot 5^3 \cdot 7^2$. 37. $2^3 \cdot 5^3 \cdot 7 \cdot 11^2$; 77, 50. 38. $2^6 \cdot 3^2 \cdot 5$.

39. $2^8 \cdot 3^4 \cdot 5^2 \cdot 7$; $5^2 \cdot 7$; $2^4 \cdot 3^2$. 40. 11. 41. 3. 45. 311.

46. 113. 47. 17. 48. 38. 49. 999. 50. 135.

Exercise VII (d) (p. 91)

1. 9. 2. 144. 3. 22 ; 121. 4. 4. 5. 10s.

6. 70 ft. 7. 10 ft. 8. 9, 11. 9. 9 in. square. 10. £10, 10s.

11. 360,360. 12. 300 yd. 13. 6 ft. 14. One o'clock. 15. 5.

16. 18. 17. 250, 2420. 18. 80. 19. 8. 20. 1399.

21. $3 \cdot 7 \cdot 11 \cdot 13 \cdot 37 \cdot 101 \cdot 9901$. 22. 17, 19. 23. 209. 25. $2^2 \cdot 3^4 \cdot 5^2 \cdot 7$.

26. $2^5 \cdot 5^3$. 27. 2. 28. 50 ft. 5 in.

Exercise VII (e) (p. 93)

1. 37, 67. 2. 216 ; 732 ; 101,001. 3. 9316 ; 999,000 ; 79,068.

4. 8622 ; 73,701. 5. 5247 ; 80,102 ; 10,120 ; 81,818. 6. 7, 8, 2 or 6.

7. 6, 0 or 9, 6, 1. 8. Yes. 9. 27. 10. 93,192 ; 13,992.

11. 59,136. 12. (i) 7 ; (ii) 13 ; (iii) 7 × 13.

Exercise VIII (a) (p. 95)

1. $2\frac{3}{5}$, $1\frac{7}{10}$, $1\frac{1}{3}$, $3\frac{2}{3}$, $5\frac{1}{4}$, $6\frac{15}{16}$, $7\frac{13}{17}$. 2. $\frac{5}{3}$, $\frac{11}{4}$, $\frac{11}{10}$, $\frac{43}{6}$, $\frac{53}{6}$, $\frac{35}{3}$, $\frac{73}{5}$, $\frac{401}{20}$.

Exercise VIII (b) (p. 96)

1. $\frac{1}{3}$, $\frac{1}{2}$, $\frac{5}{6}$; $\frac{5}{6}$; 2, 3. 2. $\frac{1}{12}$, $\frac{2}{3}$, $\frac{3}{4}$; $\frac{3}{4}$; 8, 9. 3. $\frac{1}{5}$, $\frac{1}{4}$, $\frac{3}{10}$, $\frac{3}{4}$; $\frac{3}{4}$; 4, 5, 6, 15.

4. $1\frac{3}{4}$. 5. $2\frac{5}{12}$. 6. $\frac{3}{25}$. 7. $\frac{13}{1000}$. 8. $\frac{8}{25}$.

9. $1\frac{1}{4}$. 10. $4\frac{1}{9}$. 11. $5\frac{1}{12}$. 12. $9\frac{19}{30}$. 13. $7\frac{7}{48}$.

14. $7\frac{13}{36}$. 15. $9\frac{13}{72}$. 16. $\frac{3a}{4}$. 17. $\frac{5x}{24}$. 18. $\frac{5a}{4}$.

19. $\frac{13a}{6}$. 20. $\frac{2a+b}{2}$. 21. $\frac{4a+21b}{6}$. 22. $\frac{b+a}{ab}$. 23. $\frac{3b+2}{6ab}$

24. $11\frac{7}{8}$. 25. $8\frac{23}{24}$. 26. $7\frac{22}{105}$. 27. $6\frac{1}{27}$. 28. $7\frac{10}{63}$.

29. $4\frac{1}{6}$. 30. $6\frac{2}{15}$. 31. $14\frac{1}{10}$. 32. $8\frac{13}{20}$. 33. $13\frac{1}{8}$.

34. $7\frac{17}{72}$. 35. 5.

Exercise VIII (c) (p. 97)

1. $\frac{3}{40}$. 2. $\frac{5}{36}$. 3. $\frac{5}{12}$. 4. $\frac{5}{24}$. 5. $\frac{7}{11}$.

6. $2\frac{7}{15}$. 7. $\frac{5}{8}$. 8. $1\frac{13}{15}$. 9. $\frac{9}{100}$. 10. $\frac{1}{60}$.

11. $\frac{9}{1000}$. 12. $2\frac{1}{30}$. 13. $4\frac{7}{24}$. 14. $3\frac{7}{12}$. 15. $4\frac{3}{8}$.

16. $1\frac{1}{3}$. 17. $\frac{5}{12}$. 18. $\frac{2}{3}$. 19. $\frac{a}{6}$. 20. $\frac{5a}{6}$.

21. $\frac{b-a}{ab}$. 22. $\frac{2b-3a+1}{ab}$. 23. $\frac{59}{60}$. 24. $\frac{1}{6}$. 25. $\frac{7}{15}$.

26. $6\frac{7}{8}$ ft. 27. $20\frac{1}{3}$ yd. 28. $\frac{32}{75}$. 29. 192 sq. yd. 30. 30 mi

31. $\frac{1}{15}$, $\frac{7}{60}$. 32. $\frac{9}{80}$.

Exercise VIII (d) (p. 99)

4. $\frac{4}{11}$. 5. $2\frac{1}{7}$. 6. $3\frac{3}{5}$. 7. 6. 8. $\frac{1}{2}$. 9. $\frac{5}{9}$.

10. $\frac{7}{32}$. 11. $\frac{2}{5}$. 12. $\frac{5}{54}$. 13. $6\frac{1}{2}$. 14. $13\frac{1}{3}$. 15. 10

16. $\frac{1}{100}$. 17. $\frac{1}{1000}$. 18. $\frac{1}{100}$. 19. $4\frac{7}{12}$. 20. $4\frac{23}{24}$. 21. 16

22. a. 23. $9ab$. 24. $\frac{ac}{6b}$. 25. $\frac{3mq}{2np}$.

Exercise VIII (e) (p. 99)

1. $10\frac{1}{12}$ sq. ft. 2. 476 sq. ft. 3. $23\frac{29}{32}$ sq. ft. 4. £1.

5. $427\frac{2}{9}$ sq. ft. 6. $90\frac{2}{9}$ cu. in. 7. $24\frac{39}{64}$, $28\frac{11}{16}$, $47\frac{7}{64}$, $57\frac{3}{8}$, $196\frac{7}{8}$ sq. in

8. (i), (ii), (iii). 9. $\frac{1}{2}$. 10. $3\frac{5}{24}$. 11. $15\frac{5}{8}$. 12. $\frac{9}{10}$.

Exercise VIII (f) (p. 102)

1. 10. 2. 1000. 3. $1\frac{1}{3}$. 4. $3\frac{3}{4}$. 5. $\frac{2}{9}$.

6. $\frac{1}{100}$. 7. $\frac{1}{1000}$. 8. $3\frac{1}{3}$. 9. $9\frac{1}{3}$. 10. $1\frac{5}{6}$.

11. $\frac{21}{16}$. 12. $\frac{63}{40}$. 13. $22\frac{5}{9}$. 14. $\frac{2}{3}$. 15. $1\frac{5}{12}$.

16. $\frac{ab}{2}$. 17. $\frac{3xz}{2v}$. 18. $\frac{4n^3}{pm}$. 19. 17s. 20. £3, 14s. 7d.

21. £11, 15s. 22. £21, 3s. 6d. 23. 12s. 4d. 24. £4, 9s. 7d. 25. (i), (iii).

26. $\frac{35}{6}$. 27. $1\frac{7}{10}$. 28. $31\frac{1}{5}$. 29. $39\frac{13}{16}$. 30. the first; $\frac{2}{3}$

31. 80. 32. 792 yd. 33. $2\frac{1}{2}$ ft. 34. $1\frac{7}{9}$ ft. 35. $\frac{1}{4}$, $\frac{2}{7}$.

36. $\frac{2}{3}$. 37. $13\frac{1}{2}$. 38. $\frac{32}{27}$. 39. 3. 40. $\frac{6}{7}$.

41. (i) $\frac{15}{32}$, (ii) $\frac{39}{64}$, (iii) $\frac{1}{2}$, (iv) $\frac{31}{39}$, (v) $\frac{1}{4}$, (vi) $\frac{3}{5}$. 42. $\frac{28}{33}$. 43. $\frac{7}{16}$. 44.

Exercise M (III) (p. 103)

1. 0·01 ; 3. **2.** 0·69. **3.** 8·22 cm. **4.** $1\frac{2}{5}$. **5.** $\frac{1}{2}$.
6. 0·835. **7.** $\frac{1}{870}$. **8.** 61. **9.** 14 ; 5s. **10.** 18 days.
11. 0·966. **12.** 35 lb. **13.** $3\frac{5}{6}$. **14.** 26, $\frac{1}{2}$ in. **15.** 0·88 gr.
16. 589 fr. 95 c. **17.** 104. **18.** $\frac{5}{7}$, $-0·1$. **19.** 8·4 gal. **20.** 38 sq. cm.
21. 3114. **22.** 5s. 6d. **23.** 2·4. **24.** $\frac{7}{13}$, $\frac{27}{50}$, 0·55, $\frac{69}{125}$, $\frac{4}{7}$, $\frac{19}{33}$.
25. 13. **26.** 88·9 Kg. **27.** 210 gns. **28.** 10,800. **29.** 0·146.
30. 5. **31.** 12·35 oz. **32.** Aug. 4, 11, 18, 25. **33.** £6, 1s. 4d.
34. 4 oz. **35.** £1, 16s. **36.** 144. **37.** 22·4 cu. ft. **38.** 2·27 in.
39. 1·51 Kg. **40.** 81 lb. 12 oz. **41.** 29. **42.** 1530. **43.** $74\frac{1}{3}$ sq. ft.
44. 1, $\frac{4}{25}$, $\frac{9}{25}$. **45.** $2^3 . 3^2 . 7^3 . 11^2$; $7^2 . 11$; $2 . 3 . 11^2$; $2^3 . 3 . 7^2 . 11^2$; $2 . 3^2 . 7^3$.
46. 204. **47.** 48. **48.** Less. **49.** $\frac{5}{11}$. **50.** 66 ft.

REVISION PAPERS 11–20 (p. 106)

R 11. **1.** £2250, 9s. 3d. **2.** $2\frac{1}{2}$d. **3.** $1\frac{7}{8}$ mi. **4.** 155,848 sq. yd.
5. 0·503, 0·42, 0·01. **6.** $2\frac{1}{18}$, $1\frac{55}{63}$.
R 12. **1.** A 308, B 303, C 290, D 280, E 242. **2.** £8, 1s. $7\frac{1}{2}$d. **3.** 2 min. 41 sec.
4. £873,000. **5.** 5000. **6.** 2 ft.
R 13. **1.** 3 yd. 2 ft. 8 in. **2.** 3 sq. yd. 7 sq. ft. 72 sq. in.; 2 t. 9 cwt. 2 qr. 11·2 lb.
3. 240 sq. in. **4.** 160. **5.** 1·62, 324·375 ; 314, 1262·3. **6.** 10,800.
R 14. **1.** 19,184. **2.** £38, 6s. $9\frac{1}{2}$d. **3.** £779, 12s. 6d.
4. $\frac{3}{5}$, $\frac{1}{3}$, $\frac{7}{8}$, $\frac{1}{16}$, $\frac{5}{12}$, 0·6, 0·333, 0·875, 0·0625, 0·417.
5. 616 sq. yd. **6.** £104.
R 15. **1.** 5d. 18 hr. 53 min. 20 sec. **2.** $5(x+y)-12(b-a)$. **3.** 2s. $1\frac{1}{2}$d.
4. £16, 17s. 4d. **5.** $\frac{7}{20}$, $\frac{3}{200}$. **6.** 0·062, 0·0014, 0·68.
R 16. **1.** £1, 12s. **2.** £58, 15s. **3.** 21 ; rem. 149. **4.** £11, 11s. 8d.
5. £186. **6.** 1460 cu. in.
R 17. **1.** £55, 10s. $2\frac{3}{4}$d. **2.** 0·96072, 2030. **3.** 10s. **4.** 3 mi. 800 yd.
5. 162,000 tons. **6.** $432yz$.
R 18. **1.** 5,175,497. **2.** 672. **3.** (i) £1, 8s. 11d.; (ii) 0·8708.
4. 2 hr.; 20 m.p.h. **5.** 750. **6.** 3 in.
R 19. **1.** 26. **2.** 40 ft. **3.** 9s. 4d. **4.** $2^2 . 3^2 . 23$; 19,044.
5. 640 cu. ft.; 2 ft. **6.** 15 yr. 2 mo.
R 20. **1.** 44. **2.** 1 t. 8 cwt. 2 qr. 8 lb. **3.** £64, 2s. 6d. **4.** $a-b$.
5. 374 ; 12 sq. in. **6.** 356,400 cu. ft.

PROBLEM PAPERS 11–20 (p. 110)

P 11. **1.** $12\frac{6}{13}$ min. **2.** 57 doz. **3.** 1665. **4.** 578 gr. **5.** 72.
P 12. **1.** C pays B 16s. 4d., and A £2, 11s. 4d,
2. $2 . 3$; $2^2 . 3^2 . 5$; $2^5 . 3^5 . 5^4$. **3.** £18. **4.** 8073. **5.** 900, 540 cu. in.
P 13. **1.** 5291. **2.** $5\frac{1}{2}$, $b+\frac{1}{a}$. **3.** 225. **4.** 8 sq. ft. **5.** £3, 12s., £5, 12s., £7, 12s.
P 14. **1.** 132. **2.** £2,185,000. **3.** 1120, 1536. **4.** 9s. 8d. **5.** 520 sq. ft.

P 15. 1. 2718. 2. 72°. 3. 2^{18}. 4. £9, 9s. 9d. 5. 368.

P 16. 1. 73. 2. £17, 10s. 3. 166 ft. 3 in. and 167 ft. 10 in.
4. The second. 5. 1224, 1680.

P 17. 1. 45 sec. 2. £8, 9s. 9d. gain. 3. $8\frac{1}{6}$ ft. per sec. 4. £2, 14s.
5. 1440 gm.

P 18. 1. 2190 or 2196. 2. $9\frac{1}{3}$ gal. 3. 2s. $0\frac{3}{4}$d. 4. 12,600 sq. yd. 5. 726 tons.

P 19. 2. 95,000,000 mi. 3. 6. 4. 2 hrs. later ; 147 mi. 5. 16s. 8d.

P 20. 1. 416 ; 8 red, 176 blue, 344 green. 2. £11, 6s. 3. 18 sq. in.
4. $2\frac{1}{4}$ hr. 5. 5 in.

Exercise IX (a) (p. 115)

1. 8, 40 lb. 2. $\frac{1}{8}$, 9 gal. 3. 24, 72 mi. 4. $12\frac{1}{2}$ gal.
5. 20 min. 6. 1s. 8d. 7. 220 lb. 8. $22\frac{1}{2}$ mi., 4·8 Km.
9. 12s. 10. 90 ft. 11. 7s. 6d. ; 28. 12. 40 ; 250 yd.
13. 10 hr. 14. 4000. 15. £28 ; 4 tons. 16. 26 hr. 40 min.
17. £7, 10s. 18. 42. 19. 20. 20. $3\frac{3}{4}$ lb.
21. 15 sec. 22. £3, 16s. 23. £20, 8s. 24. £4.
25. £25, £75 ; £120 ; £37, 11s. 6d. 26. £600. 27. $4\frac{1}{6}$ yr.
28. £1, 5s.; £25. 29. 2s. 1d.; 25s.; £25. 30. £8, 6s. 8d. 31. £$\frac{bx}{a}$.
32. $\frac{xz}{y}$ lb., $\frac{20xz}{y}$ lb. 33. $\frac{qa}{p}$ pence, $\frac{qa}{12p}$ shillings. 34. $\frac{mx}{t}$ shillings.
35. $\frac{180xh}{m}$ ft. 36. £$\frac{xb}{80a}$. 37. £$\frac{100y-100x}{x}$. 38. £$\frac{100a}{x-a}$

Exercise IX (b) (p. 117)

1. $4\frac{1}{2}$ hr. 2. 18 men. 3. 3 days. 5. 55 oz. 6. 6 hr.
7. 18 days. 9. 14 sec. 10. $2\frac{1}{2}$ hr. 11. 140. 13. £45.
15. £8. 16. 9 hr. 36 min. 17. 200 tons. 18. 7·2 in. 19. 2·5 in.
20. $62\frac{1}{2}$ mi. 21. 1440. 22. $\frac{ax}{b}$ hr. 23. $\frac{ax}{b}$ hr. 24. $\frac{xy}{z}$ yd.
25. $\frac{xp}{q}$ hr.

Exercise X (a) (p. 120)

1. 2 : 3, $\frac{2}{3}$. 2. 4 : 5 ; 0·8. 3. 3 : 2. 4. 4 : 5. 5. 3 : 4. 6. 6 : 5.
7. 4 : 3. 8. 2 : 3. 9. 3 : 2. 10. 5 : 1. 11. 1 : 4. 12. 1 : 2.
13. 2 : 1. 14. 3 : 2. 15. 4 : 3. 16. 1 : 2. 17. 3 : 2. 18. 8 : 3.
19. 1 : 4. 20. 2 : 3. 21. 1 : 3. 22. 1 : 4. 23. 3 : 4. 24. 1 : 3.
25. 3 : 25. 26. 1 : 2. 27. 3 : 5. 28. 1 : 4. 29. 1 : 6. 30. 4 : 1.
31. 1 : 6. 32. 1 : 6. 33. 2 : 3. 34. 11 : 52. 35. 1 : 2. 36. 4 : 3.
37. 1 : 3. 38. 5 : 7. 39. 7. 40. 10. 41. 4. 42. 1·5.
43. 1·4. 44. 21. 45. x^3. 46. x^2y. 47. $\frac{5}{4}$, $\frac{4}{5}$. 48. [illegible], $\frac{5}{4}$.

49. $\frac{4}{3}, \frac{5}{4}$. 50. 2 : 3. 51. 4 : 3. 52. 5 : 6. 53. £350.
54. 126 mi. 55. 1 : 1. 56. £180. 57. $12\frac{1}{4}$ in. 58. £600.
59. £8 ; £4, 10s. 60. 156. 61. 30. 62. £38. 63. 1·06.
64. 4 : 1, 0·25. 65. 64 : 27. 66. 3 : 22. 67. $\frac{ax}{b}$. 68. $\frac{abq}{p}$ sq. in
69. £$\frac{xb^2}{a^2}$. 70. 88 : 45. 71. 0·1986. 72. 405. 73. $\frac{3}{8}$.
74. 10° C., 149° F. 75. 9 : 5. 76. 8 : 4 : 7. 77. 3 : 2 : 1 ; 8 : 18 : 16 : 9.
78. No. 79. $\frac{32}{39}, \frac{32}{39}$. 80. $\frac{4}{3}, \frac{3}{4}$. 81. 0·475. 82. 0·628.
83. 0·675. 84. (*a*) 0·478, (*b*) 0·563, (*c*) 0·329, (*d*) 0·205, (*e*) 0·704.

Exercise X (b) (p. 123)

1. 1·5 : 1. 2. 0·87 : 1. 3. 2·5 : 1. 4. 4·4 : 1. 5. 12 : 1.
6. 0·04 : 1. 7. 3 : 2. 8. 6 : 5. 9. 11 : 9. 10. 95 : 79.
11. 2 : 3. 12. $5^2 : 3^2$. 13. 10 : 7. 14. 2 : 3. 15. 4 : 3.
16. $\frac{3}{2}$. 17. 4 : 3. 18. $\frac{5}{3}$. 19. $\frac{3}{5}$. 20. Greater ; 47 : 31.
21. Decreased. 22. 4 : 3 ; $\frac{1}{4}$. 23. $\frac{1}{4}$. 24. $\frac{19}{20}, \frac{6}{5}, 4, \frac{22}{15}, \frac{7}{3}$.
25. $\frac{1}{380,160}, \frac{1}{100,000}, \frac{1}{64,450}$. 26. $\frac{1}{100,000,000}$.
27. 3360 yd., 2·15 Km. 28. $\frac{nx}{63,360}$ mi., $\frac{nx}{100,000}$ Km.
29. 12, 8s. 30. 19·2, 4·8 in. 31. 2 : 5 ; 5 : 3. 32. £1, 10s., £3, 10s.
33. 210 ft. 34. £32, £60. 35. 8, 12, 16 in. 36. £420.
37. 15, 20, 25, 40. 38. 8 : 10 : 27. 39. 1 in 11·5.
40. £780, £1118, £702. 41. 1 in 12·5. 42. 190·2, 168·2.
43. £$\frac{ax}{x+y}$, £$\frac{ay}{x+y}$. 44. $x : (x-y)$. 45. £60, £40, £24.
46. $\frac{14}{9}$, 2s. $7\frac{1}{2}$d. 47. $\frac{35}{36}$, $5\frac{5}{6}$ ft. 48. $\frac{15}{32}$, £15.

Exercise X (c) (p. 126)

1. $\frac{7x}{5}$. 2. 132. 3. 243. 4. 6 wk.
5. £560. 6. 4900 sq. yd. 7. 0·5. 8. 3 cwt. 24 lb.
9. £140. 10. £204, 8s. 4d. 11. 623·2 c.c. 12. $\frac{1}{2}$.
13. £44. 14. 576 sq. in. 15. £12. 16. 100.
17. 1250. 18. $26\frac{2}{3}$ t.

Exercise X (d) (p. 128)

1. £2, 10s. 2. 5s. 3. 45 min. 4. 38·4 min. 5. £1, 10s. 3d.
6. 15 days. 7. 36 days. 8. £33. 9. 45 lb. 10. £3, 18s. 9d.
11. 3 days. 12. 15 ac. 13. £25, 10s. 14. 50·7 oz. 15. £40, 10s.
16. 6000. 17. 28 tons. 18. 24 tons. 19. 4 in. 20. 45 gal.

Exercise X (e) (p. 130)

1. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{7}{20}$, $\frac{3}{2}$, $\frac{1}{3}$, $\frac{1}{40}$; $\frac{x}{100}$. 2. 80, 5, 10, 66$\frac{2}{3}$, 75, 250, $\frac{100a}{b}$ per cent.
3. £15, 2 ft., 88 yd., 3 pt., 315 boys. 4. 25, 17$\frac{1}{2}$, 2$\frac{1}{2}$, 5 per cent.
5. 64. 6. £250. 7. 15 per cent. 8. 45 per cent.; 24.
9. £4, 13s. 6d. 10. £7, 10s. 11. £2950. 12. 2$\frac{1}{2}$ per cent.
13. x s. 14. £16. 15. $\frac{5}{6}$. 16. 273 cu. ft.
17. 20 gal. 18. 54 per cent. 19. 300. 20. £262, 10s.
21. $\frac{1000}{x}$. 22. $(100-y)$ per cent. 23. 25 per cent. 24. £$\frac{PrT}{100}$.
25. 44 per cent. 26. 100 : 95; £20. 27. £12, 10s. 28. £9.
29. £12, 10s. 30. £2, 5s.

Exercise XI (a) (p. 133)

1. 100, $\frac{1}{10}$ in., $\frac{1}{100}$ sq. in. 3. 360. 5. 0·0385, 0·06, 0·01 sq. m.
6. 170, 0·09, 500·04 sq. cm. 7. 15,000,000, 1·5, 300 sq. m.
8. 53·3 sq. in., 29·4 in. 9. 4 sq. ft., 8 ft. 4 in. 10. 400 sq. yd., 260 ft.
11. 17$\frac{1}{2}$ sq. ft., 17 ft. 2 in. 12. 60 sq. ft., 53 ft. 13. 1 sq. ft.; 12 ft. 4 in.
14. 6000 sq. m., 460 m. 15. 1·4 sq. m., 8·7 m. 16. 120. 17. 40.
18. 400 sq. ft.; yes; 20 ft. square; 30 rows, 40 in each row. 19. 24.
20. The first; 150 sq. in. 21. 2$\frac{1}{4}$ sq. ft. 22. 5376.
23. 1080. 24. 35; 50 sq. in. 25. 52 sq. ft. 26. 1936, 7744 sq. yd.
27. 640. 28. 45. 29. 488 sq. yd. 30. 3 yd.
31. 40 ac. 32. 18 ft. 33. 480 yd. 34. 4$\frac{1}{2}$ ft.
35. 127 ft. 6 in. 36. 1 ft. 37. 6 ft. 38. 82$\frac{1}{2}$ yd.
39. 150 ac. 40. 2520 sq. mi. 41. 2 in. the mile. 42. £18, 15s.
43. 18s. 44. 40. 45. 215. 46. 54.
47. $3xy$ sq. ft., $\frac{1}{3}xy$ sq. yd. 48. 9 ft. 49. $\frac{1}{2}y(x-2y)$ sq. ft.
50. $\frac{xy}{z}$ ft.

Exercise XI (b) (p. 136)

1. 26$\frac{3}{4}$. 2. 30. 3. 27$\frac{5}{8}$. 4. 110·25. 5. 51·8.
6. 53. 7. 58. 8. 390. 9. 1120 sq. ft., 296 ft. 10. 265 sq. ft.
11. 67·4 sq. ft. 12. 432, 180, 54 sq. ft. 13. 99$\frac{2}{3}$ sq. ft.
14. 1050 sq. in., 21$\frac{1}{4}$ sq. ft. 15. 758$\frac{2}{3}$ sq. ft. 16. 2s. 5$\frac{1}{2}$d.
17. 1800 sq. ft. 18. 63; 153 sq. in. 19. 25s.
20. 1776, 6440 sq. yd. 21. 136 sq. cm. 22. 8 rows; 280.
23. 1125, 1125 sq. ft. 24. 384 sq. in. 25. 125 sq. in.
26. $(x+2z)(y+2z)-xy$ sq. ft. 27. $ab-\left(a-\frac{2x}{3}\right)\left(b-\frac{2x}{3}\right)$ sq. yd.
28. $24z(x+y)+xy$ sq. in. 29. $\frac{5x^2}{2}$ sq. ft. 30. x^2-y^2 sq. in.

Exercise XI (c) (p. 139)

1. 37·0, 34·1, 175, 247 sq. cm. **2.** 4·4 cm., 3·5 cm., 2·8 in., 10·9 in.
3. 880 sq. in. **4.** 215 sq. cm. **5.** 370 sq. m. **6.** 180 sq. cm.
7. 40 sq. m. **8.** 168 m. **9.** 208. **10.** 4·9 m.

Exercise XII (a) (p. 140)

1. 120, $54\frac{1}{2}$. **2.** 0·018, 0·006. **3.** 5000, 0·7, 4000·5.
4. 1 Kg.; 1 Kg.; 1000 Kg. **5.** 12 cu. ft.; 72 cu. cm.; 40 cu. dm.
6. 1875 cu. ft. **7.** 12 cu. cm. **8.** 6 ft., $2\frac{1}{2}$ ft. **9.** 8 in.
10. 5. **11.** 1000. **12.** 81 lb. **13.** 4·2 lb.
14. 8 in. **15.** 2 in. rise; 4 in. fall. **16.** 2 ft. 3 in.
17. $2\frac{10}{27}$ ft. **18.** 180,000 gal. **19.** 180 lb. **20.** 728 cu. in.
21. 80 sq. yd. **22.** 41,250 gal. **23.** 12 loads. **24.** £40, 10s.
25. 5 cm. **26.** (i) 900 cu. in.; $31\frac{1}{4}$ cwt. (ii) 648 cu. in.; $22\frac{1}{2}$ cwt.
27. 57 lb. **28.** $11\frac{1}{9}$ sq. in. **29.** 27 cu. ft. **30.** 1010 cu. ft.

Exercise XII (b) (p. 143)

1. 9 in. by 7 in. by $6\frac{1}{2}$ in.; $409\frac{1}{2}$ cu. in.; $550\frac{1}{2}$ cu. in.
2. 19 by 11 by 9 cm.; 1881 cu. cm.; 519 cu. cm. **3.** 2632 cu. in.
4. 3060 cu. in. **5.** 752 cu. cm. **6.** 6160 sq. ft. **7.** 2529 cu. cm.
8. £4, 19s. 7d. **9.** 1 in. **10.** 15 in. **11.** 4 in. per hr. **12.** Equal.

Exercise XII (c) (p. 144)

1. 116·5 cu. cm.; 1910 cu. in.; 102 cu. m. **2.** 1·77 cm.; 1·33 cm.; 0·886 cm.
3. 124,000 cu. m. **4.** 1·44 dm. **5.** 557 Kg. **6.** 145,000 l. **7.** 980 sq. m.
8. 9(9·2). **9.** 930 cu. cm. **10.** 1900 cu. cm. **11.** 4·6 Kg.

Exercise M (IV) (p. 145)

1. Wed., 10 p.m. **2.** 11·5. **3.** $12\frac{1}{2}$ per cent. **4.** The first.
5. 30. **6.** $\frac{1056x}{y}$. **7.** $\frac{5}{8}$. **8.** 11·7 sq. in.
9. 30·2. **10.** 11 min. 44 sec. **11.** £3, 10s.; $5\frac{5}{16}$ per cent.
12. 59. **13.** 6 ft. **14.** £2, 8s. **15.** £1, 8s.
16. $\frac{7}{26}=·27$. **17.** 1 : 3·67. **18.** £9, 19s. **19.** 45 ac.
20. £42, 0s. 10d.; £63, 1s. 3d.; £105, 2s. 1d. **21.** 43.
22. 9·24 p.m. **23.** A wins by 9 in. **24.** $1\frac{2}{17}$. **25.** £143, 18s. 2d.
26. 19s. $10\frac{1}{2}$d. **27.** 3 hr. $33\frac{1}{3}$ min. **28.** 4·8, 11·2, 10·4 cm.
29. £70, 19s. 9d. **30.** 70 m.p.h., $3\frac{1}{5}$ hr. **31.** 6352, 13,101.
32. 3, $3\frac{3}{4}$, $5\frac{1}{4}$ in. **33.** 2200 ac. **34.** £1, 19s.
35. $\frac{1}{4}$ A. **36.** $\frac{47}{21}$. **37.** 13·25 gal.; £1, 4s. 4d.
38. 4100. **39.** 37,500 Kg. **40.** 56 per cent. **41.** 10, 20.

42. £269, 8s. 43. £2, 4s. 6d ; £1, 4s. 6d. 44. 6 : 9 : 10.
45. 63·1 mi. 46. $\frac{1}{2\cdot 3025}$. 47. 62 min.; 5 mi. 48. $\frac{11}{60}$; $5\frac{6}{11}$ hr.
49. 5164, 3161. 50. A ; 100.

REVISION PAPERS 21–30 (p. 148)

R 21. **1.** $\frac{5}{6}$, $\frac{2}{3}$, $1\frac{1}{5}$. **2.** $1\frac{1}{2}$ ft. **3.** 8 mi. 1340 yd. **4.** 4 Kg. 4 Dg.
5. £6, 4s. 8d. **6.** 4 ft. 3 in.

R 22. **1.** 44 ; 2s. **2.** $\frac{29}{30}$, 31·2. **3.** $-6+\cdot 309$, $-4+\cdot 27$, $-2+\cdot 196$. **4.** 75.
5. 120 lb. **6.** 60 mi.

R 23. **1.** ·07, ·0685 ; 1·05, 1·05 ; 10·02, 10·0. **2.** 7 ; $2^2\cdot 3\cdot 5\cdot 7$. **3.** £34, 13s. 4d.
4. 15s. **5.** 1 hr. 48 min. **6.** £36.

R 24. **1.** £308, 18s. 6d. ; £96, 18s. 6d. **2.** 48 days. **3.** 6·235 Km. **4.** $1\frac{3}{5}$, 2.
5. 5s. $5\frac{3}{4}$d. ; 9s. $8\frac{3}{4}$d. ; 14s. $11\frac{3}{4}$d. **6.** 2 in.

R 25. **1.** 113 fr. 20 c. **2.** 26 ; 50 c. **3.** £3, 4s. $4\frac{1}{2}$d. **4.** $2\frac{14}{15}$, $5\frac{13}{15}$, $\frac{3}{5}$.
5. $3\frac{5}{9}$ lb. **6.** 185.

R 26. **1.** £107, 8s. 3d. **2.** $\dfrac{36yz}{x}$ pence. **3.** £3, 13s. 9d.; 27 ; 8s. 9d.
4. 245·7228 ; 148. **5.** £8, 16s. **6.** 4 ft.

R 27. **1.** 7 hr. 4 min. a.m. **2.** £90, 12s. **3.** 2s. $11\frac{3}{4}$d. **5.** $2^4\cdot 3\cdot 5^2\cdot 7^2=58{,}800$.
6. 25·3 cm.

R 28. **1.** 6d. **2.** $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{20}$, $\frac{9}{25}$, $\frac{1}{400}$. **3.** 560 lb. **4.** $1\frac{1}{12}$, $\frac{2}{9}$. **5.** 64 ac.
6. $6\frac{7}{8}$ ft. per sec.

R 29. **1.** 347380·8. **2.** (1) ·0001, (2) $26\times 1\cdot 4$. **3.** 6s. 11d.
4. £12,291, 11s. 8d. **5.** 17. **6.** $\frac{441}{64}$.

R 30. **1.** The second ; 15 coins. **2.** $\frac{2}{7}$, $\frac{1}{3}$, $\frac{8}{23}$, 0·35, $\frac{7}{19}$. **3.** $3\frac{3}{4}$, 16·72, $\frac{1}{6}$.
4. 63 ; 3·9 cm. **5.** 2200 ft., 5500 sq. ft. **6.** 1144 ; 4112 cu. in.

PROBLEM PAPERS 21–30 (p. 152)

P 21. **1.** £46, 17s. 6d. **2.** 14·53 ft. **3.** 77. **4.** $\frac{2}{3}$. **5.** $1\frac{13}{17}$ lb.

P 22. **1.** 6746, 15,144 ; 6867, 368. **2.** 10. **3.** $2\frac{1}{2}$ hr. **4.** 64 sq. ft.
5. £4890.

P 23. **1.** £1520. **2.** 2 t. 13 cwt. 4 lb. **3.** 2·7183. **4.** 1·566. **5.** 100, 99.

P 24. **1.** P. 21, tenth in eleventh row. **2.** 1985. **3.** 812 yd. **4.** $29\frac{1}{3}$ sq. ft.
5. 987 oz.

P 25. **1.** £42. **2.** 2587 sq. mi. **3.** ·6667. **4.** 290 days.

P 26. **1.** 7 wk. **2.** 1200. **3.** 133 sq. cm.; 561 gr. **4.** £13, 7s. 6d.
5. 504.

P 27. **1.** $31\frac{1}{4}$ sq. yd. **2.** 6. **3.** $4\frac{4}{9}$ m.p.h. **4.** 42 lb. **5.** 33rd division.

P 28. **1.** 52 ft. 1 in. **2.** 15 in. **3.** 74 sq. in. **4.** £79, 17s. 2d. **5.** 4·8 in.

P 29. **1.** 0·35 ; 30, 17. **2.** $1\frac{1}{8}$ min. **3.** 102, 119. **4.** $\frac{1}{24}$ wk. ; 24 of each.
5. $\frac{11}{7}$.

P 30. **1.** 1. **2.** $\frac{6}{11}$ cm. **4.** 797. **5.** Each is 15 cu. in.

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ANSWERS. PART III

Oral Questions (p. 159)

(i) 188, 139·5, 125·5, 108 lb. per sq. in.
(ii) 4, 3·67, 3·19, 2·63, 2·35 cu. ft.

Exercise XIII (a) (p. 160)

1. Increase May, decrease Sept. to Oct.
2. Yes; 9.15 p.m.; May 29, July 24; June 9, July 15. 3. No.
4. 6·8 yr.; 33. 7. 12·3, 21, 132 sec.
8. £2, 1s. 1d.; £2, 11s. 7d.; roughly £4, 10s.
9. Decrease due to extra employment during the War. 10. No.
11. 1 hr. 52·5 min. after start; 8 m.p.h. 12. 33·3 m.p.h.; 193 mi.
13. 117 mi., 58·4 m.p.h., 12.24½ p.m., 3.24 p.m. 14. 169° F.; 38° C.
15. 72·5 mi.; 1.43 p.m. 16. 1.46 p.m.; 8·75 mi.; 1.28 p.m.; 2.3 p.m.
17. 8·41, 1160, 4·12, 1·27.

Oral Questions (p. 164)

(i) 17·6, 38·1, 70·4, 77·7 ft. per sec.
(ii) 13·6, 21·1, 34·1, 45·7 m.p.h.

Exercise XIII (b) (p. 164)

1. 9·9 lb., 3·2 Kg. 2. 68·6 cm., 15·7 (5) in. 3. 11. 4. 60.
5. £19, 3s.; £44, 9s. 6. 10 in., 5·51 in. 9. $\frac{x}{y}=3{\cdot}5$.
10. Trebles *y*, halves *x*, increases *y* by 40 per cent. 11. Yes. 13. No.
14. Yes, if pipe is full. 15. Yes, No.

Exercise XIV (a) (p. 168)

17. 1820, 270. 18. 100. 19. 10. 20. 5. 21. 25. 22. $\frac{5}{8}$.
23. $\frac{17}{24}$. 24. $\frac{35}{72}$. 25. $\frac{101}{224}$. 26. $1\frac{199}{441}$. 27. $-1\frac{35}{48}$. 28. $\frac{2}{9}$.
29. $\frac{1}{3}$. 30. $\frac{8}{9}$. 31. $\frac{1}{2}$. 32. $\frac{5}{12}$. 33. $\frac{1}{32}$. 34. $\frac{7}{20}$.
35. $\frac{17}{18}$. 36. $\frac{1}{2}$. 37. $\frac{7}{10}$. 38. $2\frac{11}{12}$. 39. $1\frac{19}{20}$. 40. $\frac{1}{30}$.
41. $\frac{8}{21}$. 42. $2\frac{1}{12}$. 43. $1\frac{2}{7}$. 44. $\frac{2}{3}$. 45. $1\frac{7}{8}$. 46. $\frac{11}{108}$.
47. $\frac{1}{2}$. 48. 2. 49. $1\frac{2}{5}$. 50. $1\frac{1}{5}$. 51. 20. 52. $\frac{2}{3}$.
53. $2\frac{4}{5}$. 54. $1\frac{1}{13}$. 55. $5\frac{8}{17}$. 56. $\frac{19}{36}$. 57. $2\frac{17}{42}$. 58. $\frac{1}{9}$.

59. 5. 60. 1. 61. $\frac{5x}{6}$. 62. $\frac{7x}{12}$. 63. $\frac{x}{6}$. 64. $\frac{a+b}{ab}$.
65. $\frac{b-a}{ab}$. 66. $\frac{3x}{2}$. 67. $\frac{a+b}{b}$. 68. $2\frac{1}{2}$. 69. $\frac{a+b}{ab^2}$. 70. $\frac{a+b}{b}$.
71. $\frac{b+1}{b}$. 72. $\frac{2b-a}{b}$. 73. $\frac{bc+ca+ab}{abc}$. 74. $\frac{b+a}{b-a}$. 75. 5.
76. 8. 77. $3\frac{1}{11}$. 78. $3\frac{2}{3}$. 79. $1\frac{2}{3}$. 80. $\frac{1}{19}$.

Exercise XIV (b) (p. 170)

1. 6 in. 2. 10s. 3. 2s. 6d. 4. £320. 5. $\frac{5}{12}$.
6. £180. 7. 25 lb. 8. 384. 9. £441, £210. 10. 48.
11. 28 ; 1 in. 12. 9 ; 3d. 13. $16\frac{1}{3}$ ft. 14. £12, 16s. 15. 2031 yd.
16. $52\frac{1}{2}$. 17. $2\frac{2}{9}$ cwt. 18. £90. 19. 21 sq. in. 20. 48 mi.
21. $\frac{47}{84}$. 22. $\frac{22}{15}$, $\frac{8}{15}$; $\frac{15}{22}$, $\frac{15}{8}$. 23. Increased ; No.
24. 60, 140 lb. 25. $26\frac{1}{4}$ gall. 26. $\frac{1}{4}$. 27. 2, $\frac{3}{4}$, $1\frac{1}{4}$ cwt.
28. 18, 15 cu. ft. 29. $\frac{6}{17}$, $\frac{5}{17}$. 30. 20, 67·5 cu. in. 31. $1\frac{3}{7}$.
32. 3. 33. $33\frac{1}{3}$. 34. $6\frac{3}{13}$. 35. 121.

Exercise XV (a) (p. 175)

1. 1, 2, 3, 4, 5, 6. 2. 1, 2, 3, 4, 5. 3. 1, 2, 3, 5.
4. (i) 2, (ii) 5, (iii) 2, (iv) 5, (v) 3, (vi) 4, (vii) 2, (viii) 3, (ix) 2, (x) 5.
5. 6, 0·0230, 0·02303, 0·023026.
6. 100 gr., 97 gr., 96·7 gr. ; 96·73 ± ·005 gr.
7. Limits are 3·076, 3·072 gr. ; 3·1 gr.
8. 12·935, 12·925 cu. cm. ; 0·005 cu. cm. 9. Varies 3×10^6 mi.
10. 0·05 in. 11. 500. 12. Less. 13. 120 ± 0·17.
14. Between 1138 and 1134 sec. 15. 0·5 cm., 0·026, 2·6.
16. 0·5 mgr., 0·0038, 0·38. 17. 50 yd., 0·015, 1·5.
18. 0·03 cu. cm., 0·00019, 0·019. 19. 2·5 min., 0·026, 2·6.
20. 0·5 lb., 0·0031, 0·31. 21. 10. 22. 0·9. 23. 0·04.
24. 0·7. 25. 3. 26. 0·5. 27. 3, 0·06. 28. 1·5 mm.
29. 1·6 ± 0·1 cm. 30. £1, 3s. 4d. 31. 14·2225, 14·2175, 0·0025 gr.
32. 1·7 ± 0·1 mi. ; 41·6 ± 0·1 mi. ; No, correct to $\frac{1}{8}$ mile.

Exercise XV (b) (p. 179)

1. 0·1. 2. 0·0006. 3. 0·0005. 4. 30. 5. ·0003.
6. 7. 7. 1. 8. 0·1. 9. 7. 10. ·006.
11. 100. 12. 10. 13. 2000. 14. ·03. 15. 3000.
16. 90. 17. 0·4. 18. ·000,002. 19. 2. 20. £200.
21. 1d. 22. 6d. 23. 90,000. 24. 60,000. 25. £4000.
26. 40,000. 27. 3,000,000. 28. 4. 29. ·004. 30. 8s.

31. 9. **32.** ·01. **33.** 0·3. **34.** ·05. **35.** 4s.
36. 200. **37.** 500. **38.** 0·3. **39.** 20. **40.** 10.
41. 600,000. **42.** 300,000. **43.** 60. **44.** 100,000. **45.** 1815.
46. 2. **47.** 20. **48.** 10. **49.** 0·1. **50.** 30.

Exercise XV (c) (p. 183)

1. 8·9 sq. in. **2.** 27 cm. **3.** 120 sq. in. **4.** 498 sec.
5. 247,356,671 tons. **6.** 175, 669.
7. 25,000, 11,600 ; 1130, 695 ; 200,000, 94,000 ; 410, 319.
8. +7·5, −1·1, +6·3. **9.** £7,435,000,000. **10.** 52·2 yd.
11. 4,030,000. **12.** £2, 17s. **13.** £1, 10s. 9d.
14. 7s. **15.** £2, 12s. 0d. **16.** 11·9. **17.** 853.
18. 32·8, 9s. 4d., £3, 4s. 9d. **19.** £108,285, 9s. 4d.
20. 320,000,000, maximum error 1,500,000. **21.** 272·68 ft.
22. 0. **23.** £3, 9s. 1d. **24.** Between £10,226,900 and £10,226,100.
25. E. W. 12·3 per cent. ; S. 11·1 per cent. **26.** 42·5, 6·29 per cent. ; 0·148.
27. 16 per thousand. **28.** £23,000,000 (2 fig.). **29.** 126·89 ft.
30. 10·5−8·1 =2·4 per cent. ; 1925, −19·7 per cent., −21·6 per cent.
31. £90,000. **32.** 9·86. **33.** 32 (32·2).
34. 0·000,009 per cent. ; the same. **35.** 2·2. **36.** 6076 ft.
37. 25,300,000. **38.** £406,500.

Exercise XV (d) (p. 187)

1. 9·870. **2.** 0·0435. **3.** 13·2. **4.** 1·62.
5. 85·0. **6.** 1·95. **7.** 8190. **8.** 0·0736.
9. 2·02. **10.** 6·82. **11.** 29·9 in. **12.** 0·080852.
13. 0·3183, 21·9 in. **14.** 1·09 in. **15.** 38·25. **16.** 71,800,000.
17. ·00737. **18.** 7041. **19.** £42, 14s. 9d.
20. 1·47d. ; £3,047,150. **21.** 7·389. **22.** 1550·01. **23.** 0·45359.
24. 54,650,000,000. **25.** £2,894,600.

Exercise XVI (a) (p. 189)

1. 144, 1·44, 14,400, ·0144, ·000144. **2.** 4900, 0·64, 90,000, ·0036, 1·21.
3. 49, 7 ; 0·49, 0·7 ; 1·44, 1·2 ; ·0004, ·02.
4. 1, 4, 9, 6, 5, 6, 9, 1, 0 ; No ; 8468, 2917, 4483, 8412. **5.** 25, 16, 9.
6. 441, 484, 529, 576, 625. **7.** 3^2, 4^2, 5^2 ; 100 ; n^2. **8.** 10.
9. $\frac{1}{9}$, $\frac{1}{100}$, $\frac{9}{16}$, 0·01, $2\frac{1}{4}$, 1·69, 6·25 ; $x>1$, $x<1$. **10.** ·01 ; 10,000 ; 100.
11. 9, 0·9, 900, 110, 1·1, ·011. **12.** $\frac{2}{3}$, $\frac{1}{10}$, 0·1, ·01, $\frac{8}{9}$, $1\frac{1}{3}$, $1\frac{1}{5}$, $3\frac{1}{2}$, 1·3 ; $x>1$, $x<1$.
13. 12, $\sqrt{290}$, $3\frac{1}{3}$, 2·5, $7\frac{3}{4}$. **14.** 1·96, 2·25, 1·9881, 2·0164.
17. 10, 100 ; 2, 2, 3, 1, 2. **18.** 3, 4, 4. **19.** 10, 0·1, 10, 100.
20. 6·3, 6·4 ; 9·4, 9·5 ; 15, 16 ; 3·2, 32, 320.

Exercise XVI (b) (p. 193)

1. 38. 2. 55. 3. 59. 4. 93. 5. 108. 6. 127.
7. 253. 8. 739. 9. 904. 10. 360. 11. 1·19. 12. 11·7.
13. 18·2. 14. 7·81. 15. 10·01. 16. 0·14. 17. 0·131. 18. 0·32.
19. ·0043. 20. ·0251. 21. 0·537. 22. 0·185. 23. 5·48. 24. 17·5.
25. 1·73. 26. 3·61. 27. 11·4. 28. 4·02. 29. 3·57. 30. 1·13.
31. 3·80. 32. 0·577. 33. 1·83. 34. 0·513. 35. ·0788. 36. 9·14.
37. ·0193. 38. 0·723. 39. 0·229. 40. 1·39.

Exercise XVI (c) (p. 195)

1. 29930. 2. 2,360,000. 3. 1701. 4. 3·334. 5. 9·873.
6. ·01610. 7. ·0004567. 8. 29310. 9. 1·026. 10. 6·083.
11. 0·7071. 12. ·07071. 13. 3·712. 14. 1·173. 15. 7·071.
16. 2·598. 17. 7·008. 18. 2·054. 19. 5·056. 20. 1·762.
21. 3·416. 22. ·05882. 23. ·007634. 24. ·04348. 25. 0·2890.
26. ·04608. 27. ·008914. 28. ·01027. 29. 60·02. 30. ·00996.
31. 0·3700. 32. ·03177. 33. 0·1307. 34. 0·1744. 35. 0·3109.
36. ·004900. 37. 0·1267. 38. ·07720. 39. 0·2548. 40. 4·812.
41. 0·1235. 42. 0·377. 43. 6·68. 44. ·00987. 45. 0·976.
46. 3·79. 47. 0·161. 48. 7·74. 49. 0·423(5). 50. ·0329.

Exercise XVI (d) (p. 197)

1. 11·1 ft. 2. 3·46 in. 3. 69·6 yd. 4. 220 yd. 5. 25·3 yd.
6. 3·16 ch. 7. 1·07 ft. 8. 5·98 ft. 9. 165. 10. 440 yd.
11. 15 min. 12. 6·27 in. 13. 10 in. 14. 9 ft. 8 in. 15. 1·29 in.
16. 1892 A.D. 18. £93, 10s. 19. 3·8 in. 20. Four. 21. 6·93 in.
22. 17·89 in. 23. 8·775 in. 24. 25·9 ft. 25. 9·43 in.
26. 13, 5, 12 ; 29, 21, 20. 27. 22·1. 28. —34·2. 29. —3·06.
30. —8·83. 31. 0·862. 32. ·0670. 33. 177·8 yd. 34. 1·23 in.
35. 8·33 in. 36. 65 ft.

Exercise M (V) (p. 199)

1. 6 hr. 49 min. 2. 2·5 in. 3. £2, 18s. 3d. 4. $2\frac{3}{11}$ per cent.
5. 8·049, 3·2. 6. 2·6 ton. 7. 1·3. 8. 12. 9. 8·01 ft.
10. £383, 0s. 0d. 11. £98. 12. 5 per cent. 13. £99, 4s. 8d.
14. 575 m., 660 f. 15. 0·365, 0·45 in. 16. 7·6 million. 17. $\frac{1}{10,000}$.
18. 29. 19. 24,900 mi., 37·7 ft. 20. 58, 11. 21. $7\frac{7}{8}$ oz.
22. $2\frac{51}{290}$. 23. 5808. 24. —15s. 25. $1\frac{1}{3}$, 3 tons.
26. 115,920. 27. $\frac{1}{22}$. 28. 126 mi. 29. £1, 2s. $6\frac{1}{2}$d.
30. $\frac{1}{300}$. 31. 306. 32. $\frac{112}{15}$. 33. 19,400 tons.
34. 63·7 35. 12·5. 36. £13,299,400. 37. 1s. 38. 10.

39. 55·0, 56·6 per cent. 40. Train. 41. 44. 42. $133\frac{1}{3}$ sq. in.
43. $\frac{xy}{100-x}$ s. 44. 40·3 m.p.h. 45. 3 in. = 76·2 min. 46. 9216.
47. 140° F., 10° C. 48. 5 tons 17 cwt. 49. 709·5 yd. 50. $30\frac{5}{9}$ per cent.

REVISION PAPERS (R 31–40) (p. 202)

R 31. 1. $-\frac{5}{6}$. 2. 2. 3. 1s. 10·4d. 4. 0·31. 5. 78 per cent. 6. 29·4 lb.
R 32. 1. 15s. 9d. 2. 200 per cent. 3. $1\frac{4}{13}$. 4. $\frac{1}{4}\pi d^2$; 10·9 sq. cm.
5. $\frac{3}{5}$; £413, 6s. 8d. 6. ·025 sq. in.
R 33. 1. $\frac{121}{144}$; ab. 2. 6·3. 3. 38; 1s. 10d. 4. 35s. 5. £5, 4s. 4d.
6. $2x$ ft.; $22x^2$ sq. in.
R 34. 1. £85, 1s. 4d. 2. £55,500, 11s. 6d. 3. 237, 0·237.
4. 13·6 per cent. 5. $2\frac{2}{3}$, 550. 6. 62·33 ft.
R 35. 1. $\frac{5}{9}$, $\frac{7}{11}$, $\frac{2}{3}$. 2. 33 yd. 3. £17,864. 4. 19·25 sq. in.
5. 40·5, 47·4, 12·1 per cent. 6. 196 yd.
R 36. 1. 22·09 Km. 2. 96 lb. 3. 6s. 0·8d. 4. £245,000. 5. 30·8 lb.
6. $\frac{53}{35}$, $\frac{22}{49}$.
R 37. 1. 2; $4\frac{64}{105}$. 2. ·032. 3. 547 yd. 4. 33 m.p.h. 5. £22, 19s. $4\frac{1}{2}$d.
6. 0·28 in.
R 38. 1. 7^2, 13, 17. 2. £37, 10s. 9d. 3. $\frac{1}{31\cdot55}$. 4. 10s. 4d. 5. £57, 15s.
6. 3·2 in.
R 39. 1. $3\frac{1}{9}$, ·0992. 2. 21·3. 3. The first. 4. 10·9 per cent. 5. 600,000.
6. 1·74 mi.
R 40. 1. 347. 2. £23, 8s. 2d. 3. £56,249. 4. ·0938. 5. 2·25 ft. per sec.
6. 5·4 Kg.

PROBLEM PAPERS (P 31–40) (p. 207)

P 31. 1. 1. 2. £7245. 3. 2·24, 2·23. 4. $3\frac{11}{18}$ m.p.h.
5. Between 68·35 and 67·5 yd.
P 32. 1. 5·54 in. 2. 1 in 8. 3. £80. 4. 91·4 5. 28·2 per cent.
P 33. 1. 19 m. 2. 20, 37, 43. 4. 7·21 m. 5. 136 sq. cm.; 108 cu. cm.
P 34. 1. 3 tons $13\frac{1}{2}$ cwt. 2. £8, 7s. 3. 9. 4. 10·6 per cent. 5. 4 : 11.
P 35. 1. £440, 10s. 6d. 2. $\frac{5}{18}$ lb. 3. 48 days. 4. 12·4 cm.
5. 14, 15; 70·02, 39·02 sq. cm.
P 36. 1. £94, 12s. 3. 340. 4. £12,000. 5. 10, $9\frac{1}{11}$ yd. per sec.
P 37. 1. 8. 2. 18. 3. 11·5 sq. in. 4. Singly; 1s. 5. £800, £1200, £1600.
P 38. 1. £2, 5s. 4d. 2. $33\frac{1}{3}$ per cent. 3. 28 min. 4. £53, 2s. 6d.
5. 75 m.p.h.
P 39. 1. 29,700. 2. 3s. $2\frac{7}{12}$d. 4. 20 ft. 2 in. 5. 2 : 1.
P 40. 1. 0·866. 2. £410, 16s. 8d.; £315; £615. 3. £4550. 4. 10 g.
5. 1430.

Exercise XVII (a) (p. 213)

2. x^7, x^4, x^6, x^7, x^6, x^{33}, x^{16}, x^{12}.
3. $y^m \times y^n = y^{m+n}$; $y^m \div y^n = y^{m-n}$; $(x^m)^n = x^{mn}$.
4. x^8, x^6, x^9, x^4, x^4, x^3, $4x^8y^4$, $3a^9b^3$.
5. 10^3, 10^2, 10^1, 10^0; 1000, 100, 10, 1. 6. 10, 4, 1, 9, 8; 3, 2.
7. 2, 3, 5. 8. 10. 9. 10. 11. Indices are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{2}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{5}$, $2\frac{1}{2}$, $\frac{5}{6}$.
12. Indices are $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{3}$, $1\frac{1}{6}$, 0, $1\frac{1}{3}$.
13. (i) 1·58, 2·51, 5·01, 7·94; (ii) 0·48, 0·70, 0·91; (iii) 1·58, 4·64, 3·98; indices are 0·301, 0·544, 0·845.
14. Indices are 1·301, 3·301, 2·477, 0·778, 1·079, 0·1505; 200, 3000, 6, 20,000, $\frac{2}{3}$, 5, $\sqrt{3} = 1{\cdot}73$.
15. 1, 2, 5, 4, 0, 4, 1. 16. $10^{1{\cdot}01}$, 11, $10^{4{\cdot}8}$, $\sqrt{1000}$, $10^{{\cdot}7}$.
17. Indices are 2·2, 1·8, 0·5, 0·85, 1, 0·3, ·08, 0·2, 4, 1·2, 0·49, $1\frac{1}{6}$.

Exercise XVII (b) (p. 215)

1. 0·6021. 2. 1·6021. 3. 2·6021. 4. 4·6021. 5. 2·8573.
6. 1·8573. 7. 0·8573. 8. 4·8573. 9. 2·8591. 10. 0·8591.
11. 1·8591. 12. 3·8591. 13. 1·8594. 14. 0·8594. 15. 3·8594.
16. 4·8594. 17. 1·7782. 18. 2·5821. 19. 1·9703. 20. 0·4728.
21. 3·4786. 22. 1·4453. 23. 2·2062. 24. 2·6030. 25. 0·6397.
26. 5·5991. 27. 3·7820. 28. 1·8923. 29. 3·0261. 30. 2·0025.
31. 4·7166. 32. 1·9035. 33. 2·9545. 34. 4·3021. 35. 0·8456.
36. 2·200. 37. 22·00. 38. 220·0. 39. 22,000. 40. 60·00.
41. 6·000. 42. 6000. 43. 60,000. 44. 2·050. 45. 80·8.
46. 35·40. 47. 3·540. 48. 1·135. 49. 762·0. 50. 76·20.
51. 7·620. 52. 1·225. 53. 5730. 54. 50·70. 55. 1·220.
56. 863·0. 57. 45·30. 58. 6·310. 59. 97·50. 60. 670,000.
61. 400·0. 62. 91,000. 63. 3·057. 64. 1·118. 65. 405·4.
66. 19·55. 67. 3·001. 68. 18·44. 69. 30,070. 70. 204·6.
71. 1·224. 72. 1·995. 73. 1·072. 74. 1000. 75. 1·007.

Exercise XVII (c) (p. 217)

1. 21·1. 2. 2190. 3. 78·4. 4. 8·71. 5. 492,000.
6. 1520. 7. 283. 8. 9060. 9. 278. 10. 6·34.
11. 7460. 12. 481. 13. 334,000. 14. 666. 15. 1·55.
16. 123,000,000. 17. 1290. 18. 1·05. 19. 264. 20. 5·75.
21. 141. 22. 87·9. 23. 9·79. 24. 28,000,000. 25. 217,000.
26. 4310. 27. 5·74. 28. 2·66. 29. 1·24. 30. 1·55.
31. 2·03. 32. 16·1. 33. 3·98. 34. 5360. 35. 536,000.
36. 5·65.

Exercise XVII (d) (p. 217)

1. 177. 2. 166. 3. 645. 4. 1075. 5. 280.
6. 172,000. 7. 13,200. 8. 266,000. 9. 44·4. 10. 110.
11. 2·16. 12. 1·18. 13. 4·51. 14. 1·84. 15. 1·54.
16. 1·71. 17. 1·36. 18. 16·2. 19. 3·30. 20. 10·1.
21. 1·61. 22. 14·5. 23. 3·36. 24. 80·1. 25. 2·23 (5).
26. 1·04. 27. 1·37. 28. 1·36. 29. 854. 30. 1·15.
31. 1·86. 32. 6·31. 33. 50·7. 34. 14·9. 35. 623.
36. 122.

Exercise XVII (e) (p. 220)

1. 1·7. 2. 3·5. 3. 1. 4. $\bar{5}$·8. 5 $\bar{4}$·5. 6. $\bar{2}$·5.
7. 3·4. 8. 2·7. 9. $\bar{4}$·4. 10. $\bar{5}$·6. 11. $\bar{1}$·8. 12. $\bar{2}$·3.
13. $\bar{4}$·7. 14. 0·7. 15. $\bar{4}$·6. 16. $\bar{3}$·4. 17. $\bar{5}$·6. 18. $\bar{8}$·8.
19. $\bar{9}$·5. 20. 1·2. 21. $\bar{1}$·4. 22. $\bar{2}$·9. 23. $\bar{1}$·8. 24. $\bar{2}$·6.
25. $\bar{1}$·68. 26. $\bar{1}$·7. 27. $\bar{1}$·7. 28. $\bar{1}$·4. 29. $\bar{1}$·9. 30. $\bar{3}$·8.

Exercise XVII (f) (p. 220)

1. 1·07. 2. 5·45. 3. 5·09. 4. ·0905. 5. 0·805.
6. ·000805. 7. 0·573. 8. 2220. 9. 717. 10. ·0000531.
11. 0·357. 12. ·00872. 13. 6·07. 14. ·00520. 15. 0·808.
16. 0·929. 17. 0·294. 18. 0·781. 19. 0·363. 20. 0·168.
21. 0·328. 22. ·0856. 23. 0·557. 24. 0·318. 25. 0·910.
26. 0·808. 27. 0·824. 28. ·000182. 29. 0·607. 30. ·0685.
31. 0·166. 32. 4·09. 33. 0·230. 34. 11·5. 35. 0·259.
36. 0·172

Exercise XVII (g) (p. 221)

1. ·00890. 2. ·0137. 3. 0·350. 4. 0·105. 5. ·00486.
6. 2·97. 7. 0·118. 8. 0·343. 9. 755. 10. 4080.
11. ·000,002,51. 12. ·000,002,59. 13. 1120. 14. 272. 15. 2·15.
16. ·0242. 17. 13·3. 18. 0·658. 19. ·00370. 20. ·0890.
21. ·000,051,6. 22. 954. 23. 0·450. 24. ·0275. 25. 1·26.
26. 1·34. 27. 0·136. 28. 21·5. 29. 22·4. 30. 6·90.
31. 0·845. 32. 0·794 ; 0·229 ; ·00426 ; 0·551. 33. 192.
34. 0·949. 35. 0·938. 36. 0·452 ; 197. 37. −7·77 ; 0·432.
38. 0·766.

Exercise XVII (h) (p. 222)

1. 27·2. 2. 1910 Kg. 3. ·000,015,8. 4. 121. 5. 35·9 cu. ft.
6. 16·2. 7. £6, 11s. 8. 57·3. 9. 904 yd. 10. 1·25 sq. ft.
11. 37·5 lb./ft.^{2}. 12. 1490 cu. in. 13. 42·9 per cent. 14. 4·54.

15. 1·60. **16.** 1·13 ft. **17.** 1·24 ft. **18.** 0·690. **19.** 1740 cu. in.
20. 201 sq. in. **21.** 90·4. **22.** 0·141. **23.** 3·91. **24.** 13·8.
25. 3·42 ft./sec. **26.** 39·3 ft. ; 342 lb. **28.** 43·4. **29.** 78·7.
30. 309. **31.** 1830. **32.** 4750 tons.

Exercise XVIII (a) (p. 227)

1. 21 sq. in. ; 4·8 in. ; 1·8 in. ; 6 in. **2.** 1·73 in. ; 3·46 sq. in.
3. 6 sq. in. ; 2·4 in.
4. 17·5 sq. in. ; 3·37 in. ; 4·8 in. ; 30, 25 in., 300 sq. in., 24, 20 in.
5. 15·6 sq. in. **6.** 20 in., 234 sq. in. ; 6·72, 12 in. **7.** 4·482 in.
8. 12·65 sq. in. **9.** 18·5 sq. in. **10.** 4·93 ac. **11.** 59·5 sq. in.
12. 5·74 in. **13.** 6·66 in. **14.** 14·7 sq. cm., 5·88 cm.
15. 61·1 sq. in. **16.** 1·13 ac. **17.** 360 sq. ft., 1080 cu. ft.
18. 453$\frac{3}{4}$ sq. ft. **19.** 16,335 cu. ft. **20.** 72 cu. cm. **21.** 4470 cu. ft.
22. 400. **23.** 2880 cu. ft. **24.** 21 tons.
25. 10,400, 9400 cu. ft. **26.** 25,200 gall. **27.** 7776 cu. ft.
28. 70,600,000 cu. ft. **29.** 4 ft. **30.** 32 cu. in. **31.** 149$\frac{1}{3}$ cu. ft.
32. 7·54 cu. in. **33.** 280 cu. ft. **34.** 9$\frac{1}{2}$. **35.** 19$\frac{1}{2}$.
36. 12. **37.** 20. **38.** 18.

Exercise XVIII (b) (p. 233)

1. 50·9 in., 206 sq. in. **2.** 6·78 in. **3.** 66·8 sq. in. **4.** 2·35 cm.
5. 2·67 in. **6.** 670 yd. **7.** 22·9 in.
8. 1810 sq. yd., 25·1 yd. **9.** 5·76 ft. **10.** 240. **11.** 5·09 ft.
12. 6·6. **13.** 36,200, 34,600 lb. **14.** 26·6 sq. cm. **15.** 4·32 cm.
16. 273 sq. yd. **17.** 18·(5) miles per sec. **18.** 224. **19.** 2·26 in.
20. 2$\frac{1}{4}$ lb. **21.** 21·5 sq. in., 31·4 in. **22.** 18·3 in. **23.** 30·28 sq. ft.
24. 24·57 in.

Exercise XVIII (c) (p. 237)

1. 1130 cu. in., 35·3 cu. ft., 254 cu. in., 122 cu. in., 1·37 cu. ft., 38·4(5) cu. in.
2. 1·03 ft. **3.** 408 sq. in. **4.** 401 lb. **5.** 4760 gall.
6. 2·84 ft. **7.** 1·84 ft. per sec. **8.** 422 cu. in. **9.** 3$\frac{5}{8}$ in.
10. 25. **11.** 1·02 mm. **12.** 699 sq. ft. **13.** 183 min.
14. 8·4 in. **15.** 209 sq. ft. **16.** 1·42 cm. **17.** 628 cu. cm.
18. 17·5 cm. **19.** 1040 sq. ft. **20.** 227 sq. ft.
21. (i) 162 cu. ft. ; (ii) 236 cu. in. ; (iii) 484 cu. in. ; (iv) 520 cu. in.
22. 377 cu. in. ; 47·1 cu. cm. **23.** 302 cu. ft., 188·5 sq. ft. **24.** 2$\frac{2}{3}$ cm.
25. 39·3 cu. cm. **26.** 905 cu. cm., 452 sq. cm. **27.** 47·6 cu. in.
28. 197,000,000 sq. mi. **29.** 211,000. **30.** $\frac{5}{8}$ in. **31.** 12·8 in.
32. 905 cu. in., 3·33 sq. ft. **33.** 680 cu. ft. ; 440 sq. ft. **34.** 37·7 sq. in.

Exercise XIX (a) (p. 241)

1. $63\frac{3}{4}$ lb. **2.** $1\frac{1}{9}$ oz. per cu. in. **3.** 2·70. **4.** 90 gr.
5. 37·5 cu. cm. **6.** 11·1 oz. **7.** 2·2 sq. ft. **8.** 0·92.
9. 48·6. **10.** 4·35. **11.** 583 lb. **12.** 1·9 gr.
13. 0·8. **14.** 7·61 in. **15.** 9·2 oz. **16.** 0·9, 0·8.
17. 0·87(5). **18.** 1·76. **19.** Copper, 2 Kg. **20.** 10·4 gr.

Exercise XIX (b) (p. 245)

1. 52, 39 gr. **2.** 43·62, 52 c.c. **3.** 0·75. **4.** $\frac{3}{5}$. **5.** 0·64, 0·91.
6. $\frac{11}{18}$. **7.** 0·67. **8.** 1·19. **9.** 64, 85 c.c. **10.** 7.
11. 195·2, 199·6 gr. **12.** 6·75 ; 152 gr. ; 0·67. **13.** 0·34. **14.** 44 gr.
15. 25 gr. **17.** 0·94. **18.** 0·87 gr. per c.c. **19.** 71·9 ft., 240 tons.
20. 590 Kg. **21.** 4·6 in. **22.** 2·7. **23.** 449 lb. per cu. ft.
24. 1·02. **25.** Gold to silver, 4·2 : 1. **26.** Sulphur to lead, 10·4 : 1. **27.** 4.
28. 0·39 in. **29.** 1·36 ft., 1·09 ft.
30. $\frac{Q-s\,.\,P}{1-s}$ gr., $\frac{Q-P}{1-s}$ c.c., $\frac{Q-s\,.\,P}{Q-P}$ gr. per c.c.

REVISION PAPERS (R 41–50) (p. 247)

R 41. **1.** 0·7 ; 500,000,000 ; 10,000. **2.** ·0251, 0·813. **3.** 1·50 Kg.
4. 3s. $10\frac{7}{8}$d. **5.** 0·4 in. **6.** 81 ; 3·8 cu. in.
R 42. **1.** 0·8167. **2.** 3·36. **3.** 2·6d. **4.** 17·3 cm. **5.** 1·45 cu. ft. **6.** 111 c.c.
R 43. **1.** 30·24 ± 0·22 sq. cm. **2.** 49·0. **3.** £2, 16s. **4.** 99 ft. **5.** 785 cu. ft.
6. ·030 sq. in.
R 44. **1.** 13. **2.** 93 sq. ft. **3.** £2, 12s. 9d. **4.** 1600 lb. **5.** 6·5 per cent.
6. 6s. 0d.
R 45. **1.** $-\frac{1}{4}$, $\frac{1}{5}$. **2.** ·00151 ; 1·39. **3.** 148 yd. **4.** £54. **5.** 400 lb.
6. 3·12 Kg.
R 46. **1.** 0·5000. **2.** 119 sq. cm. **3.** 15·5. **4.** $\frac{6}{25}$, 62·4 gr.
6. 15 cwt. 20 lb.
R 47. **1.** £595. **2.** 4·63, $5\frac{2}{7}$. **3.** 5·25. **4.** 1·53 yd. **5.** $7\frac{1}{2}$ per cent.
6. 6 cu. in.
R 48. **1.** $\frac{14}{31}$, $\frac{5}{11}$, $\frac{15}{32}$, $\frac{8}{17}$. **2.** 3·03 per cent. **3.** 59·6 c.c.
4. $\frac{1}{6}\pi d^3$; 7·45 cm. **5.** 2·6 per cent. gain. **6.** £273, 4s.
R 49. **1.** 6 : 7. **2.** 2·37 in. **3.** 27,301,807 ; 127·28. **4.** 3140.
5. 0·31 in. **6.** 3,000,000,000 cu. ft.
R 50. **1.** 149. **2.** 37·3. **3.** 455 lb. **4.** £20, 12s. 6d. ; £22, 5s. 6d.
5. 12·8 yd. **6.** £24, 15s.

PROBLEM PAPERS (P 41–50) (p. 252)

P 41. 1. +21 per cent., −19 per cent. 2. 2 gall. 3. 1·0792, 1·20. 5. $\frac{1}{3}$ in.
P 42. 1. 6·64 per cent. 2. 60 6. 3. 1809 A.D. 4. 0·823.
5. 437·9 yd.; 703, practically 704; 3.
P 43. 1. 32·7. 2. $\frac{10000}{100-x}$. 3. 11·3, 0·42. 4. 1152, 468. 5. 0·49 in.
P 44. 1. 25 mi. 2. 89·1 cu. yd. 3. 8·1. 4. 6000 tons. 5. $1\frac{7}{8}$ in.
P 45. 1. £41, 13s. 4d. 2. $21\frac{3}{7}$ yd. 3. $222\frac{2}{9}$. 4. £110, 15s. $2\frac{1}{2}$d.
5. 3 min. 15 sec. past three o'clock.
P 46. 1. $226\frac{2}{3}$ rev. per min. 2. Equal; $\frac{1}{2}\pi a$; $\left(\frac{3\pi}{2}+2\right)a$; all are equal.
3. 177 tons. 4. 599. 5. 7·21 per cent.
P 47. 1. £103, 15s. 2. $15\frac{15}{19}$ per cent. 3. 1·000463 sec. 4. 12·8 lb.
5. 6 ft.; 27·6 cu. ft.
P 48. 1. $17\frac{11}{17}$ per cent. 2. £7500, 15 per cent. 3. $7\frac{1}{2}$ mi. from one end.
4. 1 : 5. 5. $\frac{1}{6}$ gall.
P 49. 1. 0·111. 2. 3,470,400. 3. 1·25 in. 4. $47\frac{1}{7}$ ft. 5. 200·1 cu. ft.
P 50. 1. £2, 5s., £4, 10s., £1, 10s., £1, £4, £1, 5s., £3; 10s. 2. 1·12 min.
3. 12 8 in. 4. 284. 5. $33\frac{1}{3}$ per cent.

Exercise XX (a) (p. 260)

[*Note.*—If it is required to calculate angles to the nearest minute, 4-figure tables must be used.]

1. 0·47, 1·00, 2·14. 2. 21° 48′, 35°, 58°. 3. $\frac{BA}{CB}$. 4. $\frac{PR}{QP}$.
5. $\frac{ZX}{YZ}$. 6. $\frac{BC}{AB}$. 7. Tan R. 8. Tan A. 9. Tan X. 10. Tan Q.
11. 11·50 in. 12. 3·91 in. 13. 37·6 in. 14. 70° 55′.
15. 36° 52′. 16. 54° 36′. 17. 8·39 cm. 18. 2·80 cm.
19. 111·1 ft. 20. 56° 19′. 21. 250 ft. 22. N. 62° 18′ E.
23. 48° 49′. 24. 5·36 cm. 25. 5 20 in. 26. 10·2 ft.
27. 70°. 28. 23° 12′. 29. 324 ft. 30. 80·3 ft.
31. 40 4 ft. 32. 6·66 cm.

Exercise XX (b) (p. 263)

1. 0·60, 0·80, 0·89, 0·45. 2. 23°, 67°, 23°. 3. Sin C, cos B.
4. Sin P, cos R. 5. Sin Y, cos X. 6. Sin B, cos C. 7. Tan Y.
8. Sin R, cos P. 9. Tan R. 10. Tan B. 11. $\frac{AB}{BC}$.
12. $\frac{QR}{PR}$. 13. $\frac{YZ}{YX}$. 14. $\frac{AB}{BC}$. 15. $\frac{QR}{PR}$.

16. $\frac{YZ}{XZ}$. 17. $\frac{PQ}{PR}$. 18. $\frac{AC}{AB}$. 19. 8·45 cm.
20. 4·24 in. 21. 5·66, 4·11 cm. 22. 55° 9′. 23. 53° 8′.
24. 1·89 cm. 25. 4·10 in. 26. 2·57 in. 27. 74° 56′.
28. 40·9 ft.; 1 in 2·2. 29. 90·8, 178 yd. 30. 551 ft. 31. 53° 8′.
32. 3·1 ft. 33. 11°, 1 in 5·26. 34. 6·86 in. 35. 3·53 in.
36. 1·43 in. 37. 216 ft. 38. 13·1 cm. 39. 21·4 sq. in.
40. 26° 36′. 41. 6·85, 0·30 mi. 42. 9·51 cm. 43. 36° 52′, 23° 45′.
44. 66° 9′, 72° 39′.

Exercise XX (c) (p. 266)

1. $\frac{AC}{BC}$. 2. $\frac{QR}{PR}$. 3. $\frac{XZ}{YZ}$. 4. $\frac{AC}{BC}$. 5. $\frac{XY}{ZY}$. 6. $\frac{PR}{PQ}$.
7. Cosec Y, sec X. 8. Cosec A, sec C. 9. Tan Q, cot R.
10. Cosec R, sec Q. 11. Tan A, cot C. 12. Tan X, cot Y.
13. AC. 14. PR. 15. XY. 16. QR. 17. $\frac{5}{4}$, $\frac{3}{4}$, $\frac{5}{3}$, 2·4.
18. $\frac{13}{12}$, 2·6, 2·4. 19. 0·92, 0·44, 1·1. 20. sec θ. 25. 7·47 cm.
26. 220 ft. 27. 8·44 cm. 28. 18·7 ft.
29. 8·97, 7·61, 11·4 in. 30. 12·3 cm.
31. 1·3054, 9·397, 0·8391, 0·9397, 1, 2·3662.
32. $\frac{1}{2}x \sec\theta$, $x(\sec\theta - \cos\theta)$, $x\tan\theta$, $\frac{1}{2}x^2 \sin\theta\cos\theta$. 33. 8 ft. 7 in.
34. 25° 40′, 125° 27′. 35. 19·8 cm. 36. 11·8, 18·1 ft. 37. 77° 16′, 32 ft.
38. 20·2 cm. 39. 14° 40′. 40. 35° 16′; 54° 44′.

Exercise M (VI) (p. 269)

1. 9980. 2. 4·40. 3. Between 10, 12 oz. 4. 9 ft.
6. 5·82 in. 7. 121. 8. 80. 9. 7·25 sq. cm. 10. £1000.
11. 30, 31, 32. 12. 17; $1\frac{3}{4}$ pt. 13. 8d. 14. £9, 18s.
15. 3 9d. 16. £31. 17. £18. 18. 14·2.
19. 26 ft. 20. 1 per cent. 21. 153. 22. 8·45 sq. in.
23. 515 cu. ft. 24. $\frac{29}{210}$. 25. 59. 26. 49·2 in.
27. 1·2 Kg. 28. 9. 29. 323. 30. $\frac{3x}{4}+4$; 6·38 ft.
31. 10·1 ac. 32. 12·65 mi. 33. 0·823. 34. 3·5 in.
35. 3·97 cu. ft. 36. 4·36 ft. 37. 80° 22′. 38. 23·4 cm.
39. 126 ft. 40. 28° 58′. 41. 238 sq. cm. 42. 1·12 sq. in.
43. 2230 ft. 44. 16° 56′. 45. 45° 46′. 47. 5° 18′; 12° 34′.
48. 6·44, 9·66, 11·6 mi. 49. 367 ft. 50. 88° 11′.

R. B. A. N. M'S EDUCATIONAL CHARITIES,
No.........................
Date.........................
BANGALORE.

ANSWERS. PART IV

Exercise XXI (a) (p. 274)

1. £30.
2. £30.
3. £12.
4. £28.
5. £28.
6. £6.
7. £17, 5s.
8. £12.
9. £112, 10s.
10. £6, 15s.
11. £9, 3s. 9d.
12. £2, 12s. 6d.
13. £5, 8s. 10d.
14. £23, 10s. 5d.
15. £5, 13s. 5d.
16. £4, 7s. 8d.
17. £11, 8s. 11d.
18. £51, 16s. 10d.
19. £6, 7s. 9d.
20. £48, 4s. 7d.
21. £29, 13s. 6d.
22. £29, 10s. 6d.
23. £21, 3s. 5d.
24. £125, 6s. 5d.
25. £3062, 10s.
26. The first; 1s.
27. £744.
28. ½d.
29. £152.
30. £70, 17s.
31. £56.
32. £24,920,000.
33. £225.
34. 17s. 3d.
35. £2000.
36. £2, 7s. 8d.
37. £$\frac{2}{5}xyz$.
38. $\frac{xyz}{100}$s.

Exercise XXI (b) (p. 277)

1. (i) 3, (ii) 3, (iii) £15, (iv) £800, (v) 6, (vi) 1½, (vii) £6400, (viii) $\frac{100z}{xy}$, (ix) $\frac{100z}{an}$, (x) $\frac{100q}{ck}$.

2. (i) £28, 17s. 11d.; £269, 13s. 11d: (ii) 5 yr., £432, 18s: (iii) £260; £312: (iv) £250; £45: (v) £850; £89, 5s: (vi) 3½ yr.; £950, 17s. 1d.: (vii) 7 yr.; £519, 15s.: (viii) £906, 13s. 4d.; £340: (ix) £7160; £537: (x) £1733, 6s. 8d.; £2006, 6s. 8d.

3. 8 per cent.
4. 2½ yr.
5. £168.
6. 30 per cent.
7. 4¾ yr.; £5782, 10s.
8. £126.
9. £110; £90.
10. 40 per cent.
11. £62, 10s.
12. £50; 20 per cent.
13. £2000.
14. £150.
15. 8⅓ per cent.
16. 6⅔ yr.
17. 120 days.
18. £1,815,000.
19. 7⅙ per cent.
20. 8½ per cent.
21. £156, £104.
22. $\frac{z}{y-x}$ years.

Exercise XXI (c) (p. 280)

1. £34, 7s. 8d.
2. £40, 15s. 10d.
3. £6, 18s. 9d.
4. £97, 14s. 7d.
5. £33, 17s. 7d.
6. £43, 11s. 0d.
7. £71, 0s. 10d.
8. £11, 7s. 2d.
9. £16, 18s. 1d.
10. £14, 12s. 3d.
11. £3, 8s. 6d.
12. £15, 17s. 0d.
13. £7, 15s. 9d.
14. £9, 19s. 5d.
15. £12, 0s. 5d.
16. £45, 17s. 0d.
17. £4, 18s. 8d.
18. 4½ per cent.
19. £1 becomes £1, 19s. 9½d.
20. £44, 2s.
21. £67, 10s. 7d.
22. £708, 11s. 9d.
23. £580, 3s. 10d.

Exercise XXI (d) (p. 283)

1. £515, 18s. 2. £79, 17s. 4d. 3. £143, 1s. 4. £321, 6s.
5. £2334, 16s. 6. £46, 10s. 7. £1953. 8. £347, 9s.
9. £511, 10s. 10. £4228. 11. 3·4 (3·34) per cent. 12. 4·18 per cent.
13. £59 (£59, 11s. 1d.). 14. 14·2 yr. 15. £115. 16. £839, 12s. 5d.

Exercise XXI (e) (p. 284)

1. 934,900. 2. £305. 3. 880,000 gall. 4. 6·08 in.
5. 26,000. 6. 11·9 yr. 7. 1520 ft./sec. 8. £1,008,400.
9. 72 per cent. ; 14 (13·1). 10. £1560. 11. 24 per cent.
12. 59. 14. 4 per cent. 15. £194,000 ; 6·4 per cent.

Exercise XXI (f) (p. 287)

1. £6, 7s. 6d. 2. £9, 9s. 3. 42s. 9d. 4. £5.
5. £5, 14s. 6. 40 per cent. 7. £48. 8. 25 per cent. ; £11, 5s.
9. £248, 17s. 6d. 10. £798, 0s. 7d. 11. £995, 19s. 5d. 12. £360, 2s. 3d.
13. £717, 11s. 7d. 14. £9, 0s. 9d. ; £240, 19s. 3d.
15. £34, 13s. 7d. ; £385, 6s. 5d. 16. £2, 5s. 6d ; £82, 14s. 6d.
17. £869, 11s. 4d. 18. £889. 19. $6\frac{1}{4}$ per cent., $5\frac{5}{8}$ per cent.
20. 6s. 9d.

Exercise XXII (a) (p. 289)

1. 28,056. 2. £14 ; £1, 8s. ; £15, 8s. 3. $62\frac{1}{2}$ per cent.
4. 70, 65 per cent. ; the first. 5. 6d. ; £4 ; £21 ; 2s. 6d. ; 17s.
6. £48 ; £12 ; £240. 7. 20, 25 per cent. 8. 300 per cent
9. £12, 10s. 10. £7, 10s. 11. £550. 12. £87, 10s.
13. 35. 14. 5489. 15. £2674. 16. 78 per cent.
17. 18 tons per sq. in. 18. $\frac{100c}{b}$. 19. $\frac{zy}{x}$. 20. 4 per cent.
21. 180 ± 3·6 ft. 22. 10·77 per cent. 23. $566\frac{2}{3}$ per cent. 24. 89.
25. 25 per cent. 26. £7. 27. $\frac{20x}{21}$. 28. $\frac{25y}{22}$.
29. $z=x\left(1-\frac{y}{100}\right)$. 30. $\frac{6400}{100+p}$. 31. 100. 32. 30 per cent.
33. 367 per cent. 34. £2, 13s. 4d. 35. 1 per cent. 36. £277, 6s. 10d.
37. 93,000,000 ; 120,000. 38. $\frac{100y}{100-x}$. 39. £477 ; $36\frac{2}{7}$ per cent.
40. 73 sq. ft. 68 sq. in ; 8·9 per cent. 41. 30 per cent. 42. 37, 1 per cent.
43. 13 st. 2 lb. 44. 15·8 m.p.h. 45. 22 per cent.
46. $2p+0{\cdot}01p^2$ per cent. 47. 1s. 6d. per lb. 48. 4·76 per cent.
49. Labour ; 8. 50. £440. 51. $33\frac{1}{3}$ per cent. 52. 50 per cent.
54. 250. 55. £3977. 56. £180. 57. $22\frac{2}{3}$ per cent.
58. $8\frac{1}{6}$ per cent. 59. £15,514,600. 60. 4·48 ft.

Exercise XXII (b) (p. 296)

1. (i) £206, +£6. (ii) £38, −£2.
(iii) 33s., +32 per cent. (iv) 7d., $-12\frac{1}{2}$ per cent.
(v) +2s., +25 per cent. (iv) −2s., −20 per cent.
(vii) £3, 10s., $-6\frac{2}{3}$ per cent. (viii) £10, 3s. 6d., −16s. 6d.
(ix) +£4, 10s., $+12\frac{1}{2}$ per cent. (x) £75, +£9.
(xi) 15s., −2s. 3d. (xii) £30, −10 per cent.
2. £1240. 3. £64. 4. $20\frac{5}{6}$ per cent. 5. $42\frac{6}{7}$ per cent.
6. 40 per cent. 7. £7, 10s. 8. $11\frac{1}{9}$ per cent. 9. 12 per cent.
10. £26, 9s. 11. 25 per cent. 12. $55\frac{5}{9}$ per cent. 13. $56\frac{2}{3}$ per cent.
14. $56\frac{1}{4}$ per cent. 15. 3s. 4d. 16. 52 per cent. 17. $27\frac{3}{11}$ per cent.
18. $42\frac{2}{9}$. 19. £13, 10s.; £16. 20. 25s.; 32s. 6d.
21. 18s.; $33\frac{1}{3}$ per cent. 22. 25 per cent. 23. $42\frac{6}{7}$ per cent.
24. 150 per cent. 25. £2, 10s. 26. $22\frac{1}{2}$ per cent.
27. $56\frac{1}{4}$ per cent.; $40\frac{5}{8}$ per cent. 28. 110 per cent. 29. 25 per cent.
30. 7s. 6d.

Exercise XXII (c) (p. 299)

1. 180 per cent. 2. 19·6 per cent. 3. $14\frac{2}{7}$ per cent. 4. $46\frac{1}{2}$ per cent.
5. $16\frac{2}{3}$ per cent. 6. 21·7 per cent. 7. 30·0 per cent. 8. 67 per cent
9. 35 gall. 10. $+2\frac{2}{9}$ per cent. 11. $26\frac{4}{11}$ per cent. 12. 6400.
13. 2000 pins. 14. 3 females to 5 males. 15. 25 per cent.
16. $+31\frac{1}{4}$ per cent. 17. $57\frac{1}{7}$ per cent. 18. 6·26 cm. 19. 384,300.
20. $x-5y-\frac{1}{20}xy$ per cent. 21. 15 per cent. 22. 54·9 per cent.
23. 5·6 per cent. 24. $4\frac{1}{6}$ per cent. 25. £20,000. 26. 67·9 per cent.
27. 37 per cent. 28. £8, 6s. 8d.
29. Poles $66\frac{2}{3}$ per cent., Germans $33\frac{1}{3}$ per cent.
30. A, B, C, +12·0, −6·5, −5·5 per cent.

Exercise XXIII (a) (p. 305)

1. £25,000; £15,000; £1 shares (12s. paid); £2500; £10,000.
2. £101,000. 3. £8100; £14,100; £19,800. 4. 4 per cent.
5 11 per cent.; 6d.; 50 per cent. 6. £291,500. 8. £1237, 7s.
9. £803, 16s. $1\frac{1}{2}$d. 10. £1299, 13s. 11. £741, 3s. 8d.
12. $5\frac{3}{5}$ per cent. 13. $5\frac{5}{7}$ per cent.
14. Total payment £1750; receipts £750 from div., £3750 from sale; net gain £2750.

Exercise XXIII (b) (p. 308)

1. 50 shares; £50; £5, £5; $8\frac{1}{3}$ per cent. 2. £410; £30; 3s. per share.
3. £1150; £100; 5s. per share. 4. £21; £105; £299, 5s.
5. £518, 15s.; £56, 5s. 6. £712, 10s.; £37, 10s.; 5·26 per cent.

7. £475. 8. £356, 5s. ; £56, 5s.
9. £3581, 5s. ; £3506, 5s. ; £192 ; 16, £9 ; £1200 ; 5·36 per cent.
10. £220 ; £83, 15s. ; 7·98 per cent. 11. £129, 7s. 6d.
12. 258 shares ; £41, 18s. 6d. 13. £280.
14. £337, 10s. ; £281, 5s. ; £11, 5s. 15. £440 ; $5\frac{10}{11}$ per cent.
16. £754.

Exercise XXIII (c) (p. 311)

1. £546 ; £35. 2. £900 ; £62, 10s. 3. £256, 10s. ; £23, 15s.
4. £2088 ; £96. 5. £1188 ; £59, 8s. 6. £400 stock ; £12.
7. £350 stock ; £42. 8. £2000 stock ; £60.
9. £1724, 2s. 9d. stock ; £51, 14s. 6d.
10. £1197, 3s. 8d. stock ; £47, 17s. 9d. 11. $5\frac{1}{3}$ per cent. 12. 5·66 per cent.
13. 4·94 per cent. 14. 6·00 per cent. 15. 6·02 per cent.
16. 5·09 per cent. 17. $5\frac{1}{2}$ per cent. stock ; 0·49 per cent.
18. 6 per cent. stock ; 0·69 per cent. 19. 7 per cent. stock ; 0·62 per cent.
20. $7\frac{1}{2}$ per cent. stock ; 0·30 per cent. 21. 5 per cent. 22. 90.
23. £1920 stock ; +£12, 16s. 24. 96. 25. German loan.
26. £199, 10s. balance ; +£4. 27. £350.
28. £285, 14s. 3d. stock ; £7, 2s. 10d. 29. 96. 30. £56, 1s. 3d.
31. £386, 5s. 32. £818, 8s. 3d. stock. 33. £779, 8s. ; £761, 8s.
34. £8006, 5s. 35. £2162, 7s. 6d.

Exercise M (VII) (p. 313)

1. $\frac{336}{106}$; 0·89 per cent. 2. £1, 5s. 10d. 3. 540°. 4. 35·2 in.
5. 48. 6. £138, 12s. 7. £1100. 8. 23 days.
9. $\frac{90}{187}$. 10. £1326. 11. 1. 12. 1·85.
13. £5, 13s. 9d. 14. 25 per cent. 15. 65 per cent. 16. £335.
17. 81. 18. £2, 10s. 19. £900. 20. £13, 4s.
21. 3·72. 22. $7{\cdot}2 \times 10^{23}$. 23. 18. 24. 20 per cent.
25. 2·3. 26. 5 per cent. 27. 7·2 in. 28. −1 per cent.
29. £500. 30. 1 ft. 6 in. 31. 35·36 mi. 32. £2112.
33. 6·9. 34. 0·879. 35. £60, 2s. 5d. 36. 30s. per gall.
37. 0·42. 38. $11\frac{3}{7}$ per cent. 39. 27·1 yr. 40. 183,600 ; 103,600.
41. £3, 17s. 42. 1·56. 43. 3·3 cm. 44. 1,600,000 gall.
45. £4, 0s. 4d. 46. 11 ft. 4 in. 47. 69 cu. in. 48. 8·2 oz.
49. 3·5. 50. 21 oz. (21·2).

Exercise XXIV (a) (p. 318)

1. ÷2 ; ×2 ; ÷3 ; $l \propto \frac{1}{b}$. 2. $y \propto \frac{1}{x}$; 6 ; 2·4 ; $y=\frac{12}{x}$. 3. $333\frac{1}{3}$; 320.

4. No ; $\div 2$; $y=\frac{480}{x}$. 5. Straight line through origin ; $x \propto \frac{1}{y}$.
6. Varies inversely. 7. 60, 30, 20, 12, 6. 8. $\div 3$; $\div 3$; $\div 10$; $\times 5$.
9. Decreased by 20 per cent.

Exercise XXIV (b) (p. 321)

1. $\times 9$; $\div 16$; $\times 3$, $\div 10$, $\times\sqrt{2}$; increased by 20 per cent. 2. 36·75 ; 20.
3. $\times 5$, $\div 4$; $\times 81$, $\div 10{,}000$. 4. 0, 3, 27, 75, 300.
5. $A \propto r^2$; $v \propto \frac{1}{t}$; $A \propto b^2$; $V \propto l^3$; $A \propto C^2$; $V \propto h^3$.
6. $F=\frac{k}{d^2}$; $t=k\sqrt{l}$; $A=kl^2$; $V=kr^3$, $V \propto d^3$; $r=k\sqrt{h}$; $C^2=kd^3$.
7. $\times 9$. 8. $\times 10$. 9. $\times\sqrt{2}$. 10. 8 : 1 ; 512 : 1. 11. 27 qt. ; 9 : 1.
12. £3. 13. 3 oz. ; 3·47 in. 14. 2·6, 20·8, 166·4. 15. 0, 2, $2\sqrt{2}$, 10, 20.
16. $V \propto S^{\frac{3}{2}}$; $\times 125$. 17. $19\frac{5}{9}$ lb. 18. 250 sq. cm. 19. 2860 sq. mi.
20. 256 ft., 112 ft. 21. 4·78 cm. 22. $13\frac{1}{2}$ lb. 23. $\frac{1}{10{,}000}$.
24. 1·0163 : 1. 25. 225 sq. in. 26. 8 sq. ft. 27. 2·1 dynes.
28. 10·4 ft. 29. 8·16 cm. 30. $y \simeq 0{\cdot}333x^2$.
31. $y=8{\cdot}02$, $x=6{\cdot}28$; $y=8{\cdot}42$, $x=6{\cdot}93$; $y=8{\cdot}82$, $x=7{\cdot}6$. 32. 88 days.
33. Penny to halfpenny, 1·3 : 1. 34. 64 cm. 35. 0·17 cm.
36. 8·5 cm.

Exercise XXIV (c) (p. 327)

1. $a=0{\cdot}24$, $b=11{\cdot}6$. 2. $W=5P-70$. 3. $a={\cdot}095$, $b=2{\cdot}5$.
4. $n=1{\cdot}37$, $k=5000$. 5. $y=1{\cdot}3x^4$. 6. $p\,.\,v^{1\cdot3}=260$. 7. $l=10\sqrt{w}$.
8. $d=16{\cdot}1t^2$. 9. $xy=13{\cdot}2$. 10. $Q=2{\cdot}66H^{2\cdot5}$.
11. $xy=0{\cdot}5x+0{\cdot}3y$. 12. $y=0{\cdot}2x^2+4{\cdot}95$.

Exercise XXV (a) (p. 330)

1. 37·5, 7·5 sec. 2. 6·8, 2·3 sec. 3. 7 sec. 4. $4\frac{4}{9}$ m.p.h.
5. 1 hr. 20 min. 6. $6\frac{2}{3}$ min. 7. $3\frac{3}{4}$ min. 8. $32\frac{8}{11}$ min. past 6.
9. $21\frac{9}{11}$ or $54\frac{6}{11}$ min. past 4. 10. 32 m.p.h. 11. 60 miles from A.
12. 5 : 3. 13. 9·6 m.p.h. 14. 5 men. 15. 6 : 1.
16. $\frac{3}{8}$ of total journey. 17. $1\frac{3}{11}$ m.p.h. 18. 1·5 m.p.h.
19. 780. 20. 8 : 9. 21. 2.40 p.m.
22. £10 ; £13, 6s. 8d. ; £6, 13s. 4d. 23. $\frac{2}{3}$ mi. 24. 1 : 10.
25. 8.16 a.m. 26. 90° E. 27. 57·6 sec.
28. 12.35 p.m. ; $16\frac{2}{3}$ mi. from A. 29. 88 yd. ; 30 m.p.h. 30. 70 yd., 40 yd.
31. £240, £220, £200, £158. 32. $2\frac{1}{2}$ m.p.h. 33. $7\frac{29}{87}$ hr. ; £6, £4, 10s., £4.
34. $\frac{7}{13}$; $\frac{5}{8}$. 35. 8000 mi. 36. C gives B 5 points.
37. C gives B 87 or 88. 38. 2 min. 40 sec. 39. $26\frac{2}{3}$ m.p.h., 16 m.p.h.
40. $7\frac{19}{23}$ min. or $39\frac{3}{23}$ min. past 9.

Exercise XXV (b) (p. 333)

2. 5 ; 1, 3, 9, 27, 81 lb. wt. **3.** Less. **4.** $\frac{1}{101}$. **5.** $a+b=c+d$.
6. $\frac{4}{3}\pi abc$ cu. ft. **7.** B by 78. **8.** 18 hr. 9 7·2 hr. **10.** 9. **11.** 3 ft. per sec.
12. $30\frac{30}{59}$ sec. or $61\frac{1}{59}$ sec. **13.** $17\frac{7}{8}$ min. ; $35\frac{3}{4}$ min. **14.** 301. **16.** 1 : 7.
17. 12 per cent. **18.** Divisible by 13. **19.** $1\frac{7}{8}$ hr. ; $3\frac{13}{24}$ hr. **29.** 5 : 8.
21. $27\frac{1}{2}$ hr. **22.** 10, 7·5, 8 ac. **23.** 38 yd. **24.** 3 tons 15 cwt.
25. Yes ; No. **26.** A. **27.** £4890.
28 100, 70 yd. per min. ; 1 hr $57\frac{1}{3}$ min. **29.** ·0013 gr.
30. 78. **31.** 61·6 sec. **32.** 5 hr. 39 min. **33.** 2·5.
34. 1275 yd. ; 28·45 in. **35.** 2s. $0\frac{3}{4}$d. **36.** 12 sec.
37. £602,300 ; £595,500 ; £528,000 and £522,000. **38.** 12, 11·5, 6·5 cm.
39. July 14, 1874 ; $353\frac{1}{2}$ days. **40.** 0·32 in. ; 3·88.

Exercise M (VIII) (p. 337)

2. 48 gall. **3.** 4s. 9d. **4.** 0·1357. **5.** 0. **6.** 681.
7. $34\frac{2}{9}$ ft. per sec. **8.** 6 per cent. **10.** $9\frac{1}{11}$ per cent. **11.** 50.
12. 4 : 3 ; $-6\frac{2}{3}$ per cent. **13.** 16, 17, 18 cm. **14.** 390 c.c.
15. 14·4 m.p.h. **16.** 2·35 min. **17.** $\div 4$, $\div\sqrt{2}$. **18.** 42·9.
19. 2 ft. 10 in. **20.** £275. **21.** $12\frac{1}{3}$ in. **22.** 2·63 ac.
23. 15 m.p.h. **24.** 66·4 per cent., 48·4 per cent.
25. £636 stock ; £15, 18s. **26.** 5 per cent. **27.** 3·45 mi.
28. 6208 yd. **29.** A £1200, B £930. **30.** 6 men. **31.** 1 in.
32. £2, 12s. **33.** 21. **35.** Gains 10·1 min. per day.
36. $n=0{\cdot}5$, $k=1{\cdot}12$. **37.** 75·6 per cent. **38.** $10^{25}\times 9{\cdot}88$ [4-fig. tables, 9·89.]
39. $£\left(\frac{x^2}{72}+\frac{180}{x}\right)$; 18·6 m.p.h. ; £14, 10s. **40.** 1 : 1·84. **41.** 10·5.
42. 1·92. **43.** 376. **44.** 129,000 cu. ft.
45. Yes, price in 1881 lies between 45·0 s. and 45·4 s. **46.** 9 yr.
47. 60 per cent. **48.** 0·86. **49.** 27·4, 12·6 gall. **50.** £580.
51. 1757. **52.** 39 tons. **53.** 69,000,000 tons. **54.** ·027 sq. mm.
55. 3150 ft. per sec. **56.** 16·4 in. **57.** 125 days. **58.** 4·5 mm.
59. £1, 0s. 10d. **60.** 500 ; 118 ft. **61.** 20 per cent. **62.** 125 ft.
63. 10·8 gr. **64.** 57 per cent.

REVISION PAPERS (R 51–60) (p. 344)

51. **1.** 628 cu. in. **2.** $£\frac{100x}{97}$. **3.** 202. **4.** 201 tons.
5. £413, 3s. 9d. ; £19, 5s. **6.** $9\frac{1}{3}$ days.
52. **1.** 66·60 m. **2.** £1300. **3.** 3·00 in. **4.** £24, 1s. 8d. **5.** 37 ft. 9 in.
6. 1 ft. 5 in.

R 53. 1. $3\frac{19}{110}$; $2\frac{2}{5}$. 2. 22 m.p.h. 3. 10. 4. 0·170. 5. £34, 11s. 9d. 6. £824, 18s. 9d.

R 54. 1. £4, 4s. 2d. 2. 1·17 ft. 3. 4·5 per cent. 4. 25 : 1. 5. £22, 11s. 4d. 6. 4·94 per cent.

R 55. 1. 15, 22, 27. 2. 4·5 min. 3. 42. 4. 5d. [4·8]. 5. £2, 4s. 0d. 6. $13\frac{1}{3}$ per cent.

R 56. 1. 0·240. 2. 48·6 sq. in. 3. 2, 17. 4. £1040. 5. £294, 13s. 6. 794 c.c.

R 57. 1. £5·101. 2. 11·8 yd. 3. 65 per cent. 4. 420. 5. £424, 1s. 5d. 6. £28,571, 10s.

R 58. 1. $\frac{425}{891}$. 2. $27\frac{3}{7}$ in. ; 3·50 cu. ft. 3. 1·64 in. 4. 1·8 ohms. 5. 15s. 6. 1 in 8.

R 59. 1. $1\frac{91}{311}$. 2. £30, 6s. 8d. 3. £9, 15s. 4. 0·828. 5. $5\frac{1}{2}$ per cent. 6. ·0031 in.

R 60. 1. 221. 2. £40, 12s. 8d. 3. 0·664. 4. £176, 13s. 5. 23 lb. 6. 37,100.

PROBLEM PAPERS (P 51–60) (p. 348)

P 51. 1. 0·17609, 2·07918, 0·69897, $\bar{1}$·88907. 2. 30 per cent. 3. 3 : 2. 4. 240 ft. 5. £413, 1s. 0d.

P 52. 1. 754,144. 2. 9·5 per cent. 3. 983 yd. ; 3·3 m.p.h. 4. 11s. 10d. 5. £111, 12s.

P 53. 1. 39·8. 2. £1, 16s. 9d. 3. $87\frac{1}{2}$ per cent. 4. 3000 ; 381. 5. £333, 6s. 8d.

P 54. 1. 52 per cent. 2. 250. 3. $12\frac{1}{2}$ per cent. wetter. 4. £918, 0s. 5d. 5. 875 yd.

P 55. 1. 9240. 2. £272, 15s. 3. $1\frac{1}{8}$ pt. 4. At B, not at A. 5. $69\frac{5}{8}$ per cent.

P 56. 1. 10·8 min. 2. 21·5 : 1. 3. 44 ft. 4. 533 ft. 5. $33\frac{1}{3}$ days ; 40 days.

P 57. 1. 20 per cent. 2. 229 (sum of others is divisible by 9). 3. 9300 lb. 4. 36 cu. in. 5. The same.

P 58. 1. 60, 40 per cent. 2. 2 min. late. 3. 3·6 in. 4. 31·3 cu. ft. 5. 70 mi.

P 59. 1. $\frac{c^3}{b^2}$. 2. 5 ft. 3. $\frac{1}{3}$. 4. $10\frac{5}{7}$ per cent. 5. £268, 13s.

P 60. 1. 280, 315, 336. 2. 79, 21 per cent. 3. $8\frac{1}{2}$ days. 4. 9174, 8184, 7194. 5. 24, 25, 26, 27, 28.

Exercises in Rapid Computation (A–J) (p. 354)

A. 1. 641,023. 2. 17,491,653. 3. 3776 ; rem. 30. 4. £71, 7s. 6d. 5. 1958. 6. £129, 5s. 7. 623,425. 8. 11·8844. 9. 3·55. 10. £29, 18s. 2d. 11. 3, 5, 11, 17. 12. 855.

B. 1. 33,665,777. 2. 25 mi. 1663 yd. 4 in. 3. 122·1. 4. 5252 ; rem. 99.
5. £2249, 7s. 3d. 6. 52. 7. 11 ft. $2\frac{3}{4}$ in. 8. £642, 10s.
9. £48, 12s. 10. 15s. 10d. 11. 79,456. 12. 0·100.

C. 1. 2492. 2. 509 ; 40. 3. $1\frac{41}{78}$. 4. $\frac{1}{6}$. 5. ·0000501.
6. £4, 17s. $10\frac{1}{2}$d. 7. 5s. $2\frac{1}{2}$d. 8. 2187. 9. $3\frac{3}{4}$ per cent
10. 5 cwt. 65 lb. 11. £6, 3s. $10\frac{1}{4}$d. 12. £4, 18s.

D. 1. 3. 2. 17,984,625. 3. £916, 13s. 4d. 4. 102 ; rem. 3.
5. $158\frac{49}{144}$. 6. 0·886. 7. £41, 18s. $4\frac{1}{2}$d. 8. ·00141.
9. £2, 11s. 4d. 10. 374 yd. 11. $\frac{15}{28}$, $\frac{19}{35}$, $\frac{13}{21}$, $\frac{5}{7}$, $\frac{11}{14}$. 12. 405 yd.

E. 1. 139,734. 2. 203,256,563. 3. £216, 6s. $4\frac{1}{2}$d.
4. ·000104. 5. £95, 14s. 6. $\frac{25}{32}$. 7. £3, 13s.
8. £481, 4s. 9. 2197 cu. in. 10. 0·167.
11. 7·5 per cent. 12. £253, 16s. 2d.

F. 1. 6,015,336. 2. 17,100. 3. £1048, 8s. 4d.
4. $97\frac{37}{48}$. 5. 0·546. 6. £51, 11s. $11\frac{1}{2}$d.
7. 13s. 11d. 8. £1, 18s. 8d. 9. £27, 11s. 4d.
10. 16, 632. 11. 462·2 ac. 12. 5·97 cm.

G. 1. $13\frac{47}{72}$; $2\frac{37}{42}$. 2. ·0045. 3. £11, 11s. 4d.
4. 9 per cent. 5. £113. 6. $1072\frac{1}{2}$ sq. ft.
7. 11·6 in. 8. 8·2 min. 9. £367, 15s. 8d.
10. 4·73 per cent. 11. 0·362. 12. $12\frac{1}{2}$ per cent.

H. 1. 100. 2. $\frac{31}{32}$. 3. 175. 4. £794, 10s. 2d.
5. £29, 1s. 11d. 6. 3·35 cu. ft. 7. 6s. 9·4d. 8. 250.
9. 2530 sq. ft. 10. £37, 4s. 8d. 11. 62·5 sq. in. 12. £6, 5s.

I. 1. 475,074,971. 2. £26, 17s. 3d. 3. 5.
4. £4, 11s. ; £4, 17s. 6d. 6. 2178 tons. 7. 83·6.
8. 68·28 yd. 9. 28·4 in. 10. 1s. $11\frac{1}{4}$d. 11. 18·7 : 1.
12. £7, 15s. 4d.

J. 1. 464,192. 2. £20,657, 18s. 10d. 3. 2 cwt. 2 qr. $21\frac{1}{2}$ lb.
4. £1414, 13s. 9d. 5. £67, 1s. 7d. 6. 1530.
7. 4990 ($4981\frac{1}{3}$). 8. 76·5 per cent. 9. $1895\frac{5}{8}$ cu. in.
10. 596·8 cu. in. 11. £203, 12s. 6d. 12. 0·598.

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R.B. 's

LOG SINES.

H.

ADD.

	0	1	2	3	4	5	6	7	8	9		12′ ·2	18′ ·3	24′ 30′
0	− ∞	2̄·242	2̄·543	2̄·719	2̄·844	2̄·940	1̄·019	1̄·086	1̄·144	1̄·[illegible]		– 11	– 17	 2
1	1̄·240	281	318	352	384	413	440	466	490	513	3 2	7 5	10 7	1
2	1̄·534	554	574	592	609	626	642	657	672	686	2	3	5	
3	1̄·699	712	724	736	748	759	769	779	789	799	1	2	3	
4	1̄·808	817	826	834	842	849	857	864	871	878	1	2	2	
5	1̄·884	891	897	902	908	913	919	924	928	933	1	1	2	
6	1̄·938	942	946	950	954	957	961	964	967	970	0	1	1	
7	1̄·973	976	978	981	983	985	987	989	990	992	0	0	1	
8	1̄·993	1̄·995	1̄·996	1̄·997	1̄·998	1̄·998	1̄·999	1̄·999	·000	·000	0	0	0	

LOG COSINES.

S

	0	1	2	3	4	5	6	7	8	9	6′ ·1	12′ ·2	18 ·3	
0	·000	·000	·000	1̄·999	1̄·999	1̄·998	1̄·998	1̄·997	1̄·996	1̄·995	0	0	0	
1	1̄·993	992	990	989	987	985	983	981	978	976	0	0	1	
2	1̄·973	970	967	964	961	957	954	950	946	942	0	1	1	
3	1̄·938	933	928	924	919	913	908	902	897	891	1	1	2	
4	1̄·884	878	871	864	857	849	842	834	826	817	1	1	2	
5	1̄·808	799	789	779	769	759	748	736	724	712	1	2	3	
6	1̄·699	686	672	657	642	626	609	592	574	554	2	3	5	
7	1̄·534	513	490	466	440	413	384	352	318	281	2 3	5 7	7 10	1 1
8	1̄·240	1̄·194	1̄·144	1̄·086	1̄·019	2̄·940	2̄·844	2̄·719	2̄·543	2̄·242	6 –	11 –	17 –	2 –

LOG TANGENTS.

A

	0	1	2	3	4	5	6	7	8	9	6′ ·1	12′ ·2	18′ ·3	24 ·4
0	− ∞	2̄·242	2̄·543	2̄·719	2̄·845	2̄·942	1̄·022	1̄·089	1̄·148	1̄·200	– 6	– 12	– 17	– 2
1	1̄·246	289	327	363	397	428	457	485	512	537	4 3	7 5	11 8	1 1
2	1̄·561	584	606	628	649	669	688	707	726	744	2	4	6	
3	1̄·761	779	796	813	829	845	861	877	893	908	2	3	5	
4	1̄·924	1̄·939	1̄·954	1̄·970	1̄·985	0·000	0·015	0·030	0·046	0·061	2	3	5	
5	·076	092	107	123	139	155	171	187	204	221	2	3	5	
6	·239	256	274	293	312	331	351	372	394	416	2	4	6	
7	·439	463	488	515	543	572	603	637	673	711	3 4	5 7	8 11	
8	·754	800	852	911	978	1·058	1·155	1·281	1·457	1·758	6 –	12 –	17 –	

D. & F. III. BACK.